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Teaching Geometry Proofs to Digital Generation

2012 NCTM Conference, Philadelphia, PA 1

NCTM

- The National Council of Teachers of Mathematics Standards advocate a unified approach to mathematics education incorporating multiple strands in coherent focused elements. Emphasis is placed on the **use of technology**, visual thinking, and the **connection between geometry and algebra**.

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Integrating Computer Technology into Teaching Mathematics

"The existence, versatility, and power of technology make it possible and necessary to reexamine what mathematics students should learn as well as how they can best learn it. ...When technological tools are available, students can focus on decision making, reflection, reasoning, and problem solving." (NCTM, 2000)

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Technology for Teaching Geometry

- Dynamic Geometry Software (Geometer's Sketchpad) by Key Curriculum Press
 - GeoGebra – open source
 - TI-Nspire CAS by Texas Instruments
- Allows students to
- discover results for themselves
 - formulate conjectures and intermediate results
 - examine special cases
 - generate new ideas

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Features

- Geometer's Sketchpad
- Dynamic geometry capabilities that are
 - Numeric based
 - Construction based
 - CAS system is not built-in
- GeoGebra and TI-Nspire CAS (Computer Algebra System)
- Dynamic geometry capabilities
 - Numeric based
 - Construction based
 - CAS is not integrated with dynamic geometry

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Geometry Expressions™ Features

- an interactive geometry system
 - constraint**-based rather than construction-based
 - symbolic** rather than numeric
 - takes a geometric configuration and outputs algebraic expressions for quantities measured from the model
- <http://geometryexpressions.com>

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Role of Geometry Expressions

Allows students in

- Discover results for themselves
- Formulate conjectures and intermediate results
- Examine special cases
- Generate new ideas

In addition assists students in

- Verifying validity of conjectures
- Proving conjectures

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How Geometry Expressions Works?

- It provides the value of and/or an expression for goal parameters.
- It provides the value of and/or an expression for introduced parameters.
- It enables coincidence of figures by rigid transformation.



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Role of Interactive Geometry Software in Development of Proofs

GEOMETER'S SKETCHPAD, GEOGEBRA, TI-NSPIRE

- Provide a **geometric approach** to strengthening reasoning skills by allowing students to explore geometric objects visually and dynamically and to generate and confirm conjectures on the basis of their observations.

GEOMETRY EXPRESSIONS

- Provide opportunities for developing an **algebraic approach** to proofs due to its capability to produce symbolic algebraic outputs for geometric objects that can guide students as they develop strategies for proofs.

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Difficulties in Teaching Proofs

- Lack of student motivation
- Lack of an algorithm
- There is no general approach to solutions
- Difficulty in choosing a method of proof
- Where to start
- Multi-step planning
- Additional constructions
- Finding conjecture
- Problem interpretation

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Logical Thinking (Reasoning)

- Best demonstration of ability to think logically is doing proofs
- Proof-based geometry is the only course in traditional K-12 mathematics curriculum in US (if any)
- Natural connection exists between generalization and logical thinking
- Formulating generalizations are difficult for students who lack logical thinking

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Sample Problems by Type of Proof

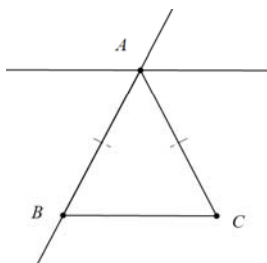
- | | |
|-------------------------------|--------------------------|
| ■ Exterior Angle Bisector | ■ Geometric method |
| ■ Segments in a Square | ■ Algebraic method |
| ■ Unexpected Locus | ■ Coordinate method |
| ■ Triangle from Three Medians | ■ Vector method |
| ■ Shortest Path | ■ Transformations method |

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Exterior Angle Bisector

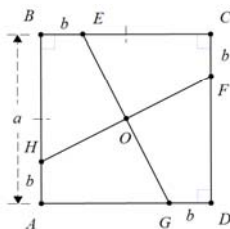
- Problem Statement:** Given isosceles triangle ABC , $AB = AC$. The angle bisector is constructed for the exterior angle at vertex A . What is relationship between the angle bisector and side BC ?
- This problem is solved geometrically without additional constructions.



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Segments in a Square

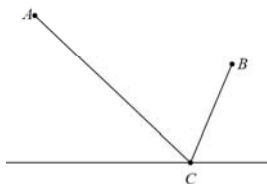
- Problem Statement:** In square $ABCD$, point E lies on BC , point F lies on CD , point G lies on DA , and point H lies on AB . Given that $BE = CF = DG = AH$, what is the relationship between segments EG and FH ?
- This problem is solved algebraically with additional constructions.



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Shortest Path

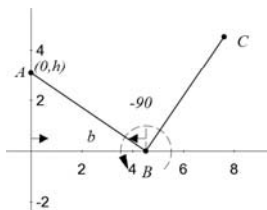
- Problem Statement:** Points A and B are on one side of a given line. Find a point C on the line, such that $AC + CB$ is smallest.
- This is a classic optimization problem with many of applications. A simple solution uses transformations, specifically a reflection.



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Unexpected Locus

- Problem Statement:** Given point A not on the given line, point B on the given line, point C is chosen so that AB and BC are equal and perpendicular. What is the locus of point C, if B moves along the given line?
- Since the problem asks for the locus of point C, a natural approach is to use the coordinate method.



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Triangle from Three Medians

- Problem Statement:** Is it possible to construct a triangle from the three medians of a given arbitrary triangle?
- The necessary and sufficient condition for constructing a triangle from three segments is given by the triangle inequalities. In this particular problem it is very difficult to verify the triangle inequalities for the medians. Thus this problem is easier to solve by vector method. In vector terms, the necessary and sufficient condition for constructing a triangle is equivalent to the condition that the sum of three non-collinear vectors be equal to the zero-vector.



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GX and Mathematical Practices

- Reason abstractly and quantitatively.
- Construct viable arguments and critique the reasoning of others.
- Look for and make use of structure.
- Look for and express regularity in repeated reasoning.

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Publications

- Lyublinskaya, I., Ryzhik, V., Funsch, D. (2009) Developing Geometry Proofs with Geometry Expressions. Tigard, OR: Saltire Software, Inc. 297p. (see booth 303)
- Lyublinskaya, I., Funsch, D. (2012) Geometry + Technology = Proof. *The Mathematics Teacher*, 105(6), 448-454 (see NCTM website)

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For more information about
Geometry Expressions™
visit booth 303 in the exhibit
hall or on-line at

<http://www.geometryexpressions.com/>

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