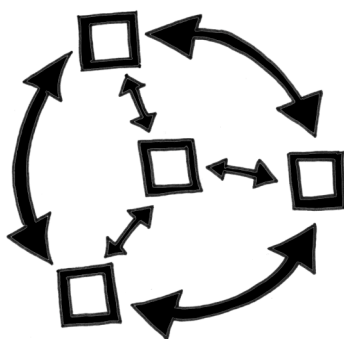


It's All Connected: Similarity as a Geometric Building Block



For more information about the materials you find in this packet, contact:

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To help students understand similarity, several activities have students enlarge and reduce shapes to create similar figures. In addition, two investigations use dynamic geometry tools to help students explore similar relationships. The examples that follow are provided from Chapter 3, Lesson 3.1.1.

3-1. WARM-UP STRETCH

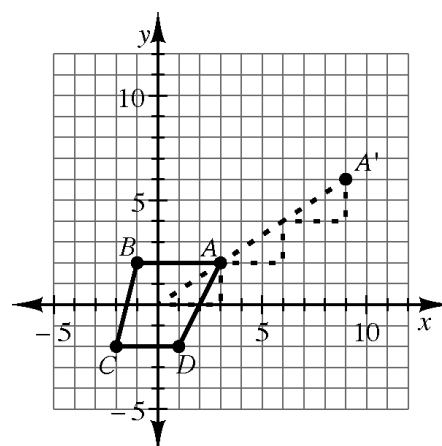
Before computers and copy machines existed, it sometimes took hours to enlarge documents or to shrink text on items such as jewelry. A pantograph device (shown in use at right) was once used to duplicate written documents and artistic drawings. You will now employ the same geometric principles by using rubber bands to draw enlarged copies of a design. Your teacher will show you how to do this.



3-2. In problem 3-1, you created designs that were **similar**, meaning that they have the same shape. But how can you determine if two figures are similar? What do similar shapes have in common? To find out, your team will need to create similar shapes that you can measure and compare.

- a. Obtain a Lesson 3.1.1 Resource Page from your teacher and **dilate** (stretch) the quadrilateral from the origin by a factor of 2, 3, 4, or 5 (each team member should pick a different enlargement factor). You may want to imagine that your rubber band chain is attached at the origin and stretched so that the knot traces the perimeter of the figure.

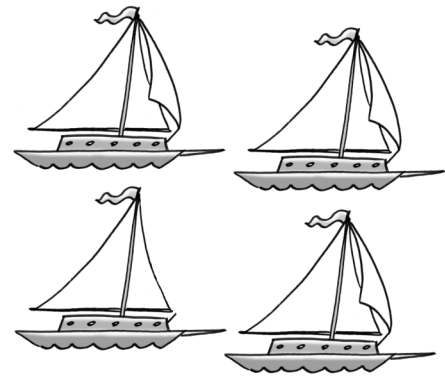
Another way to approach this task is by using slope triangles. If you draw a slope triangle from the origin to a vertex, the hypotenuse of this triangle represents the first rubber band of your chain. The hypotenuse of a second slope triangle continuing on from this vertex represents the second rubber band in your chain. Repeat this process for each vertex to create an enlargement of the original figure. Note that if you are making a figure three times as large as your original, you will need three slope triangles for each vertex. See the diagram at right.



- b. Carefully cut out your enlarged shape and compare it to your teammates' shapes. How are the four shapes different? How are they the same? As you investigate, make sure you record what qualities make the shapes different and what qualities make the shapes the same. Be ready to report your conclusions to the class.

3-3. WHICH SHAPE IS THE EXCEPTION?

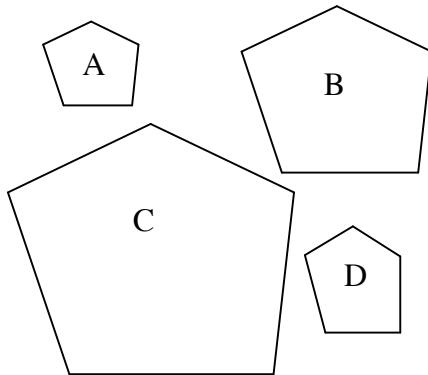
Sometimes shapes look the same and sometimes they look very different. What characteristics make figures alike so that we can say that they are the same shape? How are shapes that look the same but are different sizes related to each other? Understanding these relationships will allow us to know if shapes that appear to have the same shape actually do have the same shape.



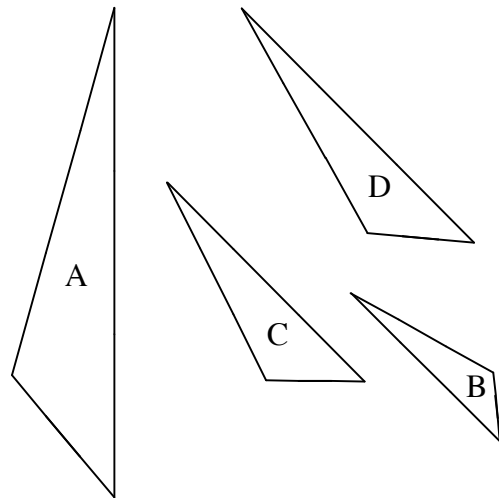
Your task: For each set of shapes below, three shapes are similar, and one is an exception, which means that it is not like the others. Find the exception in each set of shapes. Your teacher may give you tracing paper to help you in your investigation. Answer each of these questions for both sets of shapes below:

- Which shape appears to be the exception? What makes that shape different from the others?
- What do the other three shapes have in common?
- Are there commonalities in the angles? Are there differences?
- Are there commonalities in the sides? Are there differences?

a.



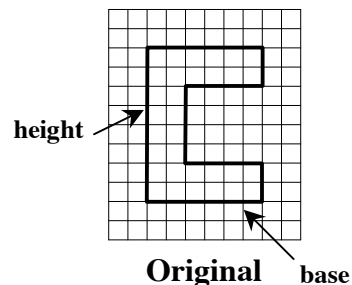
b.



While learning about similarity, students are confronted throughout the chapter with similar figures that have a similarity ratio of 1. The notion of congruence is introduced as a special case of similarity. In Chapters 3 through 6, students continue to use this understanding of congruence to prove that triangles are congruent (i.e., students first prove that the triangles are similar and provide evidence that the similarity ratio is 1). The formal Triangle Congruence Properties (such as SAS, SSS, and AAS) are introduced as shortcuts in Chapter 6.

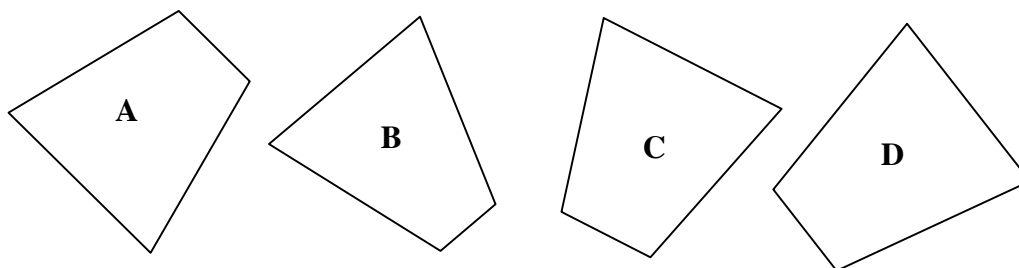
3-12. Casey decided to enlarge her favorite letter: C, of course! Your team is going to help her out. Have each member of your team choose a different zoom factor below. Then on graph paper, enlarge (or reduce) the block "C" at right by your zoom factor.

- a. 3 b. 2
c. 1 d. $\frac{1}{2}$



3-13. Look at the different "C's" that were created in problem 3-12.

- a. What happened when the zoom factor was 1?
b. When two shapes are the same shape **and** the same size (that is, the zoom factor is 1), they are called **congruent**. Compare the shapes below with tracing paper and determine which shapes are congruent.



To see a sample of an investigation from the *Geometry Connections* course, work with your team to solve the following problem. Be ready to share your method with the rest of the teachers! (Note: This problem is from Chapter 3, Lesson 3.1.4.)

3-32. GEORGE WASHINGTON'S NOSE

On her way to visit Horace Mann University, Casey stopped by Mount Rushmore in South Dakota. The park ranger gave a talk that described the history of the monument and provided some interesting facts. Casey could not believe that the carving of George Washington's face is 60 feet tall from his chin to the top of his head!



However, when a tourist asked about the length of Washington's nose, the ranger was stumped! Casey came to her rescue by measuring, calculating and getting an answer. How did Casey get her answer?

Your task: Figure out the length of George Washington's nose on the monument. Work with your study team to come up with a strategy. Show all measurements and calculations on your paper with clear labels so that anyone could understand your work.

Discussion Points

What object are you being asked to measure?

How can you use similarity to solve this problem?

Is there something in this room that you can use to compare to the monument?

What parts do you need to compare?

Do you have any math tools that will help you determine the missing parts?

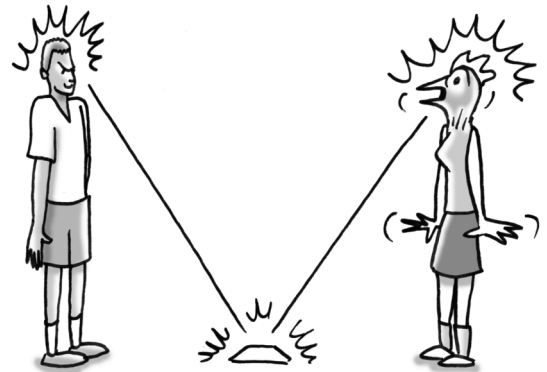
By the end of Section 3.2, students understand what similarity means, are able to determine if two triangles are similar, and can use proportional reasoning to solve for missing side lengths of similar figures.

Below is an example of a performance assessment. Performance assessments are used in each chapter as a chance for each student to assess his or her understanding.

3-93. YOU ARE GETTING SLEEPY...

Legend has it that if you stare into a person's eyes in a special way, you can hypnotize them into squawking like a chicken. Here's how it works.

Place a mirror on the floor. Your victim has to stand exactly 200 cm away from the mirror and stare into it. The only tricky part is that you need to figure out where you have to stand so that when you stare into the mirror, you are also staring into your victim's eyes.



If your calculations are correct and you stand at the *exact* distance, your victim will squawk like a chicken!

- Choose a member of your team to hypnotize. Before heading to the mirror, determine where you will have to stand in order for the hypnosis to work. Measure the heights of both yourself and your victim (heights to the eyes, of course), then draw a diagram to represent this situation. Label all the lengths you can on the diagram. (Remember: your victim will need to stand 200 cm from the mirror.)
- How many pairs of equal angles can you find in your diagram? (Hint: Remember what you know about how images reflect off mirrors.) What is the relationship of the two triangles? Explain how you know. Then calculate how far you will need to stand from the mirror to hypnotize your victim.
- Now for the moment of truth! Have your teammate stand 200 cm away from the mirror, while you stand at your calculated distance from the mirror. Do you make eye contact? If not, check your measurements and calculations and try again.

While similarity is the focus of Section 3.2, it is repeatedly connected with other big ideas of geometry throughout the rest of the course. What follows is a sample of geometry lessons and activities from Chapter 4 that connects to similarity.

The next sample also serves as an example of a complete lesson. Each lesson in the *Algebra Connections* and *Geometry Connections* courses come complete with a suggested lesson plan and ideas for closure.

Lesson 4.1.1 What patterns can I use?

Constant Ratios in Right Triangles

Lesson Objective: Students will recognize that all the slope triangles on a given line are similar to each other and will begin to connect a specific slope to a specific angle measurement and ratio.

Length of Activity: 1 day (approximately 50 minutes)

Core Problems: Problems 4-2 through 4-4

Ways of Thinking: Investigating

Materials: Rulers

Protractors

Lesson 4.1.1 Resource Pages (“Patterns in Slope Triangles”), 2 pages in all, for each student

Lesson Overview: Today’s lesson centers on students’ investigation of slope to connect slope triangles to ideas of similarity and constant ratio. Students will investigate the patterns in different slope triangles to connect ratios to specific angles. We give students time to develop a conceptual understanding of the relationship between Δy , Δx , slope angle, and slope ratio before introducing the tangent function in Lesson 4.1.4.

During this lesson students will be counting and measuring angles and slopes. **Please note:** most angles are approximated so that students will have convenient slope triangles and will be able to find the relationship between the constant slope ratio and the corresponding angle. For example, the slope of $\frac{1}{5}$ does not exactly correspond to an 11° angle, but approximating the angle allows students to focus on the ratio rather than focusing on the exact angle measure. (The actual angle is closer to 11.3° .) Later in the chapter, once students have investigated the link between slope angle and ratio, they will be required to find more exact angles and ratios.

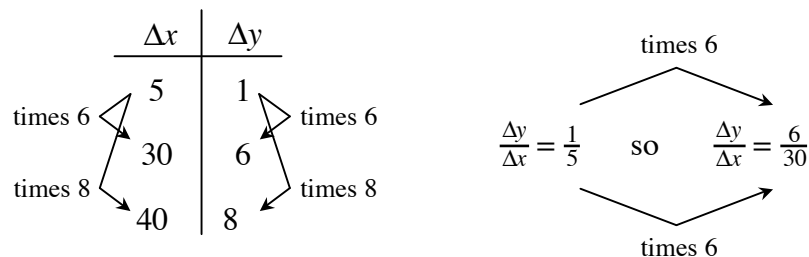
Suggested Lesson Activity:

Distribute a Lesson 4.1.1 Resource Pages (two pages in all) to each student at the beginning of the lesson.

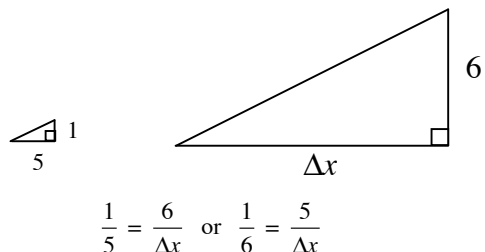
The Leaning Tower of Pisa, problem 4-1, introduces students to a context in which a right triangle is formed, yet one of the acute angles is the primary focus. This problem shows students that while they do have knowledge about the side lengths of right triangles (namely the Pythagorean Theorem), they do not yet have a way to determine the acute angles when the lengths of sides are known. This leads us into problem 4-2, where students will look at several slope triangles on a line with a slope angle of 11° to discover that the slope ratio along the same line is constant. While they can see that the angles of the triangles on this line are congruent, they may not yet connect the slope ratio to the specific angle.

As students work, circulate to be sure that teams are not limiting themselves to measuring only angles or only sides. Asking, “*What else can you measure on this triangle?*” may prompt teams to look beyond vertical and horizontal side lengths (Δy and Δx) in their measurements. While we expect students to see that the corresponding angles of their triangles are congruent, you may need to encourage them to apply the vocabulary of “corresponding angles” and “similar triangles” to these slope triangles. Part (c) of this question asks students to write slope ratios as both fractions and decimals. In order to see accurate patterns throughout this chapter, prompt them to round their decimals to three places.

Problem 4-3 will be the first place students use a slope ratio to find a missing side length. Expect students’ work to take a variety of forms, depending on the different ways they think about the ratio and the different algebraic and logical ways they can apply it. Part (a), for example, could be solved using proportional reasoning structures, such as:



Students may use the structure of similar triangles like those shown at right. Students might also recognize that Δx is always 5 times Δy , so their work would look like $6(5) = 30$.



For part (b), in addition to the strategies above, students might use the decimal form of the ratio: $(40)(0.2) = 8$. Do not expect students to write $\tan 11^\circ = \frac{x}{40}$ at this point.

After applying this ratio to find missing sides of slope triangles in problem 4-3, part (c) asks students to make conjectures about the appropriateness of using this ratio for *all* slope triangles. Require teams to discuss and write down a conjecture before moving on. Problem 4-4 asks students to investigate three new lines on their resource page in order to test their conjecture. If teams struggle to make sense of the information after they graph the three lines, ask questions such as, “*What do these lines have in common?*”, “*How are these lines different from each other?*” and “*What can you predict about a new slope triangle on this line?*” to help students look for patterns.

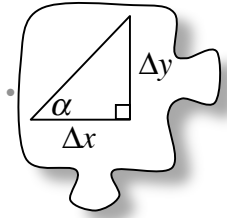
Closure:
(\approx 10 minutes)

Save 10 minutes at the end of class for a whole class discussion. Problem 4-5 asks students to revisit their conjecture about using the slope ratio for an 11° line for any slope triangle. Teams will discuss and, if necessary, rewrite their conjectures in light of their new data. With 10 minutes left, bring the class together so that teams can share their conjectures and revisions. As students share how they came to their conclusions, ask questions to elicit what information they used from the lines in problem 4-4 to confirm or contradict their original conjecture. Once the class has agreed that each line has its own unique slope ratio and several teams have shared their process, leave the class with the question, “*If you have a slope triangle, how do you decide which ratio to use to find a missing side?*” Do not expect a complete answer to this today; this question leads into tomorrow’s lesson.

Homework: Problems 4-6 through 4-10

4.1.1 What patterns can I use?

Constant Ratios in Right Triangles

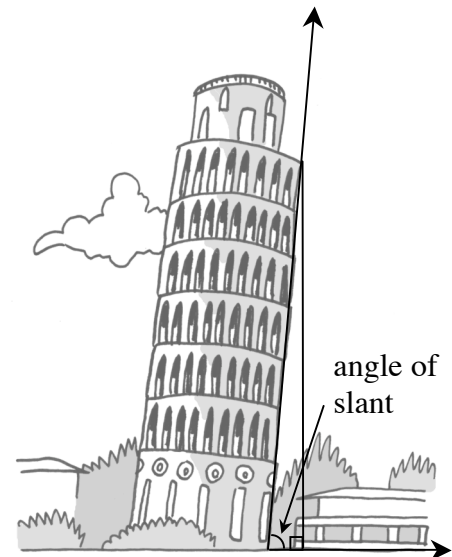


In Chapter 3, you looked for relationships and patterns among shapes such as triangles, parallelograms, and trapezoids. Now we are going to focus our attention on slope triangles, which were used in algebra to describe linear change. Are there geometric patterns within slope triangles themselves that we can use to answer other questions? In this lesson, you will look closely at slope triangles on different lines to explore their patterns.

4-1. LEANING TOWER OF PISA

For centuries, people have marveled at the Leaning Tower of Pisa due to its slant and beauty. Ever since construction of the tower started in the 1100's, the tower has slowly tilted south and has increasingly been at risk of falling over. It is feared that if the angle of slant ever falls below 83° , the tower will collapse.

Engineers closely monitor the angle at which the tower leans. With careful measuring, they know that the top of the tower is now 50 meters off the ground and they know that the original tower was 50.25 meters tall.



- With the measurements provided above, what can you determine? [**Using the Pythagorean Theorem, students can determine that the top of the tower leans ≈ 5 meters from vertical.**]
- Can you determine the angle at which the tower leans? Why or why not? [**No, not yet. We do not know the relationship between the angles and the sides of a right triangle.**]
- At the end of Section 4.1, you will know how to find the angle for this situation and many others. However, at this point, how else can you describe the slope of the leaning tower? [**The slope of the tower is $\frac{50}{5} = 10$.**]

4-2. PATTERNS IN SLOPE TRIANGLES

In order to find an angle (such as the angle at which the Leaning Tower of Pisa leans), we need to know something about the relationship between an angle and the sides of a right triangle. To investigate this relationship, we will start by studying slope triangles. Obtain the Lesson 4.1.1 Resource Pages (two in all) from your teacher and find the graph shown below. Notice that one slope triangle has been drawn for you. **Note:** For the next several lessons we will use approximate values for the angle measures.

- a. Draw three new slope triangles on the line. Each should be a different size. Label each triangle with as much information as you can.



- b. What do these triangles have in common? How are these triangles related to each other? [**They are similar because of AA~ and their corresponding sides are multiples of each other and the original triangle.**]
- c. Write the slope ratio for each triangle as a fraction, such as $\frac{\Delta y}{\Delta x}$. (Note: Δy represents the vertical change or “rise,” while Δx represents the horizontal change or “run.”) Then change the slope ratio into decimal form. [**Each should be $\frac{1}{5}$ and 0.2.**]
- d. What do you notice about the slope ratios written in fraction form? What do you notice about the decimals? [**They are all equal.**]

4-3. Tara thinks she sees a pattern in these slope triangles, so she decides to make some changes in order to investigate whether or not the patterns remain true.

- a. She asks, “What if I draw a slope triangle on this line with $\Delta y = 6$? What would be the Δx of my triangle?” Answer her question and explain how you figured it out. [**30**]
- b. “What if I want to draw a slope triangle for this line with a Δx of 40?” she wonders. “How big will the Δy be?” Find Δy , and explain your reasoning. [**8**]
- c. Tara wonders, “What if I draw a slope triangle on a different line? Can I still use the same ratio to find a missing Δx or Δy value?” Discuss this question with your team and explain to Tara what she could expect. [**At this point, students may believe that the slope ratio will stay constant, but this will only happen if the new line is parallel. Don’t correct their thinking yet. Problem 4-4 will help students determine if their conclusion was correct.**]

4-4. CHANGING LINES

In part (c) of problem 4-3, Tara asked, “What if I draw my triangle on a different line?” With your team, investigate what happens to the slope ratio and slope angle when the line is different. Use the graph grids provided on your Lesson 4.1.1 Resource Pages to graph the lines described below. Use the graphs and your answers to the questions below to respond to Tara’s question.

- On graph A, graph the line $y = \frac{2}{5}x$. What is the slope ratio for this line? What does the slope angle appear to be? Does the information about this line support or change your conclusion from part (c) of problem 4-3? Explain. [**Slope ratio: $\frac{2}{5}$, slope angle: $\approx 22^\circ$**]
- On graph B, you are going to create $\angle QPR$ so that it measures 18° . First, place your protractor so that point P is the vertex. Then find 18° and mark and label a new point, R. Draw ray \overline{PR} to form $\angle QPR$. Lastly, extend \overline{PR} across the whole paper because you will need space to draw in slope triangles. Find an approximate slope ratio for this line. [**While the results depend on an accurate drawing of the line, the slope should be $\approx \frac{1}{3}$.**]
- Graph the line $y = x + 4$ on graph C. Draw in slope triangles, and label everything you can. What is $\frac{\Delta y}{\Delta x}$ (the slope ratio)? What is the slope angle? [**1, 45°**]

4-5. TESTING CONJECTURES

Ms. Coyner’s class is writing **conjectures**, which are conclusions that are believed to be true based on observations. Below is a list of the conjectures her students wrote for problem 4-4. As a team, decide if you agree or disagree with each of them. Explain your reasoning.

- All slope triangles have a ratio $\frac{1}{5}$. [**Not true. Only lines with a slope angle of approximately 11° have a slope ratio of $\frac{1}{5}$.**]
- If the slope ratio is $\frac{1}{5}$, then the slope angle is approximately 11° . [**True**]
- If the line has an 11° slope angle, then the slope ratio is approximately $\frac{1}{5}$. [**True**]
- Different lines will have different slope angles and different slope ratios. [**Not always true. If the lines are parallel or coincide, they have the same slope ratio.**]



MATH NOTES

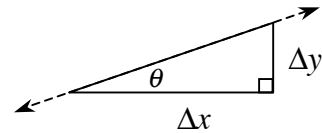
METHODS AND MEANINGS

Slope and Angle Notation

Slope is the ratio of the vertical distance to the horizontal distance in a slope triangle. The vertical part of the triangle is called Δy , (read “change in y ”), while the horizontal part of the triangle is called Δx (read “change in x ”).

When we are missing a side length in a triangle, we often assign that length a variable from the English alphabet such as x , y , or z .

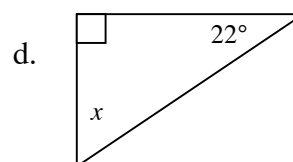
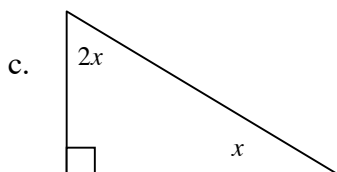
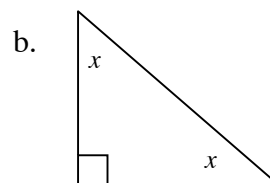
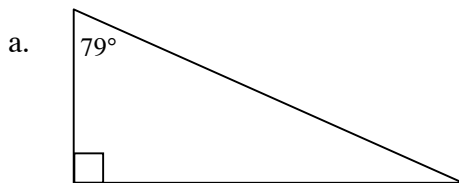
However, sometimes we need to distinguish between an unknown side length and an unknown angle measure. With that in mind, mathematicians sometimes use Greek letters as variables for angle measurement. The most common variable for an angle is the Greek letter θ (*theta*), pronounced “THAY-tah.” Two other Greek letters commonly used include α (*alpha*), and β (*beta*), pronounced “BAY-tah.”



When a right triangle is oriented like a slope triangle, such as the one in the diagram above, the angle the line makes with the horizontal side of the triangle is called a **slope angle**.

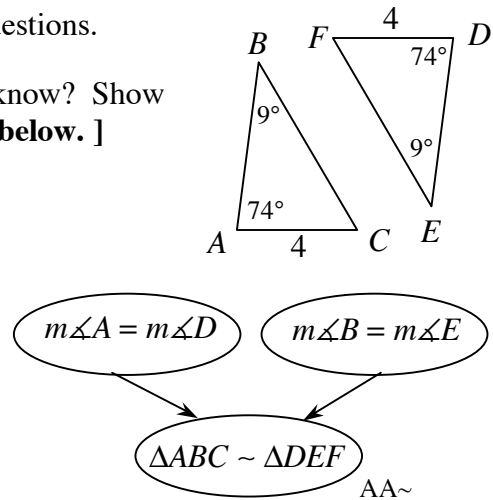


4-6. Use what you know about the angles of a triangle to find the value of x and the angles in each triangle below. [**a:** $x = 11^\circ$; **b:** $x = 45^\circ$; **c:** $x = 30^\circ$; **d:** $x = 68^\circ$]

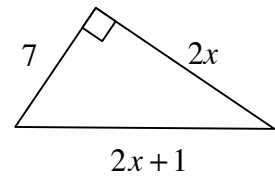


4-7. Use the triangles at right to answer the following questions.

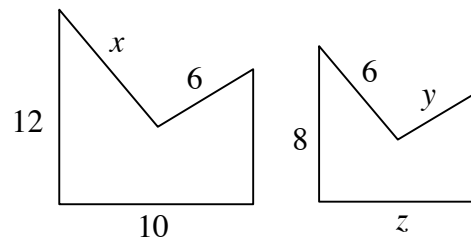
- a. Are the triangles at right similar? How do you know? Show your reasoning in a flowchart. [See flowchart below.]
- b. Examine your work from part (a). Are the triangles also congruent? Explain why or why not. [Yes, because the triangles are similar (AA ~) and the ratio of the corresponding side lengths is 1 (because $AC = DF$).]



4-8. Solve for x . Then find the perimeter of the right triangle shown at right. [$x = 12$, $P = 56$ units]



4-9. The shapes at right are similar. Find the values of x , y , and z . [$x = 9$, $y = 4$, $z = 6\frac{2}{3}$]



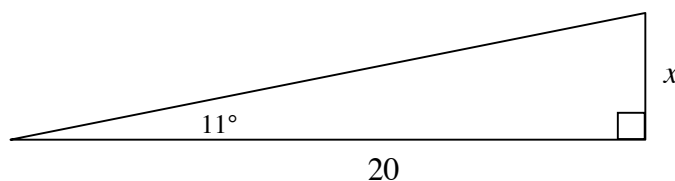
4-10. Are the lines represented by the equations at right parallel? Support your reasoning with convincing evidence. [Yes they are parallel because they have the same slope: $-\frac{3}{5}$.]

$$y = -\frac{3}{5}x + 2$$

$$y = -\frac{3}{5}x - 3$$

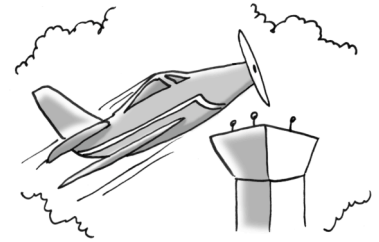
4-11 below is from Lesson 4.1.2. It launches the notion of a tangent ratio:

4-11. What do you know about this triangle? To what other triangles does it relate? Use any information you have to solve for x .



4-27 below, from Lesson 4.1.3, is an example of an application students are asked to solve involving an investigated angle (such as 11°) without yet knowing the trigonometric term “tangent.”

4-27. An airplane takes off and climbs at an angle of 11° . If the plane must fly over a 120-foot tower with at least 50 feet of clearance, what is the minimum distance between the point where the plane leaves the ground and the base of the tower?



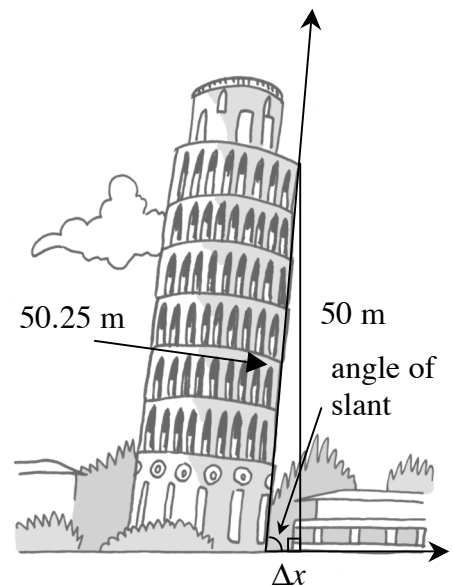
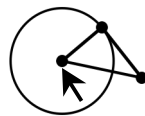
- a. Draw and label a diagram for this situation.
- b. What is the minimum distance between the point where the plane leaves the ground and the tower? Explain completely.

A dynamic tool (using either Cabri Jr., javasketch, or Geometer’s Sketchpad) is used in Lesson 4.1.3 for students to find the slope ratio for other angles. This tool helps students to connect the tangent ratio with the legs of a right triangle with a given slope angle. This also allows the students to find the slope angle when given enough information to find the tangent ratio. With this ability, students revisit and solve the Leaning Tower problem in Lesson 4.1.4.

4-30. WILL IT TOPPLE?

In problem 4-1, you learned that the Leaning Tower of Pisa is expected to collapse once its angle of slant is less than 83° . Engineers have measured the top of the tower as 50 meters off the ground, and the tower was originally 50.25 meters tall.

- a. What is the slope ratio for the tower?
- b. Use the dynamic geometry tool or your Trig Table to determine the angle at which the Leaning Tower of Pisa slants. Is it in immediate danger of collapse?



Highlights of the New *Geometry Connections* Course

CPM's new geometry course offers exciting, integrated chapters full of connections between topics. What was called a "day" in the Math 2 course is now a complete lesson containing an introduction that launches the lesson, problems that develop the topic, and suggestions for closure. Technology is woven into the course—specifically, graphing calculators and dynamic geometry tools such as Geometer's Sketchpad. For every central idea, there are investigations and labs, as well as days devoted to consolidating understanding and practice.

Course Design

- This course is designed to complement the *Algebra Connections* course, but can also be used independently.
- Each chapter contains two or three sections that may or may not be related. Each section consists of a group of lessons that study a major concept (such as transformations).
- Homework is evenly structured so that 60% is review, 20% is from new material, and 20% previews future learning.
- Each lesson contains an introduction informing students and parents of the purpose of that specific lesson. Each lesson is labeled with a title that communicates the subject matter and poses an overarching question that the lesson addresses.

Lesson and Curricular Enhancements

- The course consistently uses investigations and applications for every central idea.
- Significant emphasis is placed on understanding similarity and its connections to many other areas of geometry. Congruent triangles are first introduced as triangles that are similar with a similarity ratio of 1. Similarity is also directly connected to the development of trigonometric ratios early in the first half of the course.
- The students' ability to ask "What if..." and "What about..." questions is developed throughout the course to allow students to launch their own investigations.
- There is a consistent use of dynamic geometry tools (such as Cabri Jr. for a graphing calculator and Geometer's Sketchpad) to help students investigate geometric relationships and to help students intuitively understand how geometric shapes change when certain parameters change.
- Algebraic concepts are embedded into the geometric lessons to help students retain algebra skills.
- All topics are eventually highlighted in Math Notes boxes (formerly tool kit entries).
- The students are given questions (called "focus questions") to ask themselves and their teammates to help advance mathematical discussion. Typical questions include: "*How do you see it?*", "*Is there another way?*", and "*How can I prove it?*"

Teacher Support

- Each lesson is presented with a coherent lesson plan for the teacher that includes detailed suggestions for how to introduce the lesson, conduct the investigation, and bring the lesson to closure.
- Support for how to set up and manage study teams effectively is embedded in the teacher materials.
- Large problems (investigations, labs, etc.) are written to accommodate teachers who prefer open investigations. However, student guidance is also provided for teachers who prefer a more structured approach.
- Teachers are provided suggestions for questions to ask while circulating among study teams, such as: "*How are the triangles related?*", "*Does it matter if the polygon is convex or not? Why or why not?*", and "*What if you reflected the shape twice over parallel lines?*"