Probabilistically Correct:

Simulations & Games That Build Intuition

Dovie Kimmins and Jeremy Winters

Dovie.Kimmins@mtsu.edu; Jeremy.Winters@mtsu.edu

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Grab It Activity

Objective: To have the most items recorded on your list

Materials: Writing instrument, two pieces of paper, one die

1. *Choose a number 1-6*
2. *Decide who will roll the die first and who will write first.*
3. *Each person has their own list on which to record items.*
4. *Teacher tells class what content is to be listed.*
5. *When the teacher says, "Begin", one person begins rolling the die, and the other person begins writing his or her list.*
6. *When the person rolling the die gets the chosen number, he grabs the pencil from the other player and begins writing on his list. The other player picks up the die and begins trying to roll the chosen number.*
7. *Process continues until the teacher says "Stop".*
8. *Both players count the number of items on their list.*

Actual Student Question:

When you roll two die at once, why do you count 5,6 and 6,5 as being different; and when you select two marbles from a bag, you count Red, Green and Green Red as being the same outcome?

Learning Theory: Constructivism & Misconceptions

•Learners CONSTRUCT knowledge for themselves. Often this knowledge is plagued with MISCONCEPTIONS.

•Students hold onto (or revert back to) their misconceptions unless they experience challenges to them.

•Simply telling students that they are incorrect or even explaining why they are incorrect doesnt work in the long run.

- To eliminate misconceptions long term, students must EXPERIENCE CONFLICT between what they believe (the misconception) and the truth. Then they must CONSTRUCT the correct knowledge.
- The role of the teacher is to DESGIN EXPERIENCES which will
  - <u>reveal the misconceptions </u>(allow students to experience a conflict between the misconception and the truth)
  - allow students to <u>construct the correct knowledge</u>.

"Psychologists have shown that our intuition of chance profoundly contradicts the laws of actual random behavior. This incorrect understanding is very difficult to correct by formal instruction. Attempts to teach probability and statistical inference without adequate intuitive preparation are a major pitfall in introducing data and chance into school curricula."

David Moore in "Uncertainty" in
On the Shoulders of Giants:
New Approaches to Numeracy, p.98.

"The conflict between probability theory and students' views of the world is due at least in part to students' limited contacts with randomness. We must therefore prepare the way for the study of chance by providing experience with random behavior early in the mathematics curriculum.

- Moore, page 98.

A Solution

- One way to give the students an experience which will cause them to see a conflict between the probability misconception and the truth is to run simulations.
- Simulations are based upon the LAW OF LARGE NUMBERS: As the number of trials of an experiment increases infinitely, the % of time an event occurs in those trials approaches the theoretical probability that the event will occur. So if the number of trials is sufficiently large, the experimental probability will approximate the theoretical probability.

Agenda

- For two types of misconceptions related to probability/randomness we will
  - Identify the misconception in the context of the pretest item
  - Run simulations ITI-73; Applets on the Web; Probability Explorer Software) to show the conflict between the misconception and the truth.
- Identify and model classroom activities which will help students to construct the correct knowledge.

Remove One

From PBS Mathlines: www.pbs.org/mathline

Individually…..

•**Imagine** rolling a pair a die and recording the sum. **Imagine** doing this 15 times. Place 15 M & M's on your Remove One game board to record your prediction.

With a partner….

•Roll a pair of die. Each person removes an M&M that corresponds to the sum rolled.

•Keep rolling the die until one player has removed all the M&M's. That player wins!

Repeat the game as time permits.

The Racing Game

Navigating Through Probability in Grades 6-8

NCTM

Sample Space for Experiment of Rolling a Red and Green Die

Written as (Green, Red)

(1,1), (1,2), (1,3), (1,4), (1,5), (1,6)
(2,1), (2,2), (2,3), (2,4), (2,5), (2,6)
(3,1), (3,2), (3,3), (3,4), (3,5), (3,6)
(4,1), (4,2), (4,3), (4,4), (4,5), (4,6)
(5,1), (5,2), (5,3), (5,4), (5,5), (5,6)
(6,1), (6,2), (6,3), (6,4), (6,5), (6,6)

Sample Space for Experiment of Rolling a Red and Green Die

Sum of 2:	(1,1)
Sum of 3:	(1,2), (2,1)
Sum of 4:	(2,2), (1,3), (3,1)
Sum of 5:	(1,4), (4,1), (2,3), (3,2)
Sum of 6:	(3,3), (1,5), (5,1), (2,4), (4,2)
Sum of 7:	(1,6), (6,1), (2,5), (5,2), (3,4), (4,3)
Sum of 8:	(4,4), (2,6), (6,2), (3,5), (5,3)
Sum of 9:	(3,6), (6,3), (4,5), (5,4)
Sum of 10:	(5,5), (4,6), (6,4)
Sum of 11:	(5,6), (6,5)
Sum of 12:	(6,6)

The Fair Hopper (NCTM Illuminations)

K	J	I	H	I	J	K

Two Players: Player A and Player B

Rules of the Game: Place a chip on H. Each turn consists of tossing a coin 3 times. For each toss, if H then move right; if T then move L. After 3 tosses, player A scores a point if the chip is on I. Player B scores a point otherwise. A game consists of 10 turns.

Before Playing: Predict – do you think the game is fair?

Play and revisit prediction.

An Analysis of The Fair Hopper

K	J	I	H	I	J	K

A wins: I comes up. B wins: H, J, K comes up.

Sample space for tossing a coin three times is
HHH, HHT, THH, HTH, HTT, TTH, THT, TTT

You either get 3H or 2H or 1H or 0H. Each is a compound event.
The probability of getting 3H is 1/8 → land on K
The probability of getting 2H is 3/8 → land on I
The probability of getting a H is 3/8 → land on I
The probability of getting 0 H is 1/8 → land on K.

Impossible to land on H or J. Probability of I is 6/8 and probability of K is 2/8. Thus A has a better chance of winning.

Discussion of the Equiprobability Misconception and Associated “Pretest” Item

Simulations

Misconception
The Nature of Randomness

Pretest Item: Pretend you are actually tossing a coin 10 times. Put a H or T in each box below to indicate the results you think you might get.

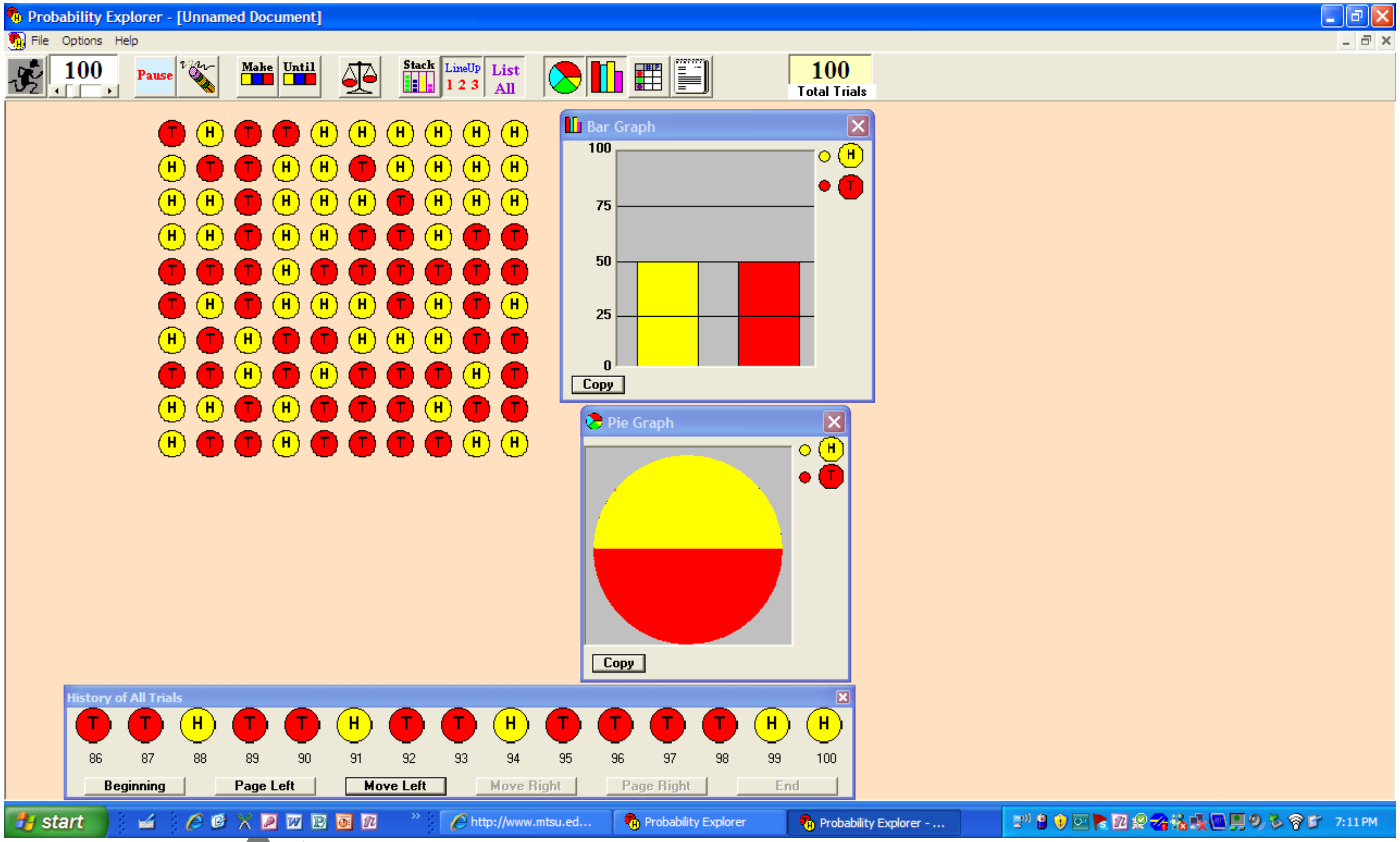
□□□□□□□□□□

For your information...

When asked to write down a sequence of heads and tails that imitates 10 tosses of a balanced coin, “most people will write a sequence with no runs of more than two consecutive heads or tails. But in fact the probability of a run or three or more heads in 10 independent tosses of a fair coin is .508, and the probability of either a run of at least three heads or a run of at least three tails is greater than .8. ... Since we don’t expect to see long runs, we may conclude that the coin tosses are not independent or that some influence is disturbing the random behavior of the coin.”

- Moore, page 120-121.

Simulation Using Probability Explorer



Strings of Heads Activity

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