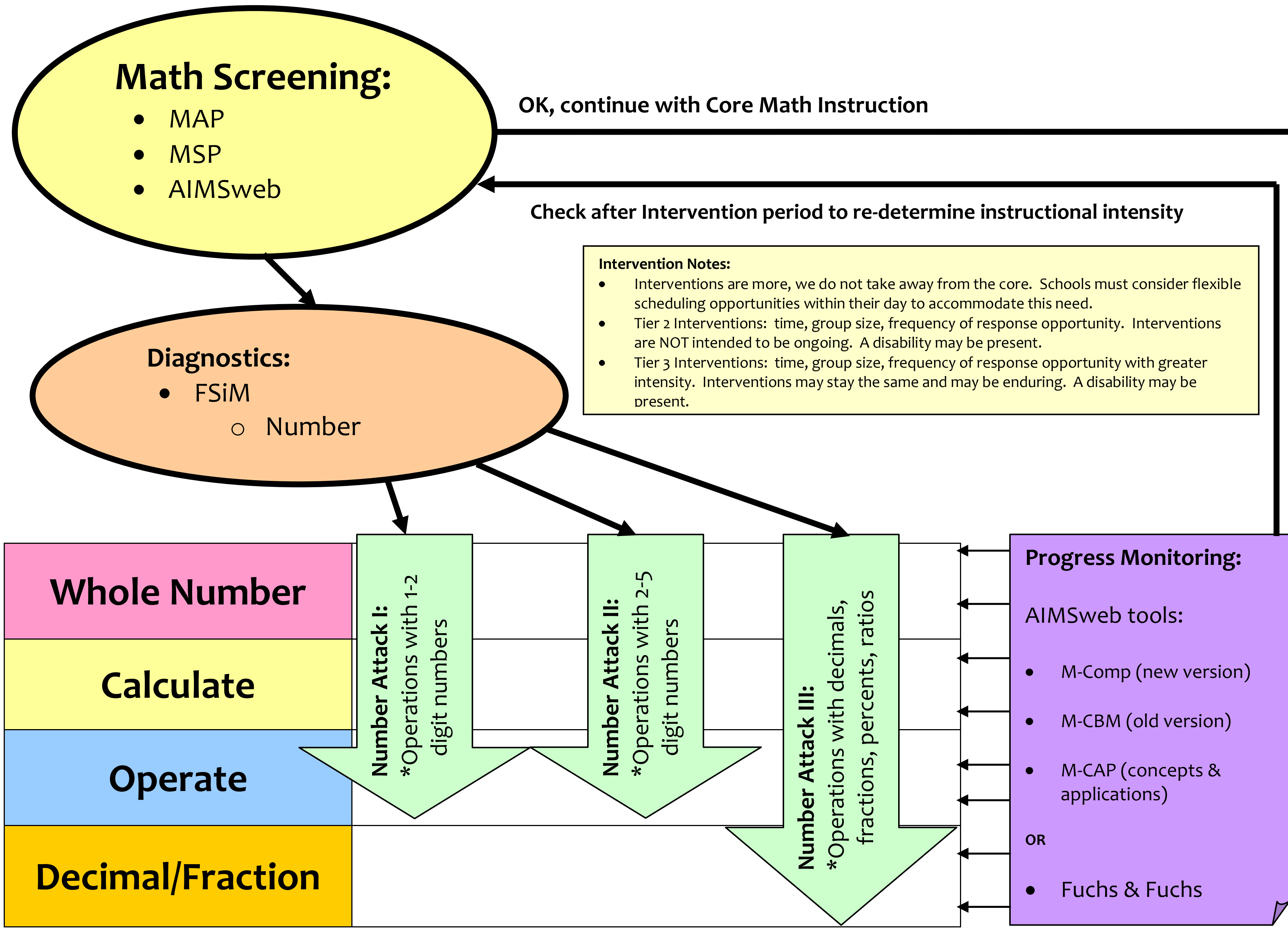
First Steps in Mathematics

How Can We Improve Students' Success with Algebra?

NCTM 2012

Improving the mathematics outcomes of students

WWSD RtI Math Flow Chart



Key Understandings

Teachers will need to plan learning experiences that include and develop the following Key Understandings (KU). These Key Understandings underpin achievement of the outcome. The learning experiences should connect to students' current knowledge and understandings rather than to their year level.

Key Understanding	Stage of Primary Schooling—Major Emphasis	KU Description	Sample Learning Activities
KU1 Adding and subtracting numbers are useful when we: • change a quantity by adding more or taking some away • think of a quantity as combined of parts • equalise or compare two quantities	Beginning ✔✔✔ Middle ✔✔✔ Later ✔	page 12	Beginning, page 14 Middle, page 16 Later, page 18
KU2 Partitioning numbers into part-part-whole helps us relate addition and subtraction and understand their properties.	Beginning ✔✔✔ Middle ✔✔✔ Later ✔✔	page 20	Beginning, page 22 Middle, page 24 Later, page 26
KU3 Multiplying numbers is useful when we: • repeat equal quantities • use rates • make ratio comparisons or changes, e.g. scales • make arrays and combinations • need products of measures.	Beginning ✔✔ Middle ✔✔✔ Later ✔✔✔	page 28	Beginning, page 30 Middle, page 32 Later, page 34
KU4 Dividing numbers is useful when we: • share or group a quantity into a given number of portions • share or group a quantity into portions of a given size • need the inverse of multiplication.	Beginning ✔✔ Middle ✔✔✔ Later ✔✔✔	page 40	Beginning, page 42 Middle, page 44 Later, page 46
KU5 Repeating equal quantities and partitioning a quantity into equal parts helps us relate multiplication and division and understand their properties.	Beginning ✔✔ Middle ✔✔✔ Later ✔✔✔	page 52	Beginning, page 54 Middle, page 56 Later, page 58
KU6 The same operation can be said and written in different ways.	Beginning ✔✔ Middle ✔✔ Later ✔✔	page 62	Beginning, page 63 Middle, page 64 Later, page 65
KU7 Properties of operations and relationships between them can help us to decide whether number sentences are true.	Beginning ✔ Middle ✔✔ Later ✔✔✔	page 66	Beginning, page 68 Middle, page 69 Later, page 71
KU8 Thinking of a problem as a number sentence often helps us to solve it. Sometimes we need to rewrite the number sentence in a different but equivalent way.	Beginning ✔ Middle ✔✔ Later ✔✔✔	page 74	Beginning, page 76 Middle, page 77 Later, page 79
KU9 We make assumptions when using operations. We should check that the assumptions make sense for the problem.	Beginning ✔ Middle ✔✔ Later ✔✔	page 82	Beginning, page 84 Middle, page 85 Later, page 86

Key

✔✔✔ The development of this Key Understanding is a major focus of planned activities.

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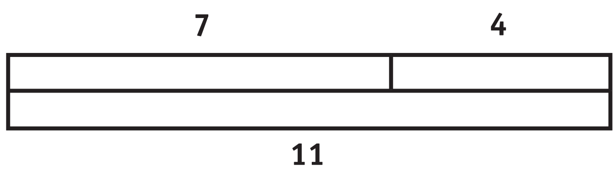
✔ Some activities may be planned to introduce this Key Understanding, to consolidate it, or to extend its application. The idea may also arise incidentally in conversations and routines that occur in the classroom.

KEY UNDERSTANDING 2

Partitioning numbers into part-part-whole helps us relate addition and subtraction and understand their properties.

A quantity, while being thought of as a whole, can also be thought of as composed of parts. That is:



$7 + 4 = 11$ and $4 + 7 = 11$
$11 - 4 = 7$ and $11 - 7 = 4$

The part-part-whole relationship shows how addition and subtraction are related, with subtraction being the inverse of addition. If the whole quantity is unknown, addition is required. If one of the other quantities is unknown, subtraction is required. This enables students to see *why* a problem that they think of as about adding, but with one of the addends unknown, could be solved by subtracting or vice-versa. (See Key Understanding 7, page 66, and Background Notes, page 91.) Linking the joining and separating of the parts that make the whole to a variety of situations also helps students to see *why* subtraction can be used to solve a take-away problem and also a comparison problem. Understanding part-part-whole relationships to represent a problem in different ways, so they can choose the most helpful.

The part-part-whole relationship is also the key to students seeing *why* addition is commutative and *why* subtraction is not. The commutativity of addition is of obvious practical use in calculating, but knowing that, and understanding why, addition is commutative and subtraction is not, helps students represent word problems with appropriate addition and subtraction sentences.

Students who have achieved Level 1 of the outcome can solve simple addition and subtraction problems for whole numbers, mostly by modelling strategies (see Key Understanding 1, page 12, and Background Notes, page 89). However, they may not link addition to subtraction or the types of subtraction to each other.

At Level 2 they link the types of addition (from Key Understanding 1) to the part-part-whole idea and so understand why the addition symbol works in each case.

Similarly, they link subtraction types to the part-part-whole idea and to the subtraction symbol. With the aid of diagrams, they can use part-part-whole relationships to link addition to subtraction and so, given $16 + \square = 34$, they could work out a related subtraction and so find the 'hidden number' on their calculator.

At Level 3, students use the inverse relationship between addition and subtraction routinely for large whole numbers, e.g. they readily say that if 35 + 65 = 100, then 100 – 65 *must be* 35, although they may still rely on imagining it in diagrams. At Level 4 this relationship has been generalised so that students can use the inverse relationship in an abstract way for any numbers including decimals and fractions. Students at Level 5 can use the relationship to solve more abstract 'algebraic' problems such as: half my number, add one, is 43, what is my number?

Diagnostic Map: Number

Most students will enter the Matching phase between 3 and 5 years of age.

During the Emergent Phase	By the end of the Emergent phase, students typically:	As students move from the Emergent phase to the Matching phase, they:
Students reason about small amounts of physical materials, learning to distinguish small collections by size and recognising increases and decreases in them. They also learn to recognise and repeat the number words used in their communities and to distinguish number symbols from other symbols. There is a growing recognition of what is the same about the way students' communities use numbers to describe collections and what is different between collections labelled with different numbers. As a result, students come to understand that number words and symbols can be used to signify the 'numerosity' of a collection.	• use 'bigger', 'smaller' and 'the same' to describe differences between small collections of the like objects and between easily compared quantities • anticipate whether an indicated change to a collection or quantity will make it bigger, smaller or leave it the same • distinguish spoken numbers from other spoken words • distinguish numerals from other written symbols • see at a glance how many are in small collections and attach correct number names to such collections • connect the differences they see between collections of one, two, and three with the number string: 'one, two, three,…' • understand a request to share in a social sense and distribute items or portions.	• may actually see at a glance how many there are in a small collection (e.g. six pebbles), yet may not be able to say the number names in order • may say a string of the number names in order (one, two, three, four, …), but not connect them with how many are in collections • may be beginning to see how to use the number names to count, but may get the order of the names wrong • may distribute items or portions in order to 'share', but may not be concerned about whether everyone gets some, the portions are equal or the whole amount is used up • can tell by looking which of two small collections is bigger; however, they generally cannot say how much bigger.

Describes students' major conceptual shift in thinking during the phase

Indicators that show what students know or can do as a result of having made the shift in thinking

Describes the learning challenges for the next phase (i.e. misconceptions, pre- and partial concepts)

Excerpts taken from First Steps in Mathematics
Department of Education and Training of Western Australia, 2007

How Much Taller?

Name James Year level 5

Jesse and Sylvia were chatting on the net. Jesse said that she was 154cm tall and Sylvia said she was 132cm tall. Jesse said I'm taller than you. Sylvia said, Yes, but not by much.
How much taller is Jesse than Sylvia? 22 cm

Explain how you worked out the answer. I subtracted sylvias height from jesses height to get the answer

Write a number sentence that you could use in a calculator to work it out.

$$\begin{array}{r} 154 \\ -132 \\ \hline \end{array}$$

How Much Taller?

Name Tanya Year level 5

Jesse and Sylvia were chatting on the net. Jesse said that she was 154cm tall and Sylvia said she was 132cm tall. Jesse said I'm taller than you. Sylvia said, Yes, but not by much.
How much taller is Jesse than Sylvia? 22cm taller

Explain how you worked out the answer.
If Jesse is taller than Sylvia she is 22 cms taller. I worked it out by counting it on my fingers.

Write a number sentence that you could use in a calculator to work it out.
If you could have a guess of which number goes between it, you might be right that's if it's 22.

$$\begin{array}{r} 132 \\ +22 \\ \hline 154 \end{array}$$

Diagnostic TASK

FOCUS
Understand Operations
- Key Understanding 2

Empty Boxes

Years/Grades 5–7

Purpose

To see whether students are able to use the inverse relationship between addition and subtraction to solve open number problems.

Producing work samples

Individual interview, small group or whole class

- Explain to students that they are to write what they would put into a calculator to solve the problem, rather than just the answer.
- Do not allow students to use calculators for this task.

If using this as a whole class task, follow-up interviews to clarify what some students are thinking may be necessary.

Empty Boxes

Name ______________________________ Year/Grade _______________ Date ______________

What numbers and symbols would you use on the calculator to solve the following problems?

$17 + \boxed{} = 36$ ____

$\boxed{} - 27 = 34$ ____

$35 = \boxed{} + 16$ ____

$43 - \boxed{} = 16$ ____

$468 + \boxed{} = 842$ ____

$283 = 674 - \boxed{}$ ____

$\boxed{} - 15.78 = 12.43$ ____

Empty Boxes

Name: Jake Year/Grade: 8th Grade

What numbers and symbols would you used on the calculator to solve the following problems?

$17 + \square = 36$	17+19=
$\square - 27 = 34$	48·27=
$35 = \square + 16$	18+16=
$43 - \square = 16$	43-32=
$468 + \square = 842$	468+342
$283 = 674 - \square$	674-483
$\square - 15.78 = 12.43$	3.33-15.78

Empty Boxes

Name: Julieana Year/Grade: 8^{th} Grade

What numbers and symbols would you used on the calculator to solve the following problems?

$17 + \boxed{19} = 36$	36-17=
$\boxed{61} - 27 = 34$	34+27=
$35 = \square + 16$	35-16=
$43 - \square = 16$	43+16=
$468 + \square = 842$	842-468=
$283 = 674 - \square$	674+283=
$\square - 15.78 = 12.43$	12.43+15.78=

Empty Boxes

first steps

Name: Nataly Year/Grade: 8^{th} Grade

What numbers and symbols would you used on the calculator to solve the following problems?

$17 + \boxed{} = 36$ 36-17=

$\boxed{} - 27 = 34$ 34-27=

$35 = \boxed{} + 16$ 36-16

$43 - \boxed{} = 16$ 43-16

$468 + \boxed{} = 842$ 842-468

$283 = 674 - \boxed{}$ 674-283

$\boxed{} - 15.78 = 12.43$ 15.78-12.43

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