

**Multisensory Algebra:
Using Manipulatives & the
Concrete-Representational-Abstract Sequence**

The NCTM 2012 Annual Meeting and Exposition
Philadelphia PA

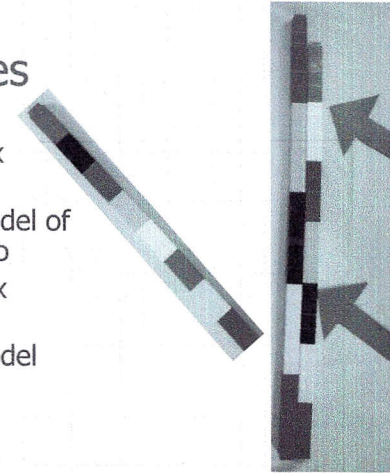
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Instructor, Math Specialist



Multisensory Training Institute of ASDEC
Rockville, MD www.asdec.org
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Manipulatives

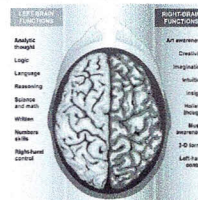
- Using the unifix cubes
- Construct a model of multiples of two
- Using the Unifix cubes
- Construct a model growth by 2^x
- And, 3^x
- Compare



Multisensory Math

• Why

- Learning occurs in many parts of the brain
- But memory is highly associative
- All students benefit from hands-on instruction
- For those students with learning differences, it can provide an essential link



Why: Multisensory

• Information

- can be processed on a modality-specific basis [visual, auditory, kinesthetic etc.]
- converges and is integrated in the brain

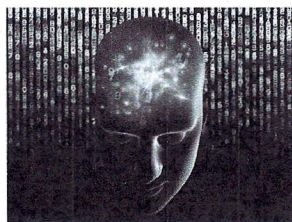
• Performance enhancement

- is larger for multisensory than unisensory stimuli

Paul J. Laurienti, M.D., Ph.D. Department of Radiology,
ANSIR, Advanced Neuroscience Imaging Research, Wake
Forest University

Meaning:

- The more of the brain that is involved
- The stronger the associations
- The stronger the implications for learning & memory



Multisensory Mathematics

- Addresses the needs of all students
- Uses manipulatives to teach, enhance, integrate and reinforce concepts
- Is research based
- Adapts to any curriculum and to the implementation of common core standards

Manipulatives Must Be. . .

- Efficient
 - For the concept being taught
- Effective
 - At illustrating the concept
- Reproducible/ Retrievable
 - In memory, making the concept visualization portable

CRA: An Instructional Sequence

- **Concrete:** Illustrates the concept
 - using hands-on instruction, manipulatives
- **Representational:** Pictorial,
 - illustrates the concept in a retrievable or reproducible format
- **Abstract:** Uses only numerals, computational algorithms

At All Levels of Math Instruction

- One study of algebra students
 - found that **those who used manipulatives** in specific math applications
 - **“outperformed peers** receiving traditional instruction on both post-instruction and follow-up tests.”
- Bradley S. Witzel, Cecil D. Mercer, M. David Miller (2003)
Learning Disabilities Research & Practice 18 (2), 121–131.



Math Deficits

- **Numeracy:** Ability to recognize quantity and/or quantity relationships
 - This is perhaps a core deficit leading to math disabilities
 - Dehaene
- **Language Processing:** Difficulty in the organization of, memory for, retrieval of, expression of language of arithmetic
 - Shaywitz, et al
- **Processing Speed**
 - Dial Up

The Multiplication Shuffle

- In quantity comparison, addition/subtraction, estimation...
- activation is in both hemispheres w/ slight preference for the right (**non-language**) hemisphere.
- Blakemore, Sarah-Jayne; Frith, Uta; (2005) **The Learning Brain, Lessons for Education**, Malden MA, Blackwell Publishing

Implications from Research

- “During multiplication, brain activity shifts toward the **left/language hemisphere. [language]**
- This fits with the notion that **multiplication is dependent on regions in the L.H. associated with language**”

Sarah-Jayne Blakemore & Uta Frith, The Learning Brain: Lessons for Education, 2005 Blackwell Publishing, Malden MA



CRA

- LD students "needed an average of **three experiences** at the concrete level before moving on to the representational level (1993)."

Flores, Margaret: Using the Concrete-Representational-Abstract Sequence to Teach Subtraction With Regrouping to Students at Risk for Failure, Remedial and Special Education Volume XX Number X February 2009



Manipulatives: Pre-Algebra

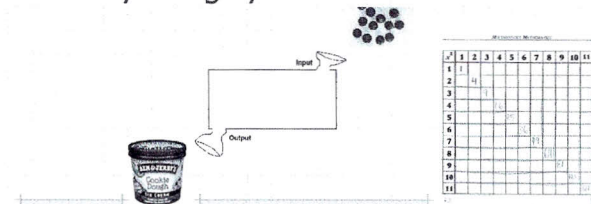
- Vocabulary & Concepts
 - Prime & Composite Numbers
 - Factors & Multiples
 - Prime Factorization
 - Mean
- Craft Sticks
- Pipe cleaners or strings & beads

Pre-Algebra

- Positive & Negative Numbers
 - Craft sticks
 - Images and concept verbalization
 - Non-math examples
 - Number lines – with wrapping paper rolls and unifix cubes
- Properties of Real Numbers
 - Unifix Cubes

Make Math Meaningful

- Link math concepts to non-math examples if it clarifies meaning or vocabulary.
- Memory is highly associative.



Functions and $f(x)$ Notation

- **Non-math examples**
- **Linear Functions & Vocabulary**
 - Unifix Cubes
 - Modeling real life story problems and solving them with manipulatives
- **Polynomial Functions**
 - Vocabulary
 - Base Ten Blocks – Multiplication, Division, Factoring
 - Translations



What do we mean by Negative?



- How much dirt is in a hole 2 feet by 4 feet by 4 feet?

Common Factor?

(blond man + blond girl + blond dog)

Greatest Common/Shared Factor is "Blond"

(blond man + blond girl + blond dog) =
blond (man + girl + dog)

Linking the Known to the New

- **Multiplication & Exponential Growth**
 - Unifix Cubes
 - Dry beans in cups or bowls
- **Perfect Squares and Roots**
 - Base Ten Blocks
 - Multiplication Arrays-
modified to illustrate squares
 - Graphic Organizers
- **Combining "Like" Terms**

\sqrt{x}	1	2	3	4	5	6	7	8	9	10	11
1	1										
2		4									
3			9								
4				16							
5					25						
6						36					
7							49				
8								64			
9									81		
10										100	
11											121

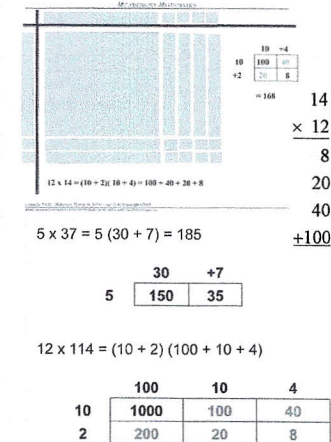


Known to New

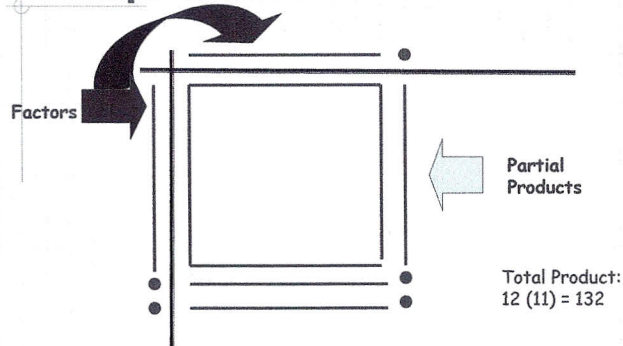
- Multiplication
- Expanded Notation
- Mental Math
- Place Value Based Multiplication
 - Foil or Box
- Polynomial operations
- Operations with radicals or imaginary/complex numbers

Multiplication

- **Build Algebra Skills Early**
- **Model partial products with arrays**
- **Transition to the abstract**

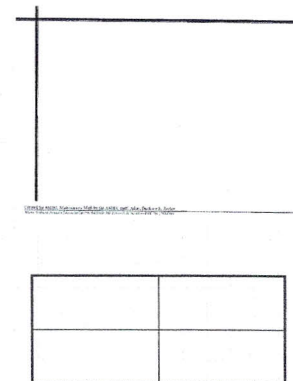


Activity: Representational Array Multiplication & Division



Activity

- Using the array
- Multiply a two digit number by a two digit number in expanded notation form
- Map that onto a "box" mechanism
- Discover the partial products



Multiplication

$(x-2)(x-4) = x^2 - 6x + 8$

- Use similar models
 - to demonstrate multiplication with binomials
 - The difference of squares
 - The sum of squares

Factoring

- **In division**
 - Assemble the dividend
 - Align blocks w/ the divisor
 - Discover the quotient
 - Discover the "left overs" or remainder
- **In factoring**
 - Assemble the product
 - Discover the factors

To the Abstract

- Link polynomials to expanded notation
- Combine "like" terms on the diagonals

$5 \times 37 = 5(30 + 7) = 185$

	30	+7
5	150	35

$12 \times 114 = (10 + 2)(100 + 10 + 4)$

	100	10	4
10	1000	100	40
2	200	20	8

$(x + 5)(x^2 + 3x - 2)$

	x^2	$+ 3x$	-2
x	x^3	$+ 3x^2$	$-2x$
5	$5x^2$	$+ 15x$	-10

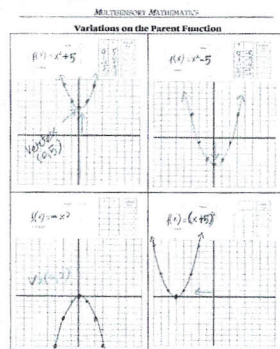
Patterns & Graphic Organizers

	3	$+ i$
3	9	$+ 3i$
$- i$	$- 3i$	$- i^2$
		$- i^2 = -(-1) = 1$

Once again, combining "like terms on the diagonal!"
Solution: $9 + 1 = 10$

Repetition and Linkages

- **The Parent Function**
- **Coding:**
Recognizing a pattern, labeling for meaning and applying to a useful purpose.
- **Tracking**
 - Pipe cleaner & bead



Summary

- CRA
 - Concrete: concept
 - Representation: Retrievable Memory
 - Abstract: Computations
- Manipulatives
 - Efficient, Effective, Essential

The Multisensory Training Institute

- Dedicated to certifying Academic Therapists and training teachers, tutors & parents
- In research based methods for helping students with learning differences
- www.asdec.org
- The *Atlantic Seaboard* Dyslexia Education Center
- 22 W. Jefferson St Rockville MD 20850
- 301-762-2414

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