Judith Kysh
jkysh@sfsu.edu

ALGEBRA TASKS FOR GROUP DISCUSSION

Some ideas for turning routine algebra tasks into discussion generating questions. In searching for and collecting problems that I have seen lead to good group discussions, some categories emerged. What follows is a list of categories along with a few examples for each. Note: there are many problems that are much more "group-worthy" and engaging; this list is limited to the algebra skills that generally do not lend themselves to interesting discussions.

Using Errors:

1. Jonah and Graham are working together. Jonah claims that $(x+y)^2 = x^2 + y^2$. Graham is sure Jonah is wrong, but he cannot figure out how to prove it. Work with your team to help Graham find as many ways as possible to rewrite $(x+y)^2$ correctly and convince Jonah that he is incorrect.

2. EXPONENTS: Select a correct answer for each expression and explain why that answer is correct. Just writing down a rule is not enough. Show why that rule would make sense for this problem.

   a. $a^2a^3 =$
      1) a^6
      2) a^5
      3) $2a^5$

   b. $(a^2)^3 =$
      1) a^6
      2) a^5
      3) a^8

   c. $2a^{-1} =$
      1) $-2a$
      2) $1/2a$
      3) $2/a$

   d. $(8a^6)/(2a^2)=$
      1) $4a^3$
      2) $4a^4$
      3) $6a^4$

   e. $a^0 =$
      1) 0
      2) 1
      3) a

   f. $(-2a^2b^2)^3 =$
      1) $1/(2a^6 b^6)$
      2) $-6a^6 b^6$
      3) $-5a^3 b^5$
      4) $-8a^6 b^6$

   Discuss with your team: For each of the above problems, what wrong answer do you think students would choose most often? What mistake would the student have made?

Rewriting Equivalent Expressions:

3. For each of the following expressions, write at least three equivalent expressions. Be sure to **justify** how you know they are equivalent.

   a. $(x+3)^2 - 4$ b. $(2a^2b^3)^3$ c. $m^2n^5 \cdot mn^4$ d. $(\frac{3p^2q}{q^3})^2$

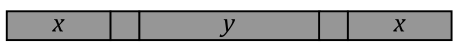
4. In each group which expressions are equivalent for x-values where both expressions represent real numbers? (in other words where denominators are not zero.) Test each expression by writing it as a function and using your graphing calculator to see and compare their graphs. For each graphically equivalent pair, show algebraically that the expressions are equivalent.

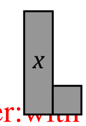
a. $\dfrac{x^2-4x-21}{x-7}$, x^2-1 , and $x+3$ b. $\dfrac{x^2+4}{x}$, $x+4$, and $x+\dfrac{4}{x}$

Starting with a Visual Model:

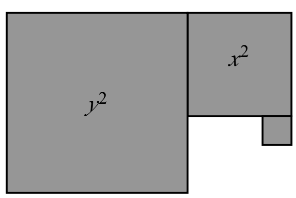
5. For each of the shapes formed by algebra tiles below:

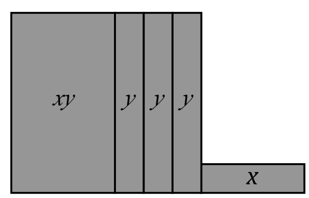
- Use tiles to build the shape.
- Sketch and label the shape on your paper and write an expression that represents the perimeter.
- Simplify your perimeter expression as much as possible.

a. 

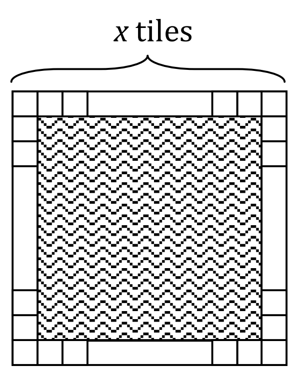
b. 

{ XE "Algebra tiles" }{ XE "Tiles, algebra" }{ XE "Perimeter:with algebra tiles" }

c. 

d. 

6. Kharim is designing a tile border to go around his new square swimming pool. He is not yet sure how big his pool will be, so he is calling the number of tiles that will fit on each side x, as shown in the diagram at right.

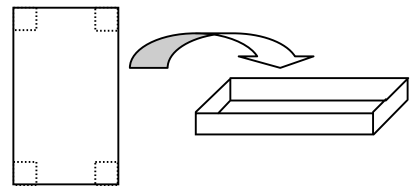


a. How can you write an algebraic expression to represent the total number of tiles Kharim will need for his border? Is there more than one expression you could write? With your team, find as many different expressions as you can to represent the total number of tiles Kharim will need for the border of his pool. Be prepared to share your **strategies** with the class.

b. Find a way to demonstrate algebraically that all of your expressions are **equivalent**, that is, that they have the same value.

c. Explain how you used the Distributive, Associative, and Commutative Properties in part (b).

7. Jill and Jerrell came up with two different expressions for the volume of a paper box made from cutting out squares of dimensions x centimeters by x centimeters. Jill's expression was $(15-2x)(20-2x)x$, and Jerrell's expression was $4x^3-70x^2+300x$.



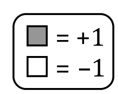
a. Are Jill's and Jerrell's expressions equivalent? **Justify** your answer.

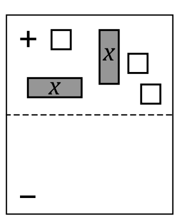
b. If you have not done so already, find an algebraic method to decide whether their expressions are equivalent. What properties did you use? Be ready to share your **strategy**.

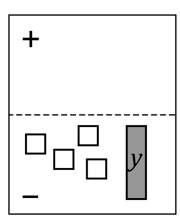
c. Jeremy, who was also in their team, joined in on their conversation. He had yet another expression: $(15-2x)(10-x)2x$. Use a **strategy** from part (b) to decide whether his expression is equivalent to Jill's and/or Jerrell's. Be prepared to share your ideas with the class.

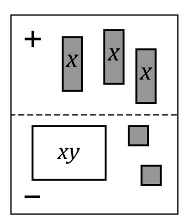
d. Would Jeremy's expression represent the dimensions of the same paper box as Jill's and Jerrell's? Explain.

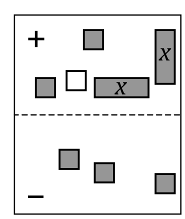
8. Write an expression from a diagram of algebra tiles. Working with a partner, write algebraic expressions for each representation below. Start by building each problem using your algebra tiles.



a.


b.


c.


d.


e. Patti, Emilie, and Carla are debating the answer to part (d). Patti wrote $2-1+2x-3$. Carla thinks that the answer is $2x+2-4$. Emilie is convinced that the answer is $2x-2$. Discuss with your team how each person might have arrived at her answer. Who do you think is correct? When you decide, write an explanation on your paper and **justify** your answer.

Working Backward:

9. Use an expression mat to create each of the following expressions with algebra tiles. Find at least two different representations for each expression. Sketch each representation on your paper. Be prepared to share your different representations with the class.

a. $-3x+4$ b. $-(y-2)$ c. $-y-3$ d. $5x-(3-2x)$

{ XE "Equivalent:expressions" }

10. Now you are going to **reverse** the process. Your teacher will give your team a simple equation that you need to "complicate." Change the equation to make it harder, but still equivalent to the original equation. Your team will have one of the following equations:

$\mathbf{4x-2=6x}$ $\mathbf{1-2x=5}$ $\mathbf{5-3x=-7}$

$\mathbf{2x+1=5}$ $\mathbf{x-6=3+2x}$ $\mathbf{4-x=-1}$

a. Verify that your new equation is equivalent to the one assigned by your teacher.

b. Share your new equation with the class by posting it on the overhead projector or chalkboard.

c. Copy down the equations generated by your class on another piece of paper and show how to use equivalent equations to solve them.

Confronting the "Sticky" Issues:

11. Is there more than one factored form of $3n^2+9n+6$? Why or why not?

a. Why does $3n^2+9n+6$ have more than one factored form while the other quadratics in (the previous problem) only have one possible answer? Look for clues in the original expression ($3n^2+9n+6$) and in the different factored forms.

b. **Without factoring**, decide with your group, which quadratic expressions below may have more than one factored form. Be prepared to defend your choice to the rest of the class.

i. $12t^2-10t+2$ *ii.* $5p^2-23p-10$

iii. $10x^2+25x-15$ *iv.* $3k^2+7k-6$

12. What do you know about the number 1? Brainstorm with your team and be ready to report your ideas to the class. Create examples to help show what you mean.

13. Mr. Wonder now tries to simplify $\frac{4x}{x}$ and $\frac{4+x}{x}$.

a. Mr. Wonder thinks that since $\frac{x}{x}=1$, then $\frac{4x}{x}=4$. Is he correct? Substitute three values of x to **justify** your answer.

b. He also wonders if $\frac{4+x}{x}=5$. Is this simplification correct? Substitute three values of x to **justify** your answer. Remember that $\frac{4+x}{x}$ is the same as $(4+x)\div x$.

c. Compare the results of parts (a) and (b). When can a rational expression be simplified in this manner?

d. Which of the following expressions below is simplified correctly? Explain how you know.

i. $\frac{x^2+x+3}{x+3}=x^2$ *ii.* $\frac{(x+2)(x+3)}{x+3}=x+2$

e. Write three different rational expressions that are equivalent to each of the original expressions in part (d).

14. Angelica and D'Lee were working on finding roots of two quadratic equations: $y=(x-3)(x-5)$ and $y=2(x-3)(x-5)$. Angelica made an interesting claim: *"Look,"* she said, *"When I solve each of them for $y=0$, I get the same solutions. So these equations must be equivalent!"*

D'Lee is not so sure. *"How can they be equivalent if one of the equations has a factor of 2 that the other equation doesn't?"* she asked.

a. Who is correct? Is $y=(x-3)(x-5)$ equivalent to $y=2(x-3)(x-5)$? How can you **justify** your ideas using tables and graphs?

b. Is $0=(x-3)(x-5)$ equivalent to $0=2(x-3)(x-5)$? Again, how can you **justify** your ideas?

Most, but not all, of the problems included here are adapted from CPM Algebra Connections or CPM Algebra 2 Connections