Teaching Geometry Proofs to Digital Generation

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Teaching reasoning and proofs in high school geometry is one of the challenging tasks that we face today. Can technology help us with this task? In this presentation we share set of problems that uses symbolic geometry software that can be used to develop students' proofs skills.

Activities:

Problem Name	**Geometry Topic**	**Pre-requisite knowledge**	**Method of proof**	**Level of difficulty**
Exterior Angle Bisector	Parallel lines	Measure of the exterior angle of a triangle	Geometric without additional constructions	1
Shortest Path	Segment length. Optimization	none	Transformations. Reflection	1
Unexpected Locus	Lines	None	Coordinate	2
Segments in a Square – Algebra for All	Comparison of angles Comparison of segments	Law of Cosines; Pythagorean Theorem	Algebraic with additional constructions	2
Triangle from Three Medians	Reconstruction	None	Vector. Addition of Vectors	3

Level of difficulty:

Level 1 is characterized by problems whose solution/proof is accomplished

- without the use of auxiliary elements
- straightforward use of the measurement expressions from *Geometry Expressions*

Level 2 is characterized by problems whose solution/proof requires

- the use of additional, auxiliary constructions
- more advanced analysis of the measurements expressions from *Geometry Expressions*

Level 3 is characterized by problems whose solutions/proof requires

- the use of additional, auxiliary constructions – particularly those not obvious in the problem context
- advanced analysis of the measurement expressions from *Geometry Expressions*
- the introduction of additional constrains and/or parameters.

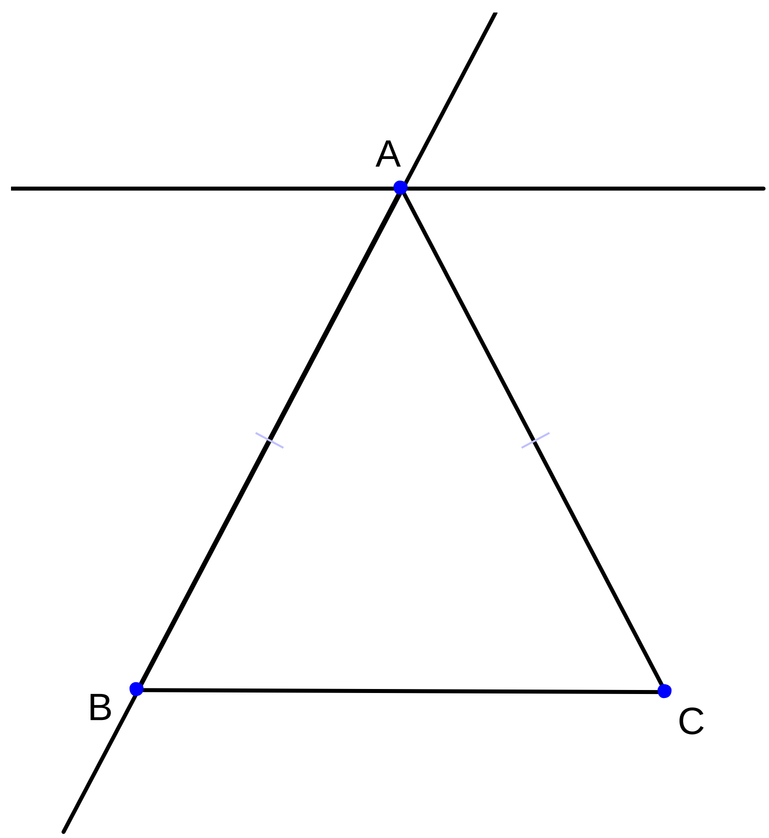
Exterior Angle Bisector

Problem Statement: Given isosceles triangle ABC, AB = AC. The angle bisector is constructed for the exterior angle of vertex A. What is relationship between the angle bisector and side BC?

This problem is solved geometrically without additional constructions.

INVERSTIGATION

1. Use **Toggle Grid and Coordinate Axes** to hide the axes and grid.
2. Choose **Draw** → Polygon and draw triangle ABC. Select and delete the polygon's interior.
3. Constrain sides AB and AC to be congruent by selecting each side and choosing **Constrain** → Congruent.
4. Extend side AB by drawing a line through these two vertices. Choose **Draw** → Infinite Line. Click on point A and then on point B.
5. Select segment AC and line AB **in this order**. Choose **Construct** → Angle Bisector.



Q1. What is the relationship between the exterior angle and its adjacent interior angle?

Q2. What is the relationship between the exterior angle and the two remote interior angles?

Q3. Formulate your conjecture.

PROOF

Q4. Prove that the angle bisector is parallel to BC.

1. Constrain angle ABC = α by selecting segments AB and BC and choosing **Constrain →** Angle.

Q5. What angle do you need to measure to prove that the angle bisector and BC are parallel.

CHALLENGE

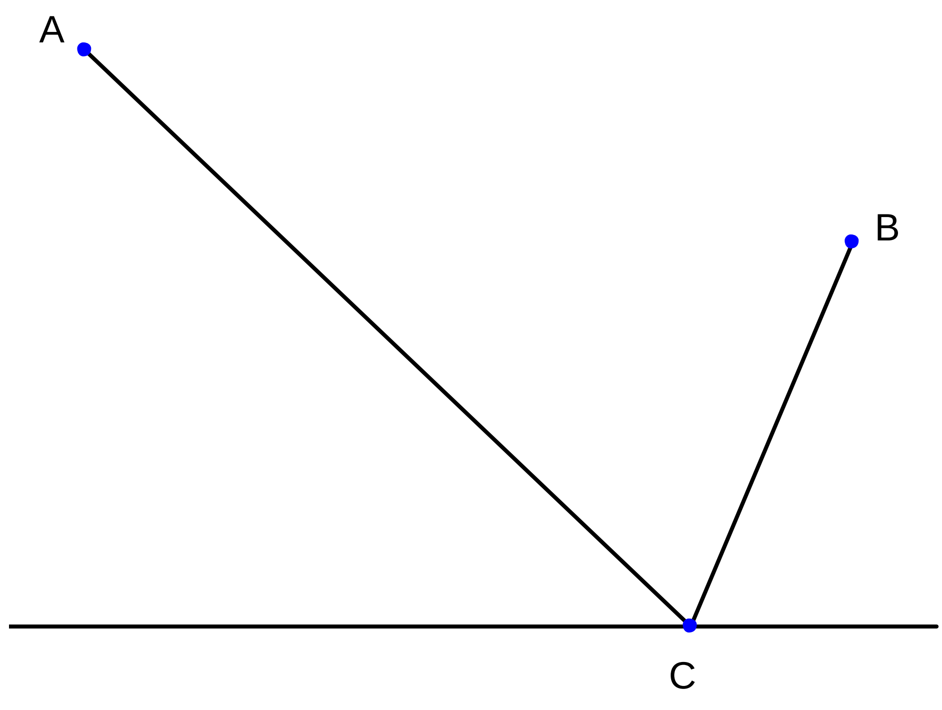
Q6. If the angle bisector of an exterior angle is parallel to the side of the triangle, will the triangle be isosceles?

Problem Statement: Points A and B are on one side of a given line. Find a point C on the line, such that AC + CB is smallest.

This is a classic optimization problem with many applications. A simple solution uses transformations, specifically a reflection.

INVESTIGATION

1. Use **Toggle Coordinate Axes and Grid** to turn off axes and grid.
2. Draw line by selecting **Draw** → Infinite Line. Select the line and constrain its equation to $y = 0$. Click on the equation and choose **View** → Hide.
3. Draw points A and B above the line at different distances from the line by choosing **Draw** → Point. Constrain coordinates of point A to (-5,6) and coordinates of point B to (3,4). Select and hide coordinates.
4. Draw point C on the line. Draw segments AC and BC.



Q1. Drag point C along the line. Where should point C be placed for the path AC + BC to be shortest? Where should it definitely not be placed?

PROOF

1. Reflect segment AC over the line. Select segment AC. Choose **Construct** → Reflect. Click on the line and image of AC will be displayed.

Q2. Drag segment AC or BC and observe when AC + BC is minimal. Describe your conjecture about point C.

Q3. Construct C and prove that at this location AC + BC is minimal.

CHALLENGE

Q4. If the distances from points A and B to the line are equal, describe the position of point C that minimizes the distance AC + BC.

Q5. Is it possible that point C is equidistant from points A and B and also minimizes the distance AC + BC? If so, when is it possible?

Problem Statement: Given point A lies not on the given line, point B lies on the given line, point C is chosen so that AB and BC are equal and perpendicular. What is the locus of point C, if B moves along the given line?

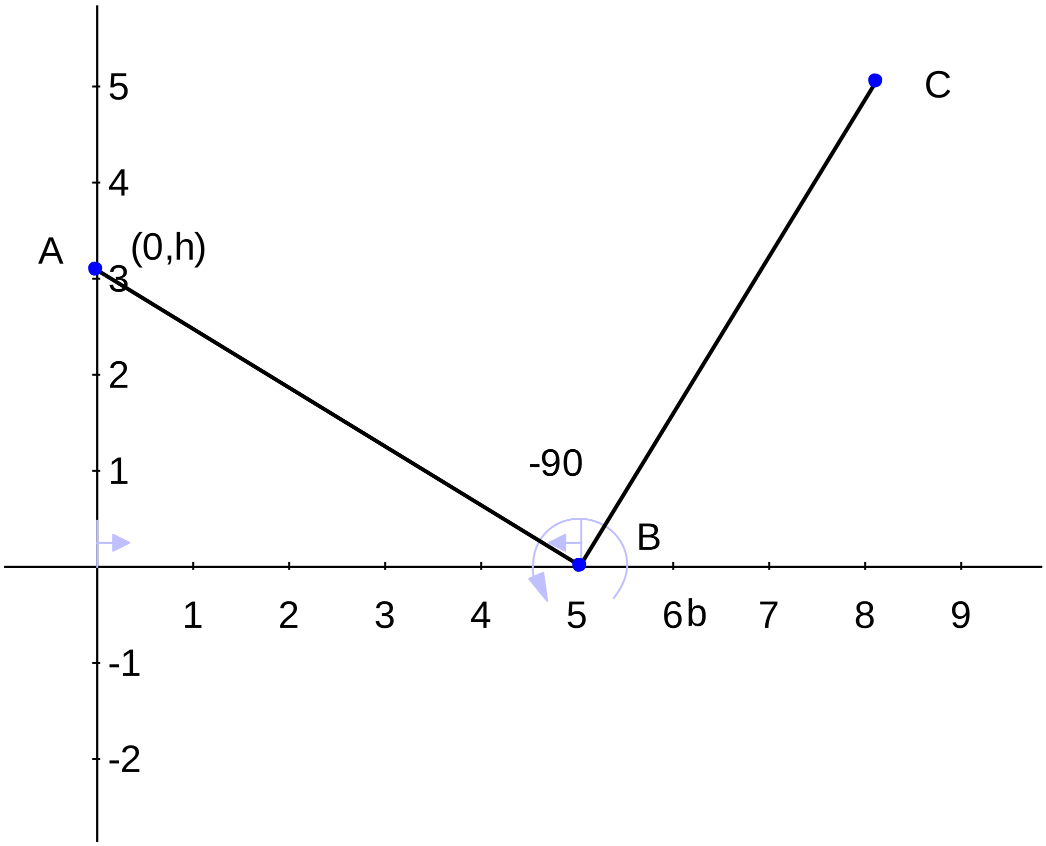
Since the problem asks for the locus of point C, a natural approach is to use the coordinate method.

INVESTIGATION

1. Use **Coordinate Axes and Grid** to display the coordinate axes without the grid.

For simplicity we will use the *x*-axis as the given line and we will construct point A on the *y*-axis.

2. Draw point A on the *y*-axis by selecting **Draw** → Point. Constrain the coordinates of point A to $(0, h)$ by choosing **Constrain** → Coordinate, and type 0, *h*.
3. In the **Variables** panel choose any value of $h > 0$. Lock this value.
4. Draw point B on the x-axis. Select point B and the *x*-axis and choose **Constrain** → Proportional. In the open window type *b*. Thus, point B is constrained to the *x*-axis, where *b* is its *x*–coordinate.
5. Draw segment AB.
6. Select segment AB and point A. Click on **Construct** → Rotate, then click on the center of rotation, point B. In the open window type the angle of rotation -90. The image of the segment will be displayed. Label the end point of the image as C.



Q1. What is the locus of point C when B moves along the x-axis?

Q2. Drag the point B and observe the motion of the point C. Do you need to modify your conjecture? Restate it if necessary.

7. In order to display the locus, select point C and choose **Construct** → Locus. The software will ask for a parametric variable. Choose *b* and select a start value of -10 and an end value of 10. The locus will be displayed.

Q3. Formulate your conjecture.

PROOF

Q4. Prove your conjecture about the locus and provide an equation for it.

CHALLENGE

Q5. Find the equation of the locus of point C we rotate segment AB around point B counterclockwise 90 degrees?

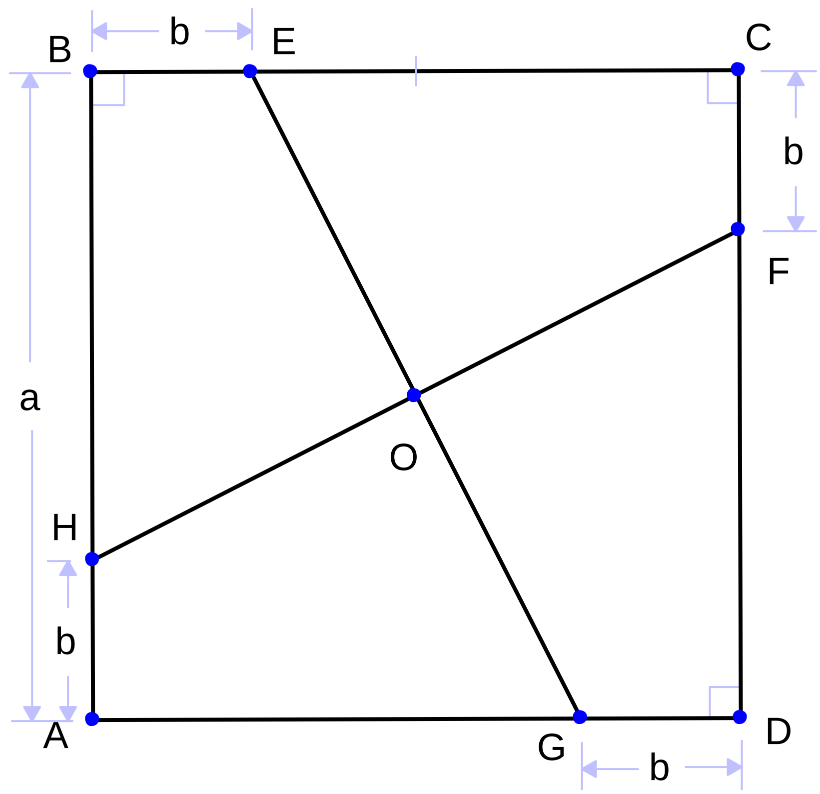
Segments in a Square – Algebra for All! Student Worksheet

Problem Statement: In square ABCD, point E lies on BC, point F lies on CD, point G lies on DA, and point H lies on AB. Given that BE = CF = DG = AH, what is the relationship between segments EG and FH?

This problem is solved algebraically with additional constructions.

INVESTIGATION

1. Use **Toggle Grid and Coordinate Axes** to hide the axes and the grid.
2. Choose **Draw** → Polygon and draw quadrilateral ABCD. Select and delete the interior of the quadrilateral.
3. Constrain segments AB and BC to be perpendicular by selecting both segments and choosing **Constrain** → Perpendicular. In the same way constrain BC ⊥ CD and CD⊥ AD.
4. Constrain AB = *a* by selecting the segment and choosing **Constrain** → Distance/Length.
5. Constrain BC = AB by selecting both segments and choosing **Constrain** → Congruent.
6. Draw point E on segment BC by choosing **Draw** → Point. Similarly, draw point F on CD, point G on DA, and point H on AB.
7. Constrain distance BE = *b* by selecting points B and E and choosing **Constrain** → Distance/Length. . Constrain CF = DG = AH by selecting the segments and choosing **Constrain** → Congruent.
8. Draw EG and FH by choosing **Draw** → Segment.
9. Select segments EG and FH and choose **Construct** → Intersection. Label the point of intersection O.



Q1. What laws allow you to calculate angle measurements from segment lengths in a triangle?

10. Use *Geometry Expressions* to calculate angle between segments EG and FH. Select both segments and choose **Calculate** → Angle.

Q2. What theorems allow you to find lengths of segments?

11. Calculate lengths of segments EG and FH. Select each segment, one at a time, and choose **Calculate** → Distance/Length.

Q3. Formulate final conjecture.

PROOF

Q4. Prove your conjecture.

1. Draw segments EH, AO and BO.

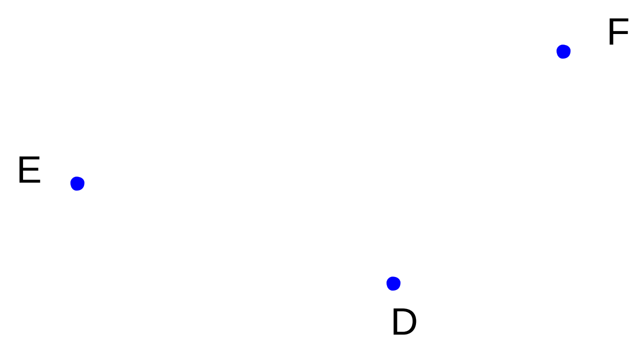
40. Triangle from Three Midpoints – Student Worksheet

Problem Statement: Construct a triangle and the midpoints of each side. Hide the sides and vertices of the original triangle. Is it possible to reconstruct the original triangle from the midpoints of its sides? Prove that the reconstructed triangle is identical to the original.

This problem is solved geometrically with additional constructions.

INVESTIGATION

1. Choose **Draw** → Polygon and draw triangle ABC. Select and delete the triangle's interior.
2. Choose each side and select **Construct** → Midpoint. Label them D, E, and F on sides AC, AB, and BC respectively.
3. Select the triangle's sides and vertices and choose **View** → Hide.



Q1. What are properties of the mid-segment of a triangle?

PROOF

Q2. How can you use the mid-segment of a triangle to re-construct it?

1. Select **View** → Show All and confirm that your triangle coincides with original triangle.
2. The proof is by the fact that the original triangle and the constructed triangle coincide.

CHALLENGE

Q3. Can you think of a different method of re-construction?

Q4. Is it possible to reconstruct a quadrilateral from the midpoints of its sides?