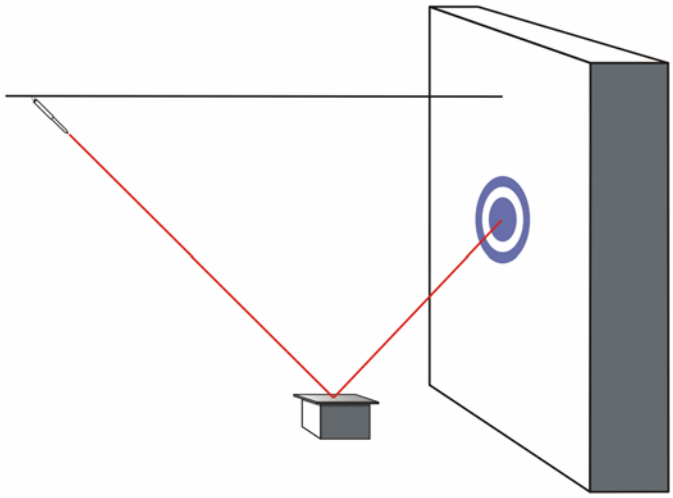
Light It Up!

NAME _____

Light It Up is the newest attraction at the Springfield Fair. In this game, a laser pointer slides along a string at a height of 1.5 m from the ground. A 10-cm platform is positioned 25 cm from a wall, and a mirror is placed on top of the platform, parallel with the ground. The object is to slide the laser pointer so that when the beam is reflected off the mirror, it will hit a target that is 1 m off the ground, as shown below.



A player is given just one attempt to hit the target. If she does, she then tries to hit a target that is 3 m off the ground. And if she hits that one, too, she tries for a target that is 10 m off the ground. If she hits all three, she wins the Grand Prize: a DVD player and a DVD copy of *Itchy and Scratchy's Greatest Hits*.

Bart and Lisa have played the game several times, yet they haven't been able to hit the target. Lisa is sure that there must be a mathematical equation that would allow them to determine the correct position for the laser pointer.

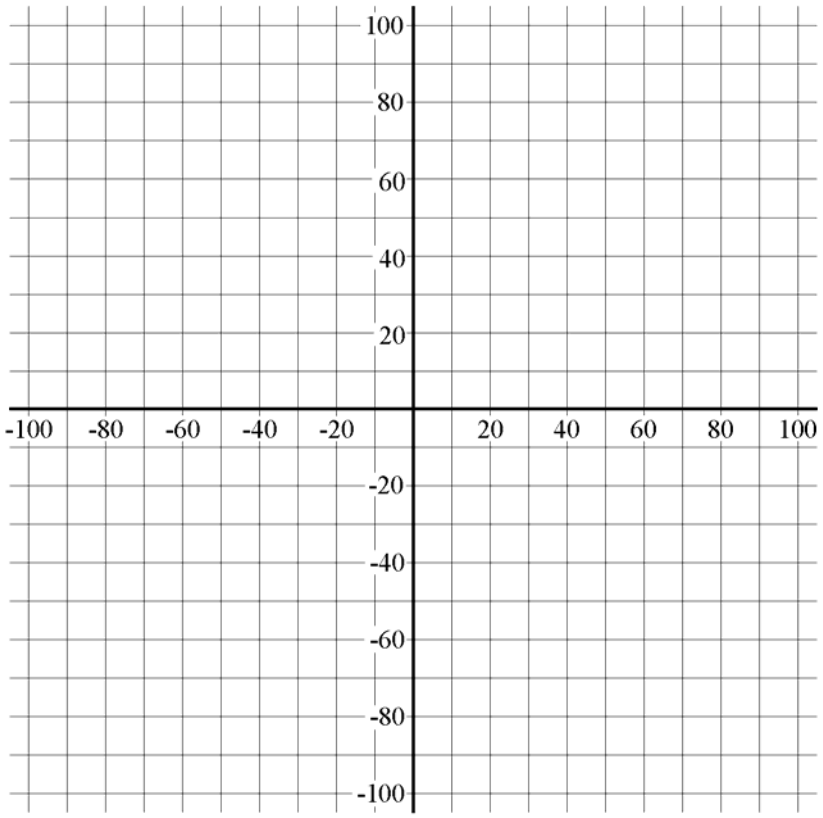
There *is* such an equation, it is a special kind known as a *rational function*. Before you can help Lisa and Bart win this game, you must first understand the behavior of these functions.

Climbing the Wall

1. The *Climbing Wall* is a popular event at the Springfield Fair. The wall is 100 meters high. Bart can climb at a rate of 8 meters per minute, but other participants climb at different speeds.

a. Complete the table to show that the time it takes to reach the top of wall depends on the climber's speed.

Rate (Meters per Minute)	Time (Minute)
0	
2	
4	
5	
10	
13	
20	
25	
40	
50	
75	
100	



b. Discuss the results with your teammates. Talk about how you determined the numbers in the table. Then, find an equation that describes the table.

Equation: _____

c. On the axes above, graph your equation. Then, extend your graph to include negative values in the domain.

d. Discuss the behavior of the function with your teammates. You should discuss the domain, the range, and any limiting values of the function. Summarize your discussion below.

_____

_____

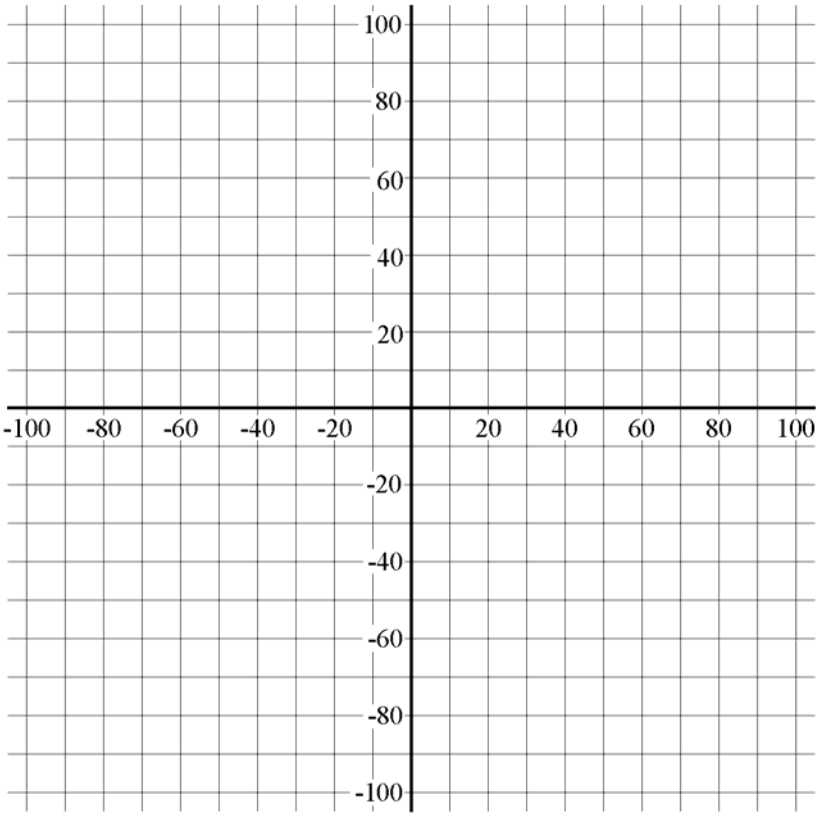
_____

Trip to the Fair

2. Ms. Crabapple, a teacher at Springfield Elementary, is sponsoring a school trip to the fair. The total cost for transportation, parking and entrance fees is $1200. The total cost will be divided among all the students who go on the trip.

a. Complete the table to determine how much each student must pay to cover the expenses.

Number of Students	Cost per Student
0	
10	
20	
30	
40	
50	
60	
80	
100	
120	
150	
200	



b. Discuss the results with your teammates. Talk about how you determined the numbers in the table. Then, find an equation that describes the table.

Equation: _____

c. On the axes above, graph your equation. Then, extend your graph to include negative values in the domain.

d. Discuss the behavior of the function with your teammates. You should discuss the domain, the range, and any limiting values of the function. Summarize your discussion below.

_____

_____

_____

3. With your teammates, discuss the following questions. Be as specific as you can in your explanations.

a. How are the equations in Questions 1(b) and 2(b) the same? How are they different?

b. How are the graphs in Questions 1(c) and 2(c) the same? How are they different?

The functions in Questions 1 and 2 are known as ***rational functions***. Rational functions are quotients of two polynomial functions. In other words, if $f(x)$ is a rational function, then

$$f(x) = \frac{p(x)}{q(x)},$$

where $p(x)$ and $q(x)$ are polynomial functions, and $q(x) \neq 0$.

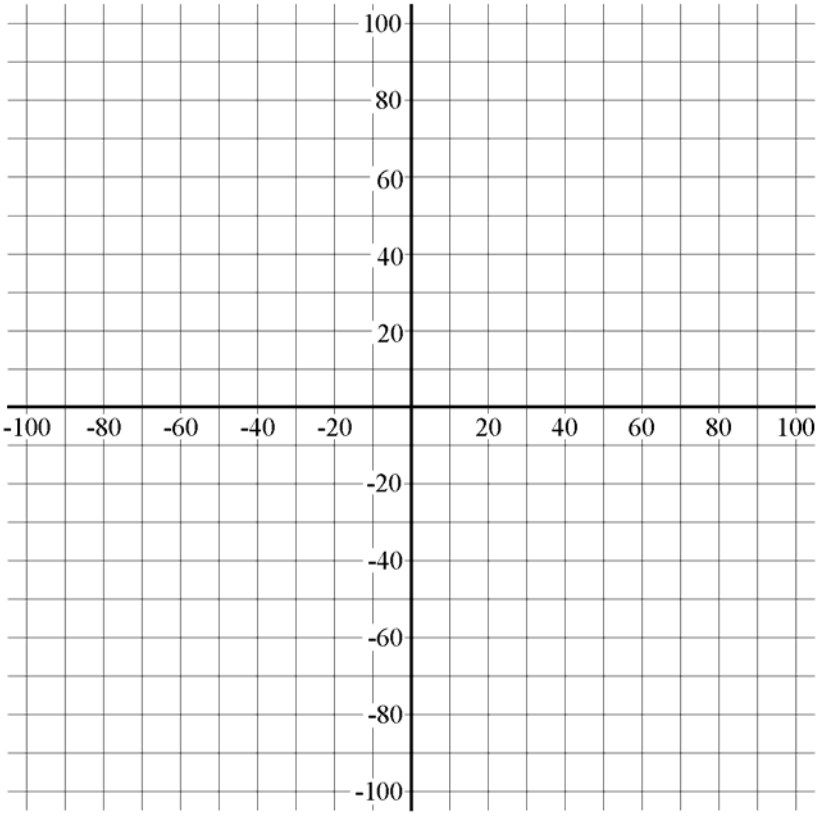
In this activity, we will only look at cases in which $p(x)$ and $q(x)$ are linear functions.

A Closer Look at the Trip to the Math Fair

4. Ms. Crabapple was just informed that the new attraction, *Light It Up*, will cost an additional $5 per student.

a. Discuss with your team members how this new information will change the amount each student will have to pay. Complete the table below to reflect this new information.

Number of Students	Cost per Student
0	
10	
15	
20	
30	
40	
50	
60	
80	
100	
120	
150	
180	
200	



b. Discuss the results with your teammates. Talk about how you determined the numbers in the table. Describe your method in the space provided.

c. Use the information from part (b) to write an equation that describes the table.

Equation:_____

d. Some rational functions can be written in the form

$$y = \frac{a}{x-b} + c\,,$$

where a, b and c are constants, and b, $c \neq 0$. If your equation from part (c) isn't already written this way, rewrite your equation in this form.

Equation: _____

e. Sketch a graph of your equation on the axes on the previous page.

5. Discuss the following questions with your teammates. Be very specific with your explanations.

a. What is the domain of the function you found in Question 4(d)? How does it relate to the problem situation?

_____

_____

b. What is the range of the function in Question 4(d)? How does it relate to the problem situation?

_____

_____

c. Use your equation in Question 4(d) to extend the graph to include negative values. Describe the difference between the graphs in Questions 2 and 4. How is this difference reflected in their equations?

_____

_____

d. Study your graph. How are the values of b and c from your equation in Question 4(d) reflected in your graph?

_____

_____

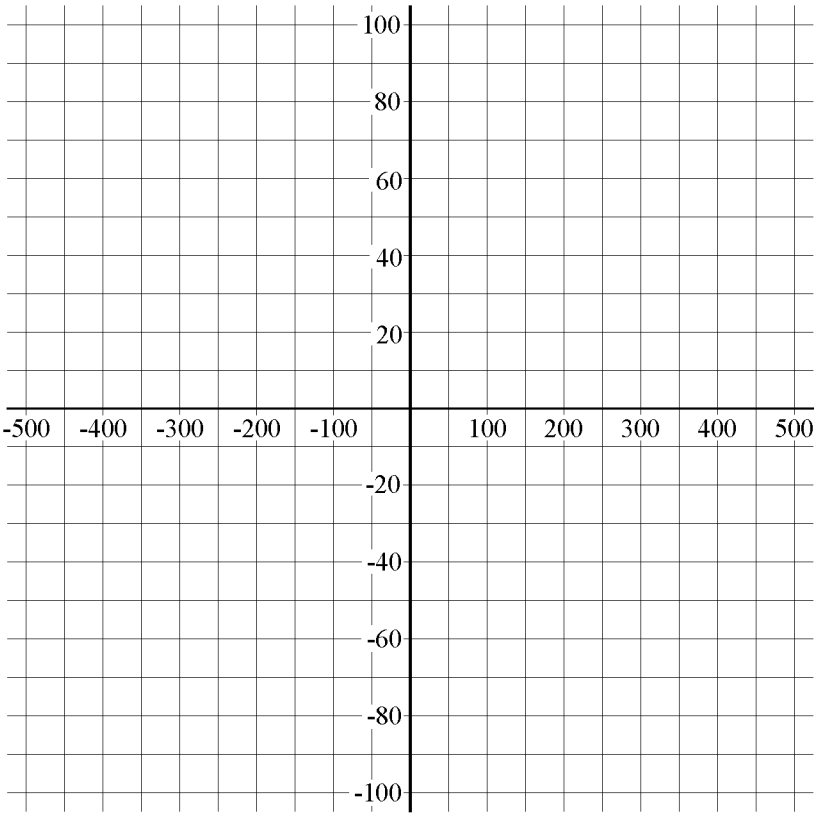
_____

An Even Closer Look at the Trip to the Fair

6. Ms. Crabappple just received some more information. The trip must include chaperones. She decides that ten sixth-graders will serve as chaperones, and the chaperones will not be required to pay for their trip. The expenses of the chaperones will be covered by the other students.

a. With your teammates, discuss how this new information will change the amount each student has to pay. Complete the table below to reflect this new information.

Number of Students	Number of Paying Students	Cost per Paying Student
10	0	
15	5	
20		
50		
60		
90		
130		
160		
210		
310		
410		



b. Discuss the results with your teammates. Talk about how you determined the numbers in the table. Describe your method in the space provided.

_____

_____

_____

c. Use the information in part (b) to find an equation that describes the table.

Equation: _____

d. On the axes above, sketch a graph of your equation.

7. Discuss the following questions with your teammates. Be very specific in your explanations.

a. What is the domain of the function in Question 6(c)? How does it relate to the problem situation?

_____

_____

_____

_____

b. What is the range of the function in Question 6(c)? How does it relate to the problem situation?

_____

_____

_____

_____

c. Use your equation from Question 6(c) to extend the graph to include negative values. Describe the difference between the graphs in Questions 2, 4 and 6. How is this difference reflected in their equations?

_____

_____

_____

_____

d. If your equation from 6(c) is not in the form $y = \frac{a}{x-b} + c$, rewrite it so that it is.

Equation: _____

e. How are the values of *b* and *c* from your equation from Question 7(d) reflected in your graph from Question 6?

_____

_____

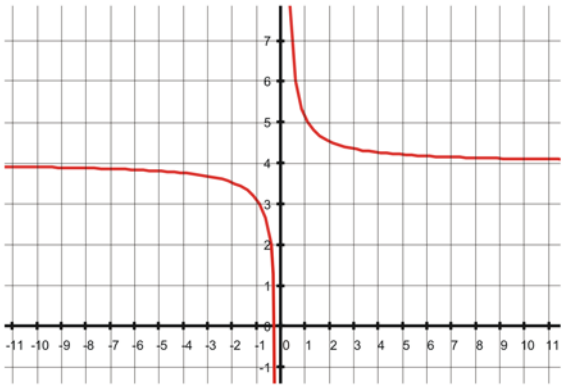
_____

_____

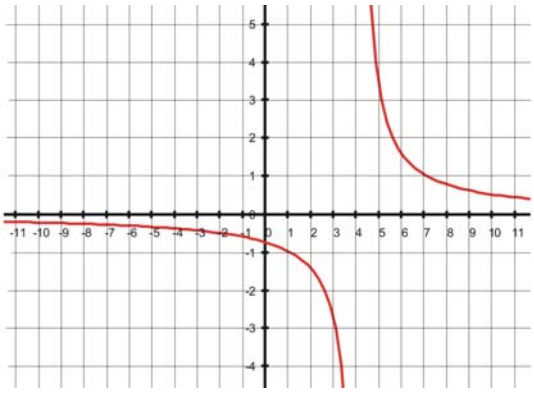
8. Write an equation in the form $y = \frac{a}{x-b} + c$ for the following rational functions.

Check your equations using a graphing calculator.

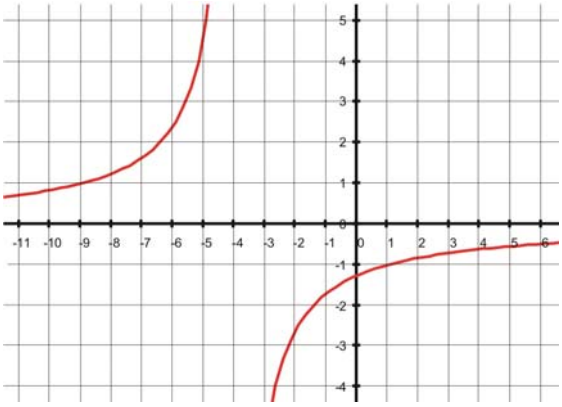
a. _____



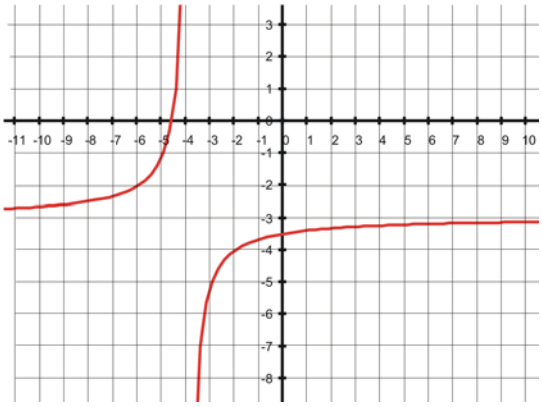
b. _____



c. _____



d. _____



e. Algebraically, how can you determine the value of a in the equation $y = \frac{a}{x-b} + c$ for the graphs in 8(a) through 8(d)?

_____

_____

f. Use your explanation in part (e) to determine the value of a in the equation $y = \frac{a}{x-b} + c$ for the graphs in 8(a) through 8(d).

8(a): a = _____ 8(b): a = _____

8(c): a = _____ 8(d): a = _____

g. What do you notice about the value of a for your answers to 8(c) and 8(d) that is different from your answers for 8(a) and 8(b)?

_____

What affect does this have on your graph? _____

_____

The *Light It Up* Game

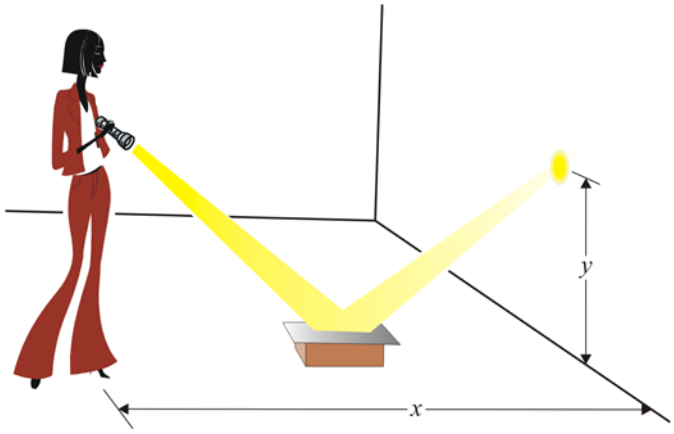
9. For this experiment, you will need:

- Laser pointer (or a small flashlight)
- Small, flat mirror
- Tape measures (2)
- Wooden block, about 10 cm tall (or a thick book)
- Tape
- Graphing calculator (optional)

a. Tape one of the tape measures to the wall, beginning at floor level. Place the other tape measure along the floor to measure the distance from the wall to the flashlight.

Place the block 25 cm from the wall. Place the mirror on top of the block.

Stand on the side of the mirror opposite the wall. Aim the laser pointer toward the center of the mirror so that its image is reflected onto the tape measure attached to the wall. (See figure at right.)



It is very important that you hold the laser pointer (or flashlight) at the same height throughout the experiment. Let x represent the distance from the wall to the laser pointer. Let y represent the distance that the reflection appears up the wall. Measure both x and y in centimeters.

b. What is the distance from the wall to the center of the mirror? _____

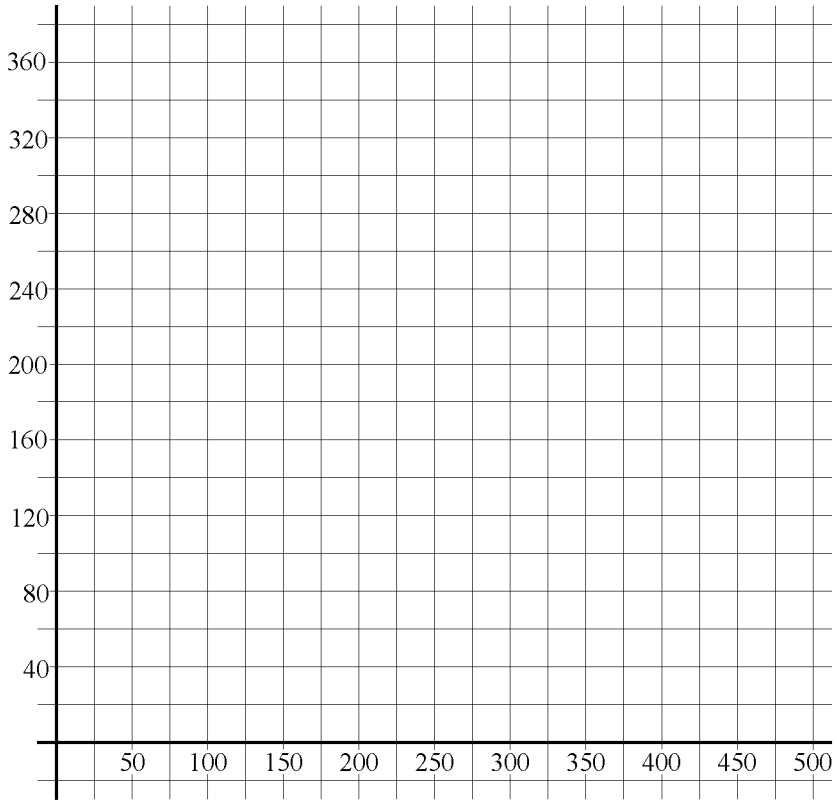
What is the distance from the laser pointer to the floor? _____

What is the distance from the floor to the mirror? _____

c. Collect at least eight data points and enter them in the table below.

DISTANCE FROM THE WALL TO THE LASER POINTER (x)								
DISTANCE FROM THE FLOOR TO THE REFLECTION (y)								

d. Sketch a graph of your data using the grid to the right.



e. Use the information in parts (a) and (b) to determine the values of b and c in the equation $y = \frac{a}{x-b} + c$.

b = _____

c = _____

f. Use your graph from (d) and the information from (e) to determine an equation for your graph.

Equation: _____

g. If the information in parts (a) and (b) is also true for Bart and Lisa's situation, determine the distance that they should stand from the wall if the target is . . .

i. 1 meter from the floor. _____

ii. 3 meters from the floor. _____

iii. 10 meters from the floor. _____

Cooling Down

10. At the fair, Lisa stumbled upon a game called *Cool Down*. In this game, there is a container with 8 cups of hot water (at a temperature of 50ºC). The object is to determine how many cups of cold water (at a temperature of 20ºC) to add to the container to reduce the temperature to 40ºC.

a. Find an equation expressing the temperature (in ºC) of the water in the container as a function of the volume (cups) of cold water added.

Equation: _____

b. Explain how you obtained the equation in part (a) above.

_____

_____

_____

c. Use your equation to determine the amount of cold water that must be added to reduce the temperature to 40ºC.