Standards for Mathematical Practice and Problem Solving: Time for Change

Dr. Linda Dacey

Word Problems - Operations

Grade	Building Expectations
Kindergarten	Adding to, taking from, putting together, taking apart; numbers to 10
Grade 1	Comparing; numbers to 20
Grade 2	One- and two-step; numbers to 100
Grade 3	Equal groups, arrays, and measurement quantities; two-step; numbers to 1,000
Grade 4	Times as many as, interpret remainders; multi-step; multi-digit
Grade 5	extends computations and types of numbers

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

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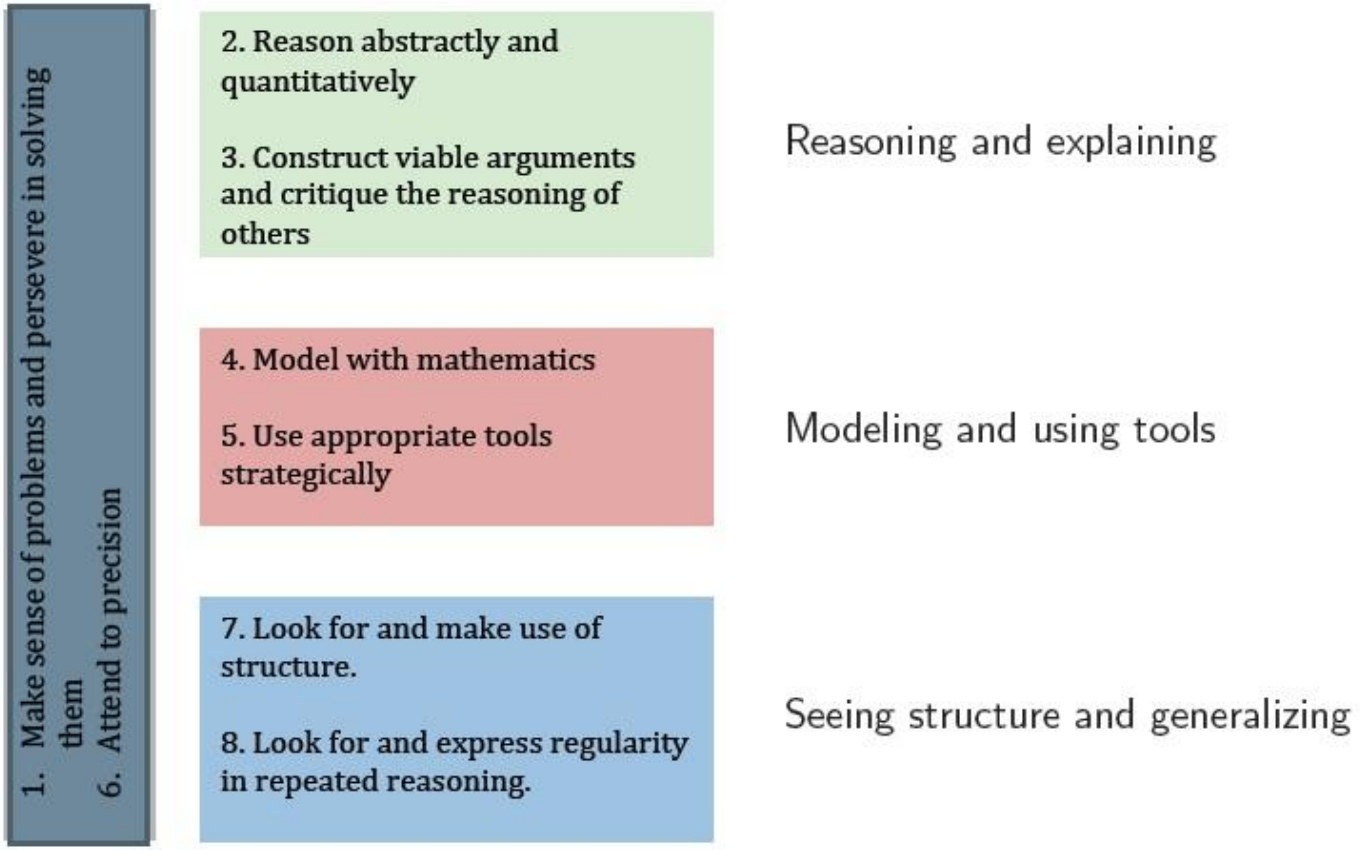
Grouping the SMPs



(McCallum, 2011)

Make Sense, Persevere, and Be Precise

- ..\..\Videos\Chimpanzee Problem Solving.mp4

What To Do

- Offer interesting problems in which significant mathematical ideas are embedded
- Put our pencils away
- Provide differentiated tasks and strategies that provide just the right balance of challenge and support
- Tap students' interest

What Not to Do

- Teach strategies in isolation
- Devote time to problems that are not mathematically relevant
- Limit problem solving to Fridays
- Use as enrichment for the few
- Just focus on answers to problems
- Limit problem solving to developing conceptual understanding of operations

Add Wonder

- How many pies can we make from this year's biggest pumpkin?
- How many candles do you need for your family's birthdays next year?
- What problems can you pose about this setting?

Goldilocks Problems

Just the right level of challenge:

- Increase success for a range of learners
- Develop readiness
- Support classroom community

Level the Tasks by Adjusting the:

- Complexity of the language
- Amount of scaffolding
- Presentation of data
- Setting
- Number of solutions to be found
- Number of conditions to be met
- Size or types of numbers

Start in the Middle

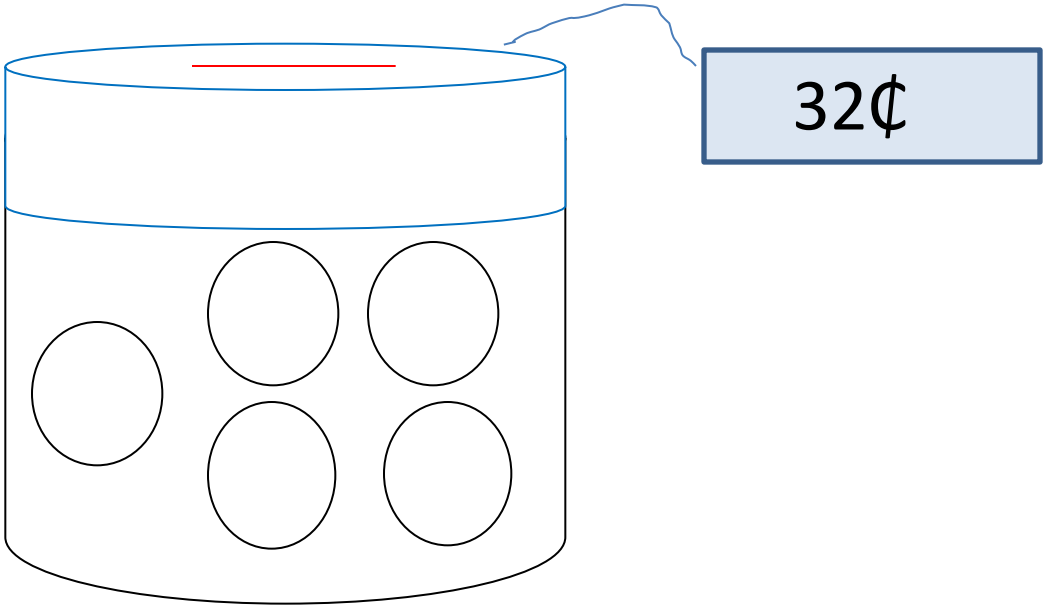
Manny has 7 coins.

He has 40¢.

He has no nickels.

What coins does Manny have?

Simplify



Make More Complex

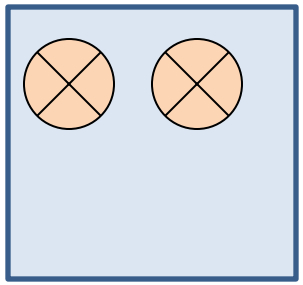
Janelle has 30¢.

She has at least 1 penny, 1 nickel, and 1 dime.

What coins could she have?

Find three possible answers.

Start in the Middle



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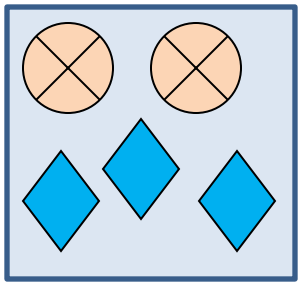

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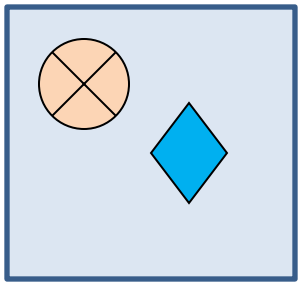

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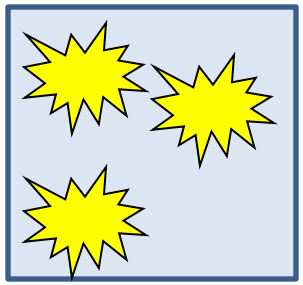
Simplify



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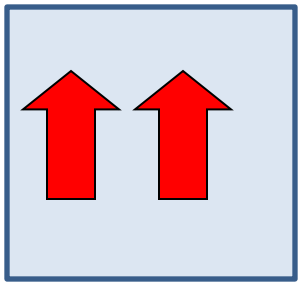

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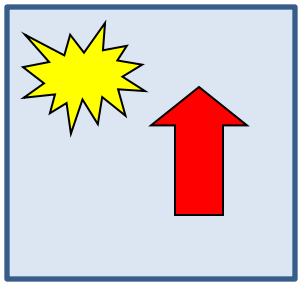

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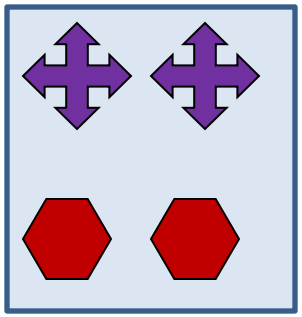
Make More Complex



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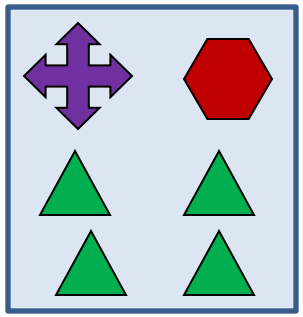

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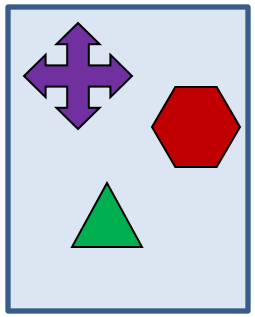

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Reasoning and Explaining (2 & 3)

- Make sense of quantities and relationships
- Decontextualize and contextualize
- Make conjectures
- Consider cases and counter examples
- Use logical reasoning to justify and evaluate conclusions

Citizens Bank Park

_____ tons of soil were excavated so that the field could be _____ feet below street level. The roof is _____ feet above street level. There are _____ sapphire blue seats in the stadium. _____ of these seats are in the Hall of Fame Club. The Park opened on April _____, _____.

2 2,500 134 43,647 23 594,000 2004

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A Format that Builds Confidence

All about Jed

6 2 8 3

Jed has _____ brothers.

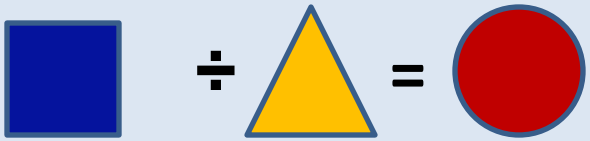
Jed has _____ sisters.

Jed has more brothers than sisters.

There are _____ children in Jed's family.

There are _____ people in Jed's family.

Analysis of Abstract Representations



A ÷ B = C

Write as much as you know about A, B, and C.

- A has to be bigger than B
- B has to be smaller than A
- C could be bigger or smaller than B
- C is smaller than A (sometimes true)
- A = B × C

$56 \div 8 = 7$ or $56 \div 7 = 8$

$72 \div 12 = 6$ or $72 \div 6 = 12$

$108 \div 12 = 9$ or $108 \div 9 = 12$

$21 \div 7 = 3$ $21 \div 3 = 7$

- A or B cannot be zero
- A could be zero

Connect to a Context

Solve 453 ÷ 21.

20 x 21 = 420
1 x 21 = 21
441
+ 12
453

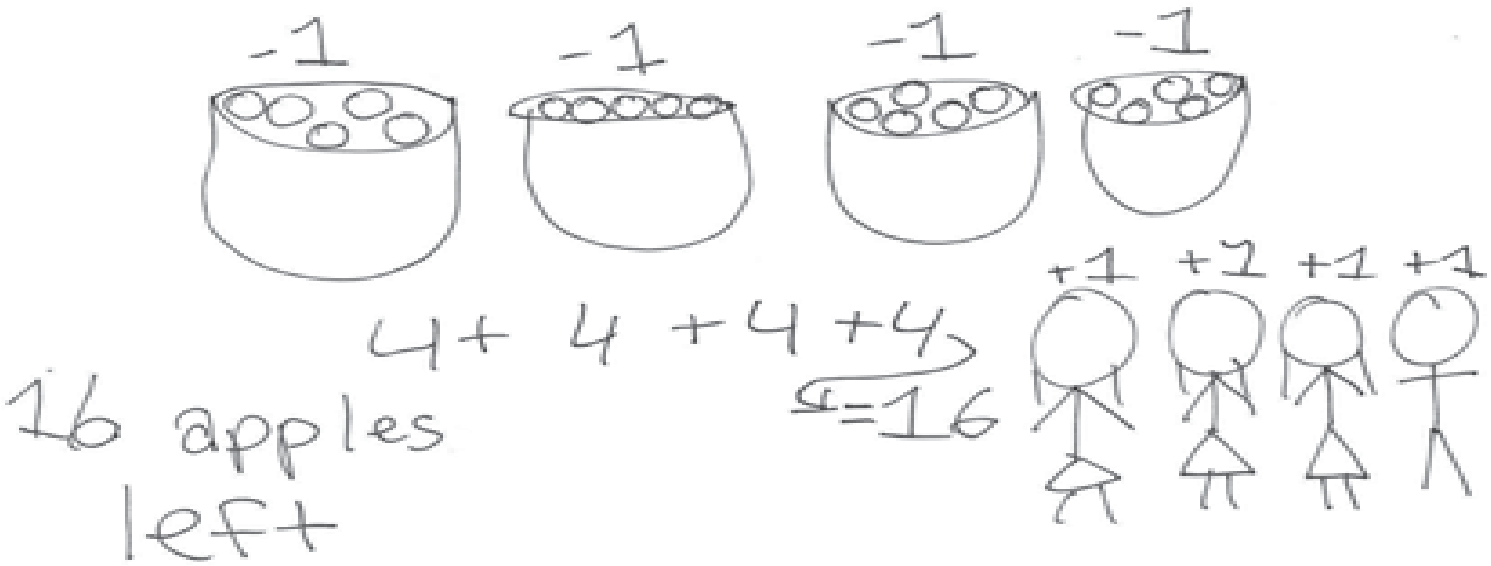
$21\overline{)\,12}$

Write a word problem for 453 ÷ 21.

There are 453 clams in a pail. If I put them evenly in 21 boxes, how many clams go in each box? $21\overline{)\,12}$

Have Students Record Thinking

There are 4 baskets. There are 5 apples in each basket. Jamie takes an apple from each basket to give to her friends. In all, how many apples are left?




December
January
March
May
June
July
August
September
October
November

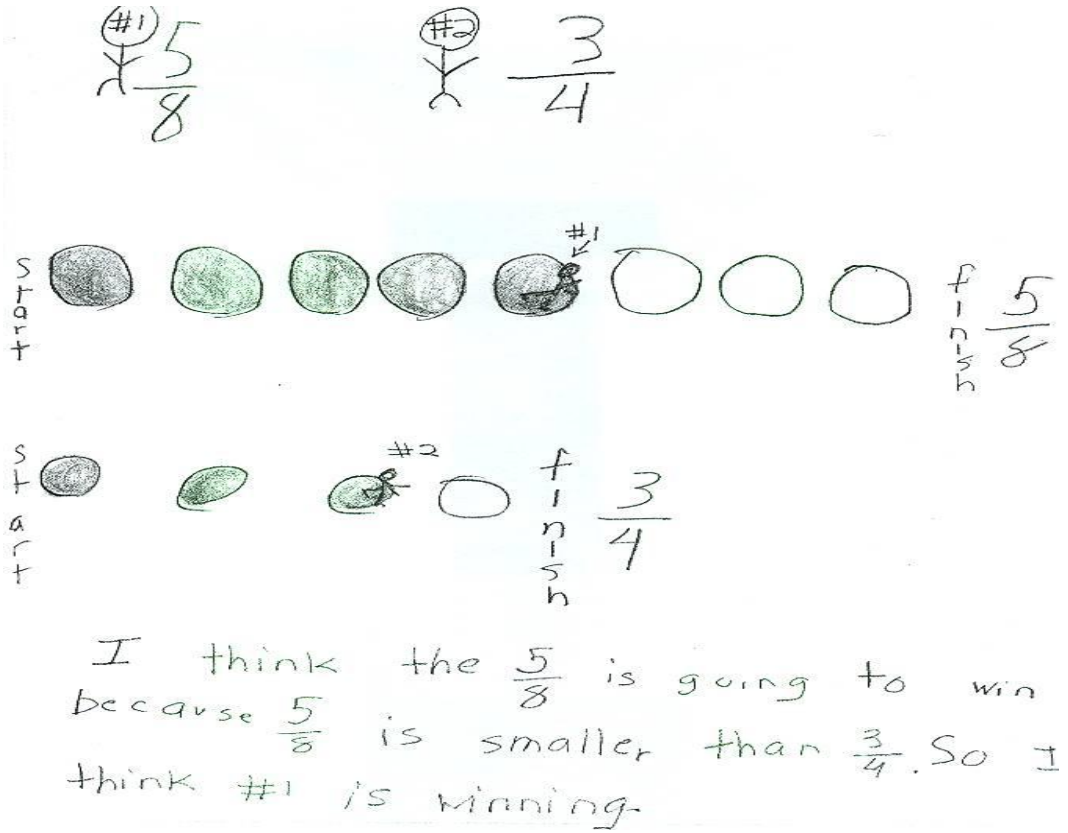
We Need to:

- Expect students to share their thinking
- Ask questions to clarify student thinking
- Teach students the social skills needed to listen with respect, build on the ideas of others, and be open to critique

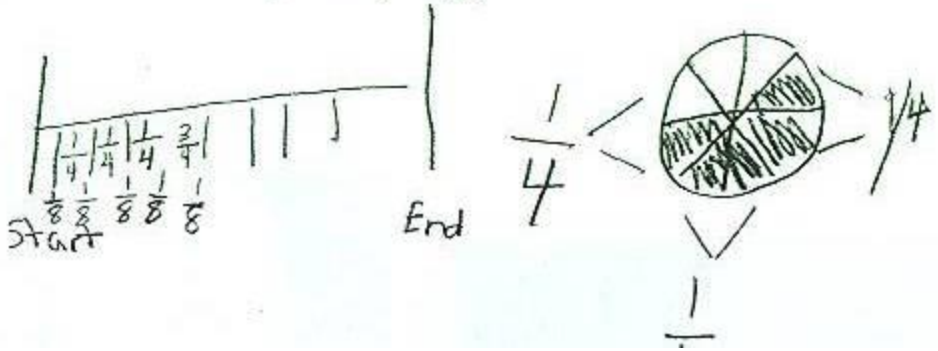
Modeling and Using Tools (4 & 5)

- Apply math to everyday life
- Detect errors
- Use diagrams, tables, charts, and formulas
- Use technology to deepen their understanding of concepts

At Olympics Day, two friends are running in a race. One friend is 5/8 of the way to the finish line and the other friend is 3/4 of the way. Who is winning?



$\frac{2}{4}$ is bigger because $\frac{2}{8} = \frac{1}{4}$ and there are $\frac{5}{8}$ and if $\frac{2}{8} = \frac{1}{4}$ $\frac{5}{8}$ would only $= \frac{2}{4}$ and $\frac{1}{8}$ but we need $\frac{3}{4}$ so one of the friends is loosing by $\frac{1}{8}$.



Answer $\frac{3}{4}$'s

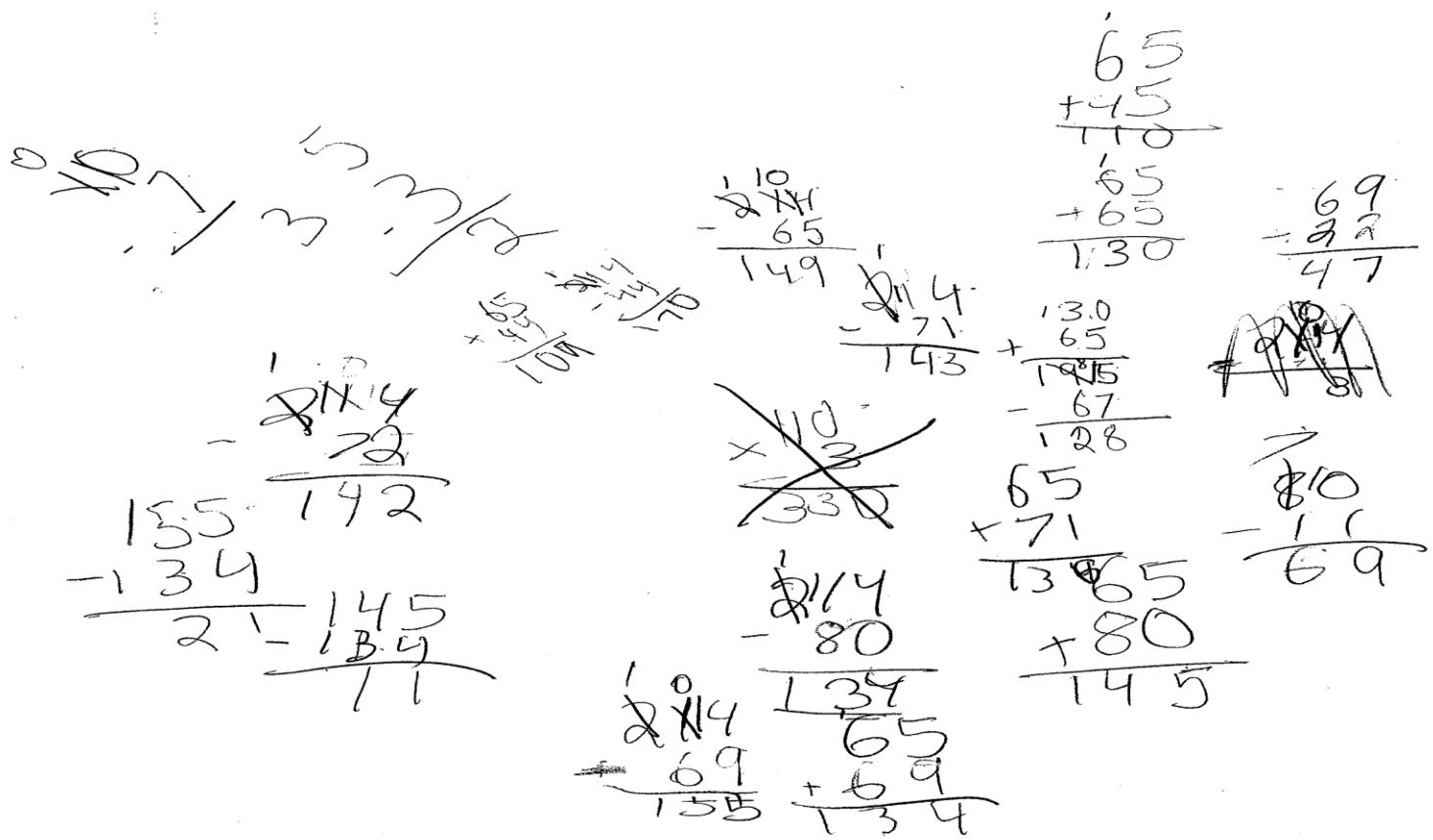
What's the Number?

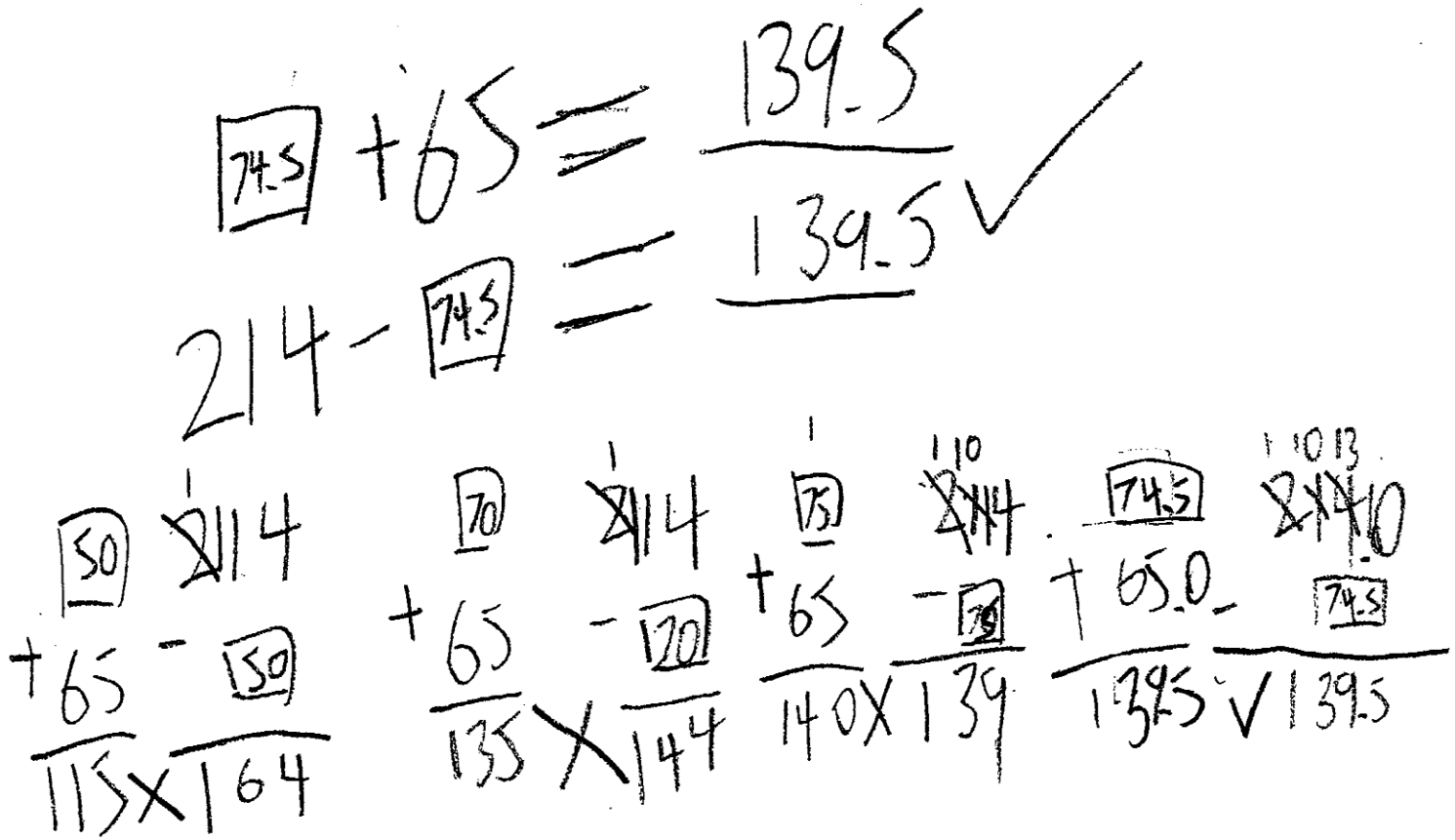
Shane added the number to 65.

Chris subtracted it from 214.

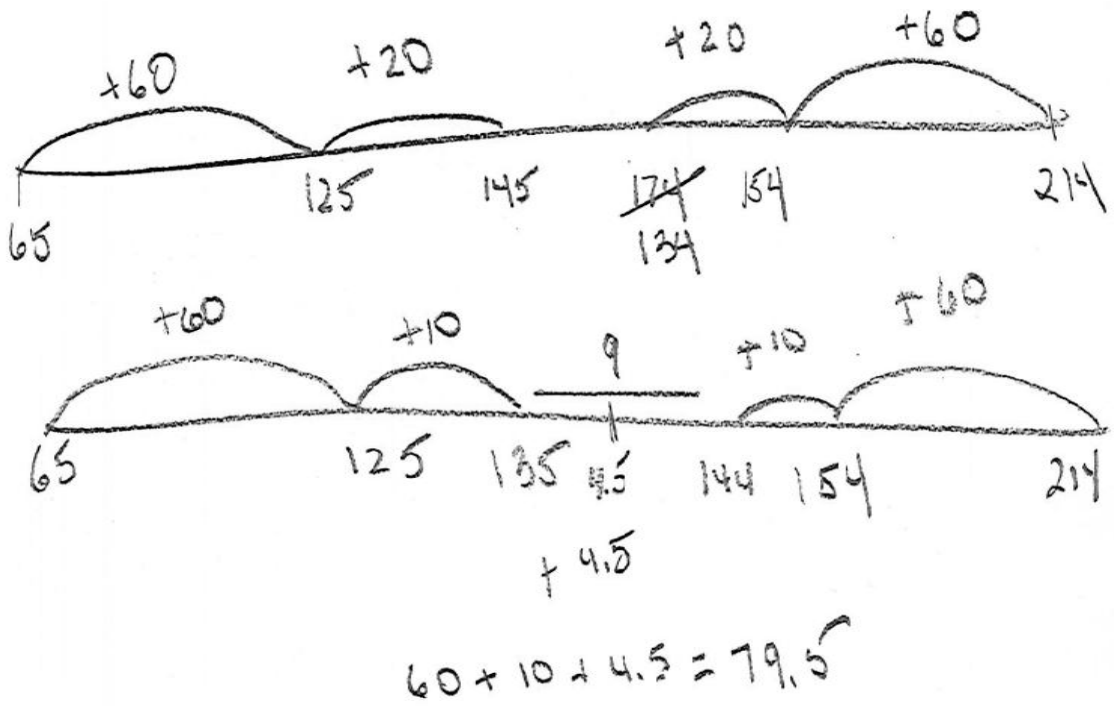
They got the same answer.

What was the number they used?




74.5 + 65 = 139.5
214 - 74.5 = 139.5
50 214
+65 - 50
115 X 164
70 214
+65 - 70
135 X 144
73 214
+65 - 74
140 X 139
74.5
+65.0
139.5 ✓
74.5
214.0
- 74.5
139.5

Number line model



Ask about Relationships in Models

Solange has 623 ones. Jed has 62 tens and 4 ones. Who has more?

We Need to:

- Have students draw their own models
- Expect students to provide a variety of representations
- Provide problems/tasks that tap misconceptions

Structure and Generalizations (7 & 8)

- Discern patterns and structures
- Note repeated calculations
- Maintain oversight of process while attending to details

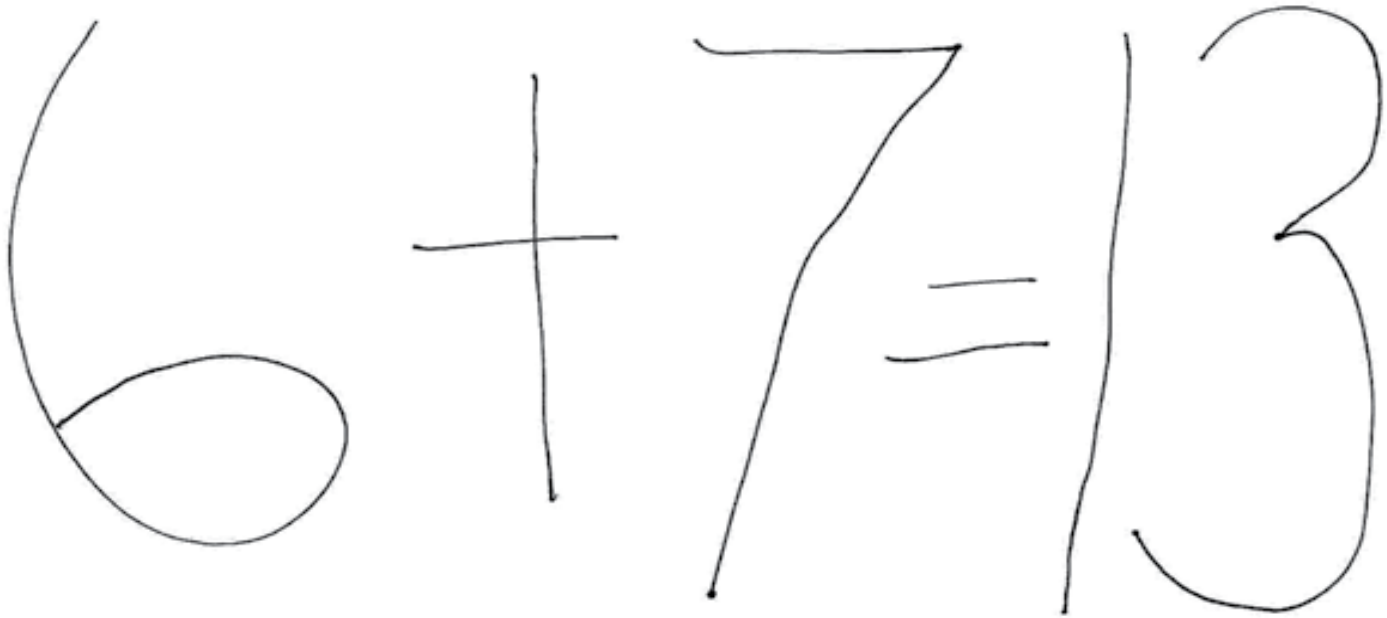
Jillian buys 13 candles. Some of the candles are red. The other 6 candles are blue. How many candles are red?

13 − 6 = 7 candles

I drue 13 candles

I wote a b on 6 of them and 7 were left.

I put 6 in My hed and caunted with My fingrs



Penny Problem

I have between 30 and 50 pennies. When I put them in piles of five, I have 1 penny left over. When I put them in groups of four, I have 1 penny left over. How many pennies do I have?

	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49
÷5	6R1	6R2	6R3	6R4	7	7R1	7R2	7R3	7R4	8	8R1	8R2	8R3	8R4	9	9R1	9R2	9R3	9R4
÷4	7R3	8	8R1	8R2	8R3	9	9R1	9R2	9R3	10	10R1	10R2	10R3	11	11R1	11R2	11R3	12	12R1

1, 2, 3, 4, 0, 1, 2, 3, 4, 0, 1, 2, 3, 4, 0, 1, 2,

3, 0, 1, 2, 3, 9, 1, 2, 3, 10, 1, 2, 3, 1, 1, 2, 3,

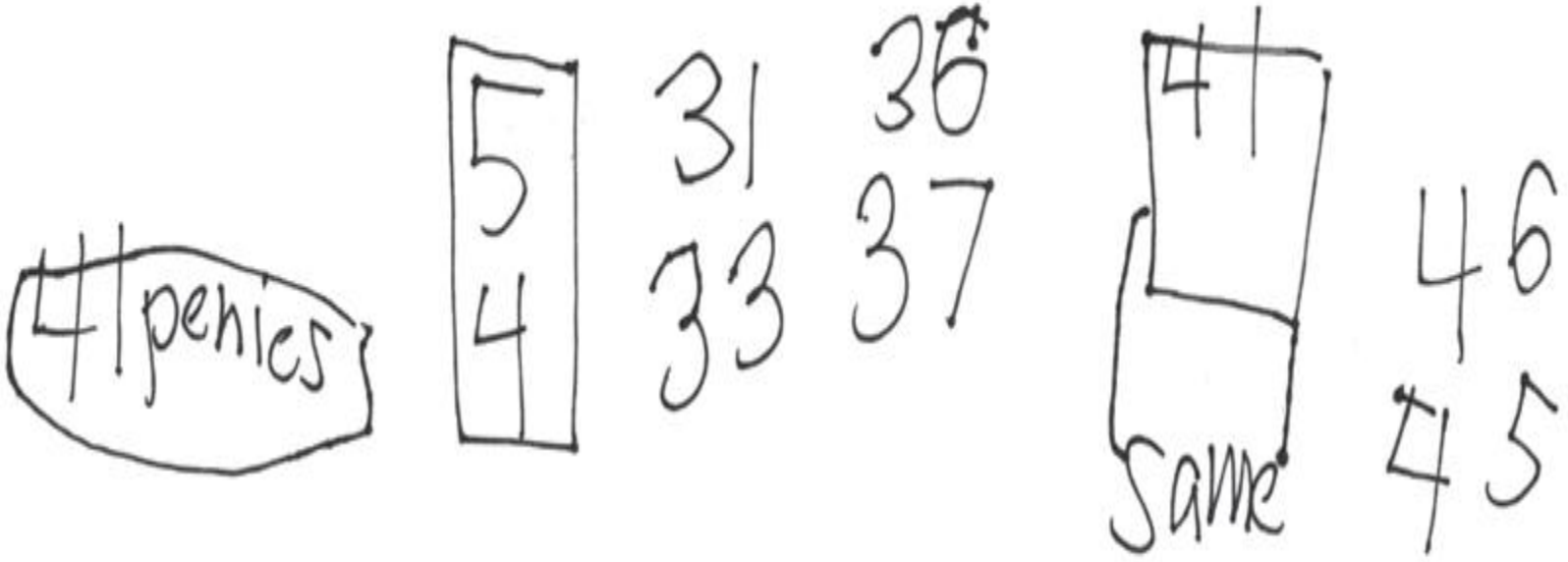
I made a list and wrote 30–50 and divieded all of them by 5. Then, I made another list of All the #s and divieded them by 4 and got 41.

$31 \div \cancel{X} = 7R3$

$36 \div \cancel{X} = 9$

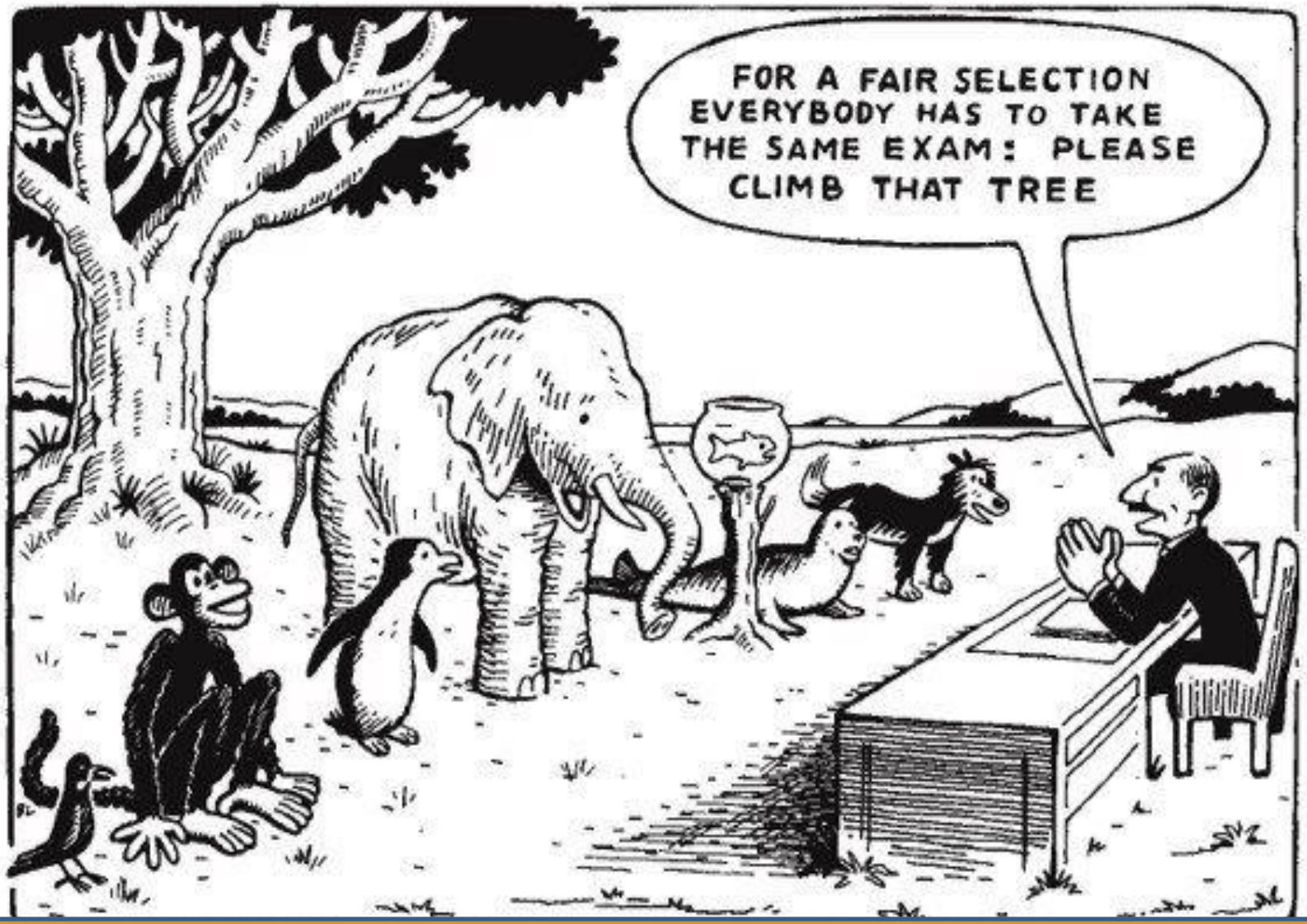
$41 \div 4 = 10R1$

$46 \div \cancel{X} = 11R2$


41 pennies
5
4
31
33
36
37
41
same
46
45

We Need to:

- Give problems that highlight structures and regularity
- Ask questions to refocus students' attention to the regularity within their work
- Prompt students to note commonalities among different solutions


FOR A FAIR SELECTION EVERYBODY HAS TO TAKE THE SAME EXAM: PLEASE CLIMB THAT TREE

My contact information:
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Sources: Problems and Responses

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