NCTM Annual Meeting
Philadelphia, Pennsylvania
April 28, 2012

Build Bridges
from
Concrete City
to the
Land of Abstraction

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The Common Core State Standards demand deeper understanding of underlying concepts at all grade levels. Moving from manipulatives to abstraction is a goal, but one not easily achieved without careful reflection about their connections and the appropriate visual models to make those connections possible.

Our goal for **all** students is mathematical thinking, not just ability to calculate.

Mathematical thinking changes the focus

from the **numbers to** the **ideas!**

Mathematics

is

about

ideas!

SMART

by
Shel Silverstein

"Smart" is a story about a boy who got the numbers...but missed the ideas!

My dad gave me a one-dollar bill
'Cause I'm his smartest son,
And I swapped it for two shiny quarters
'Cause two is more than one!

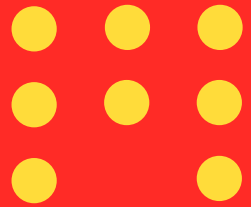
And then I took the quarters
And traded them to Lou
For three dimes –
I guess he don't know
That three is more than two!

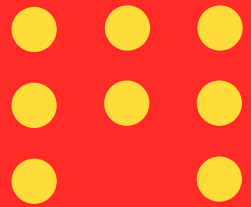
Just then,
along came old blind Bates
And just 'cause he can't see
He gave me four nickels
for my three dimes,
And four is more than three!

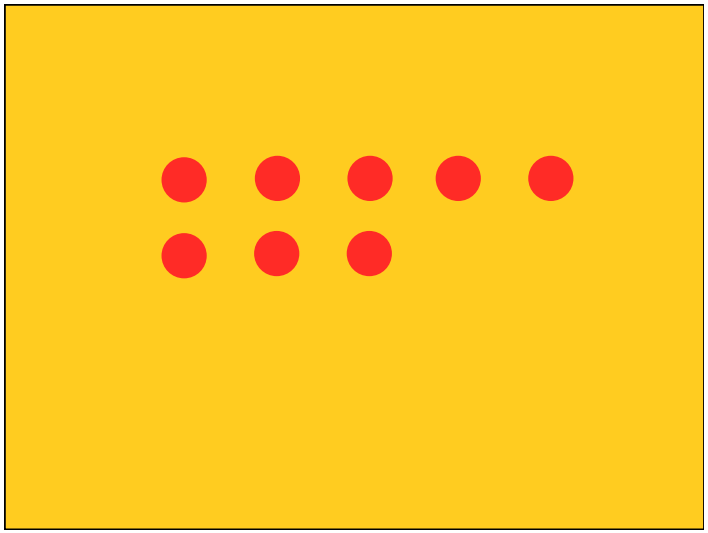
And I took the nickels to
Hiram Coombs
Down at the seed-feed store,
And the fool gave me five
pennies for them,
And five is more than four!

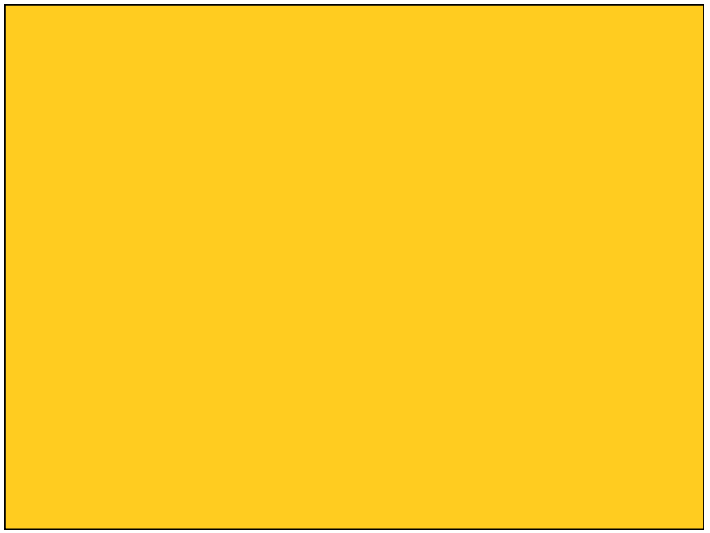
And then I went
and showed my dad,
And he got red in the cheeks
And closed his eyes and
shook his head--
Too proud of me to speak!

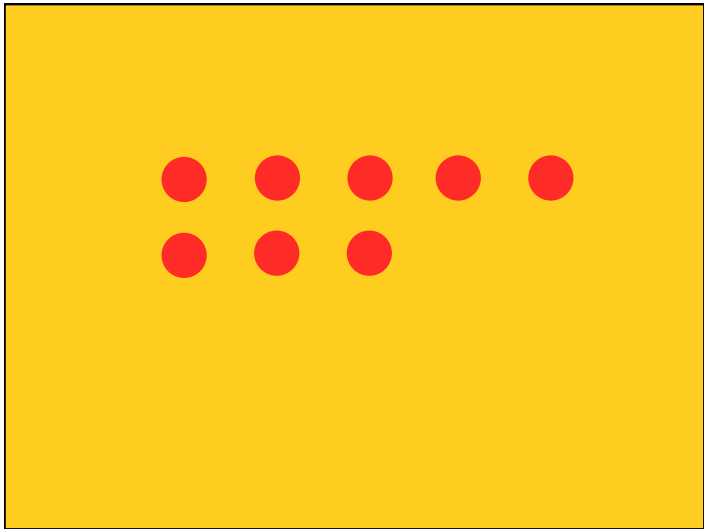
FLASH!

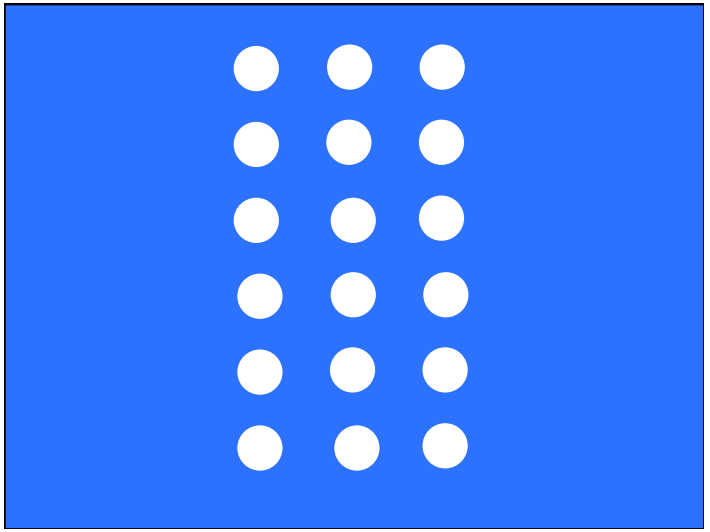


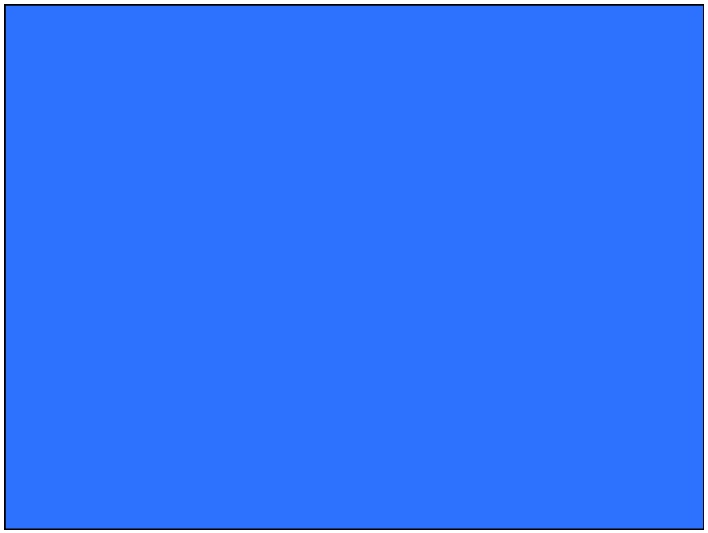


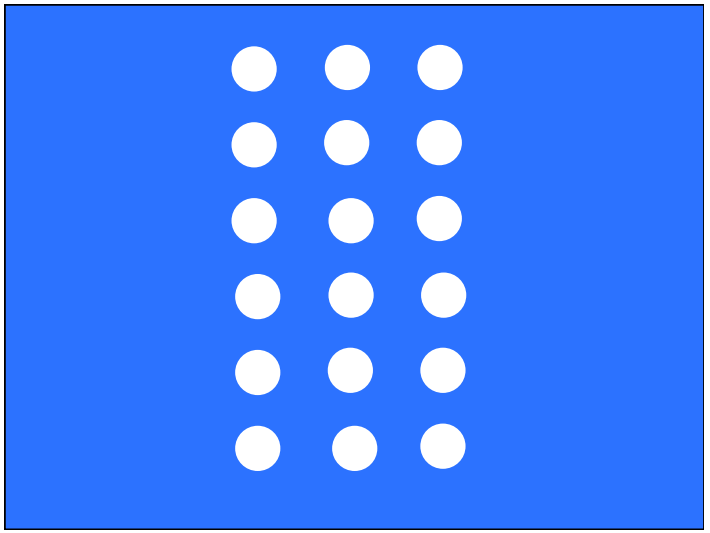


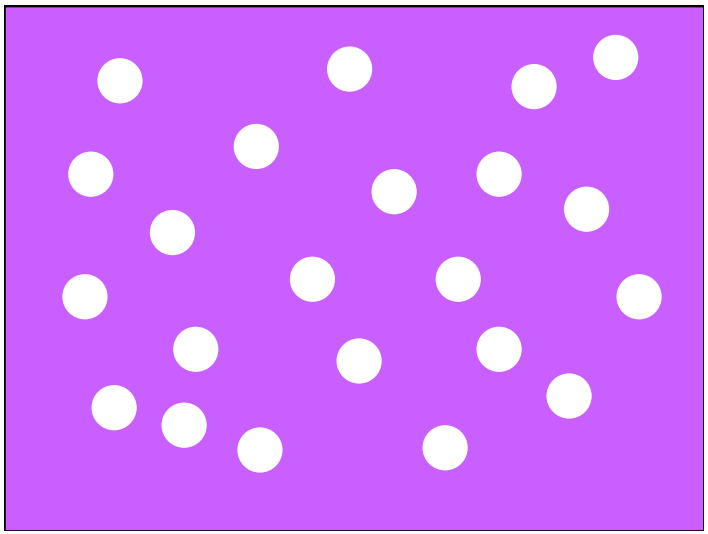


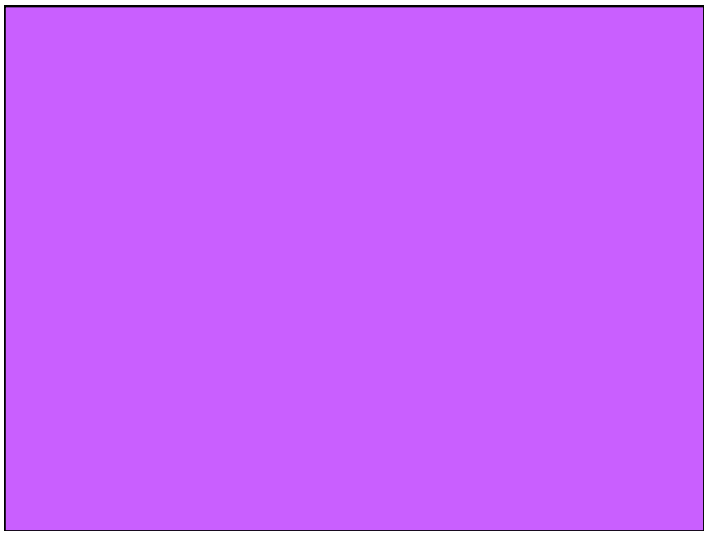


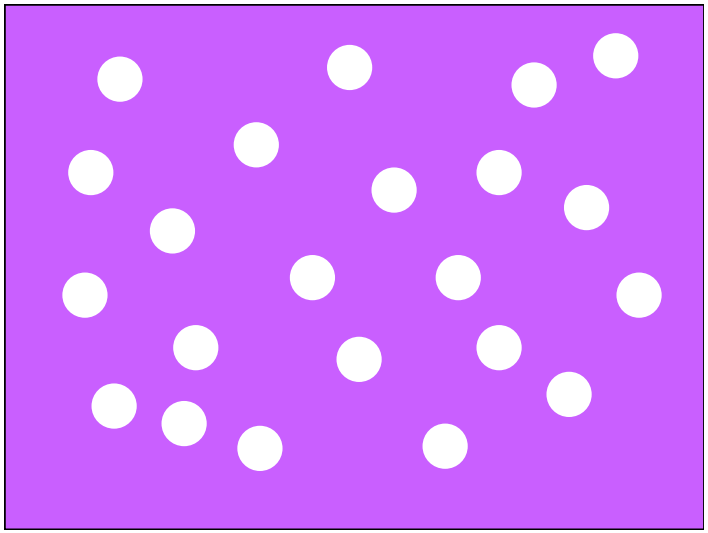










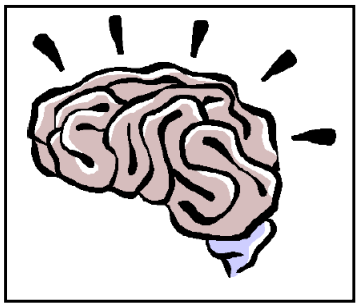


The brain is a sense-making organ.

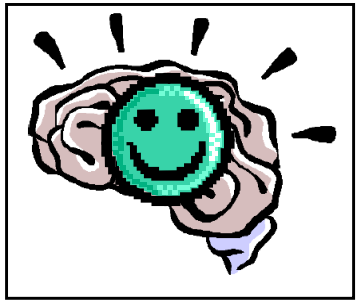
Math is a sense-making activity.

So math is brain food!

THIS IS YOUR BRAIN.



THIS IS YOUR BRAIN
ON MATH!



The Bridge

concrete  abstract

C.....**R.**.....A

IT'S NOT ONLY TRAVELED ONCE

AND

IT'S NOT A ONE-WAY STREET

Some bridges are more reliable than others.

Some are harder to use!

Where you are going?

WHAT IS ABSTRACTION?

Attention to ONLY that which is essential for your purpose.

Example:

WHAT IS ABSTRACTION?

Attention to ONLY that which is essential for your current purpose.

(Ignoring that which is not.)

The Bridge

concrete  abstract

C.....**R.**.....A

IT'S NOT ONLY TRAVELED ONCE

AND

IT'S NOT A ONE-WAY STREET

Bridges?

Pictures and stories!

Number Ideas PreK-6

Whole numbers in Base Ten

- Counting, then counting objects
- Ordering
- Expressing equivalence
- Adding/Subtracting (joining/separating, comparing)
- Multiplying/Dividing

**Then repeat all of the above for common and decimal fractions.

Time makes us have to choose, not just manipulatives but also bridging materials.

five and ten frames, number lines, base ten blocks, balance scales, hundreds charts, multi-hundreds charts, arrays, area models, pictures and stories

Story and Picture
for
Subtraction
and
Division
Strategies and Algorithms

There's a Penny in My Shoe

"I was on my way to the story to buy _____ that cost $2.79. I had $5 in my pocket. I remembered my mom's telling me to make sure I had the right change. My subtraction skills are not so good. So I took a penny out of my pocket and put it in my shoe. It was uncomfortable, but now I had an easier subtraction problem because I no longer had $5.00 in my pocket."

What did I have then?

"I had $4.99. I could easily do my subtraction...$4.99

- 2.79

My change: $2.20? On the way home my food was bothering me! Oh, a penny was in my shoe. Putting it with the $2.20 I got $2.21, and that was my change."

Reaction:

- Could you use this?
- If so with whom?
- Is it a story that is connected to a strategy or an algorithm or both?
- What strategy or algorithm is connected to?
- Is it valuable or dangerous?

Problem: 73 - 27

- Use your base ten blocks to solve the problem and sketch your solution.
- Do all sketches look the same? Do they need to?

- Now try 306 – 178
- Is your method the same?
- Is your sketch different from anyone at your table?

Suggestion:

When moving into problems that could involve regrouping, consider writing the whole ten above the digit in the ones place rather than just the 1 representing the ten. Then in the subtraction, have students subtract the ones place subtrahend from the ten in the minuend and then add it the other ones in the minuend to arrive at the difference in the ones column.

Discuss a benefits of writing the whole 10.

(The benefits do assume that our students know their combinations to ten before they every start subtraction with regrouping.)

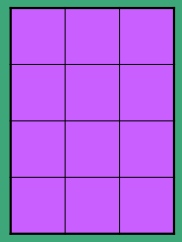
Much work has to be done before teaching or helping kids understand an algorithm. Teaching the algorithm before understanding is counterproductive

Third Grade

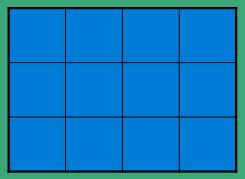
"Three groups of four is twelve, so then, four groups of three will be twelve,

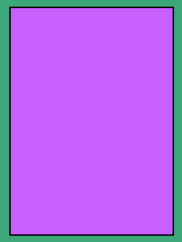
of course."

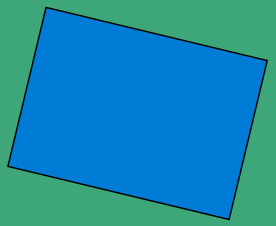
4 by 3 array

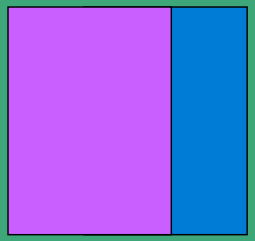


3 by 4 array

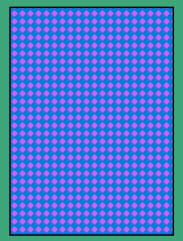








3 X 4 REALLY DOES EQUAL 4 X 3!



CONGRUENCE PROVES COMMUTATIVITY!

OF COURSE!

3 X 14

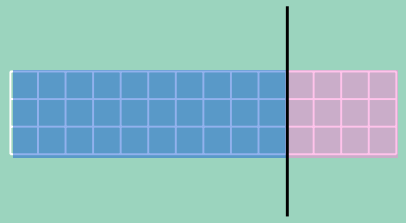
Which the distributive property says is the same as

(3 X 10) + (3 X 4)…

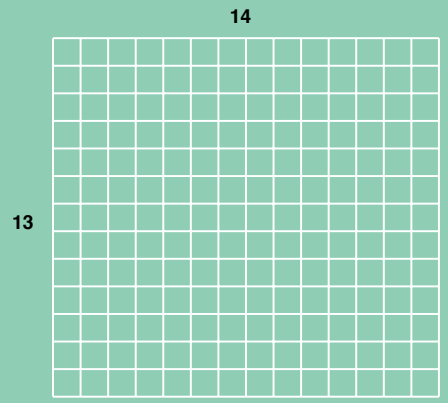
of course???

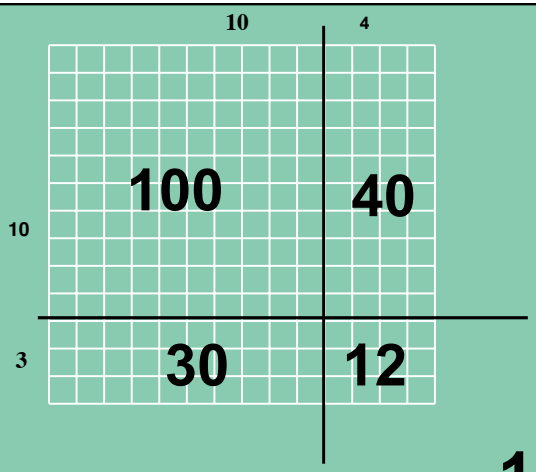
So 3 X 14 really does equal (3X10) + (3 X 4)!

Use a model to represent 3 X 14.



The distributive property rules!





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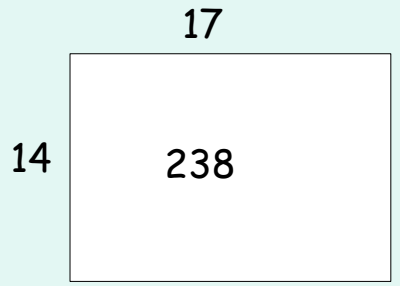
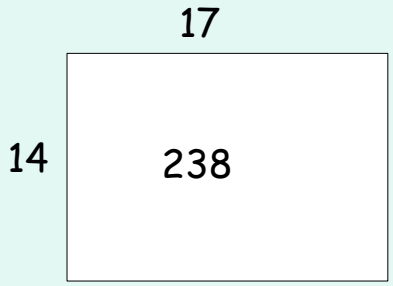
Arrays and area models to teach multiplication have become more common.

The area model to teach division?

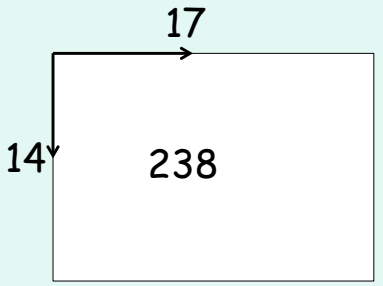
Not so much!



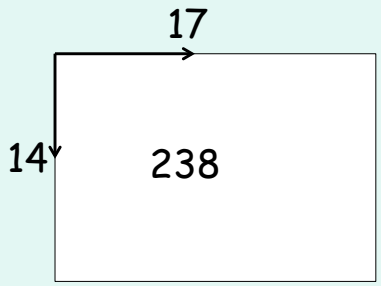
This is the picture of the previously found product by the use of area model and the distributive property.



In multiplication we are given the dimension of the rectangle and our job is to find the area.



Our symbol for division can be looked at as the top left corner of a rectangle.



For division they tell us the area and how long our rectangle is. Our job is to find the width of the rectangle.

We can find the width of the rectangle by using tiles, one-by-one to build the rectangle. We can use base ten blocks, paper representation of base ten blocks, or even graph paper to build the rectangle's width.

We also can see how many squares we have left over out of how many we would need to complete another column of width—therefore a fractional representation of the remainder.

Story as representation for division:

While walking in the park 3 friends found an envelope with 4 hundred dollar bills, 5 ten dollar bills, and 6 one dollar bills.
They shared the proceeds of the fine equally. How much was each person's equal share.

This can be done physically first (a few times). Then physically with notations of how the division was done, and then more experiences of the same with different types of notation.
One of the types of notation is an actual division algorithm.

Mathematics

is

about

ideas!

We can help students get those ideas by using story and picture BEFORE we do algorithms. Then use the stories ond pictures to build the algorithms.

If we don't teach for transfer, most students will not see the connection.

We really didn't have enough time to finish. I am happy to entertain questions via e mail. Make sure you put NCTM Philadelphia in the subject line so I don't toss your e mail before answering it!! I can only upload the handout, not the powerpoint. If you want the powerpoint please e mail me.

Thank you for your kind attention and for all the nice things so many of you said at its conclusion.

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Or
sumtchrinc@gmail.com

(The program book says I am from PA. But I really am from NY.) I do travel to schools all over the US to work with teachers on a per diem basis. But my schedule fills fast, and is almost full for next