TEACHER'S PEDAGOGICAL CONTENT KNOWLEDGE AND STUDENT'S UNDERSTANDING OF INTEGER OPERATIONS

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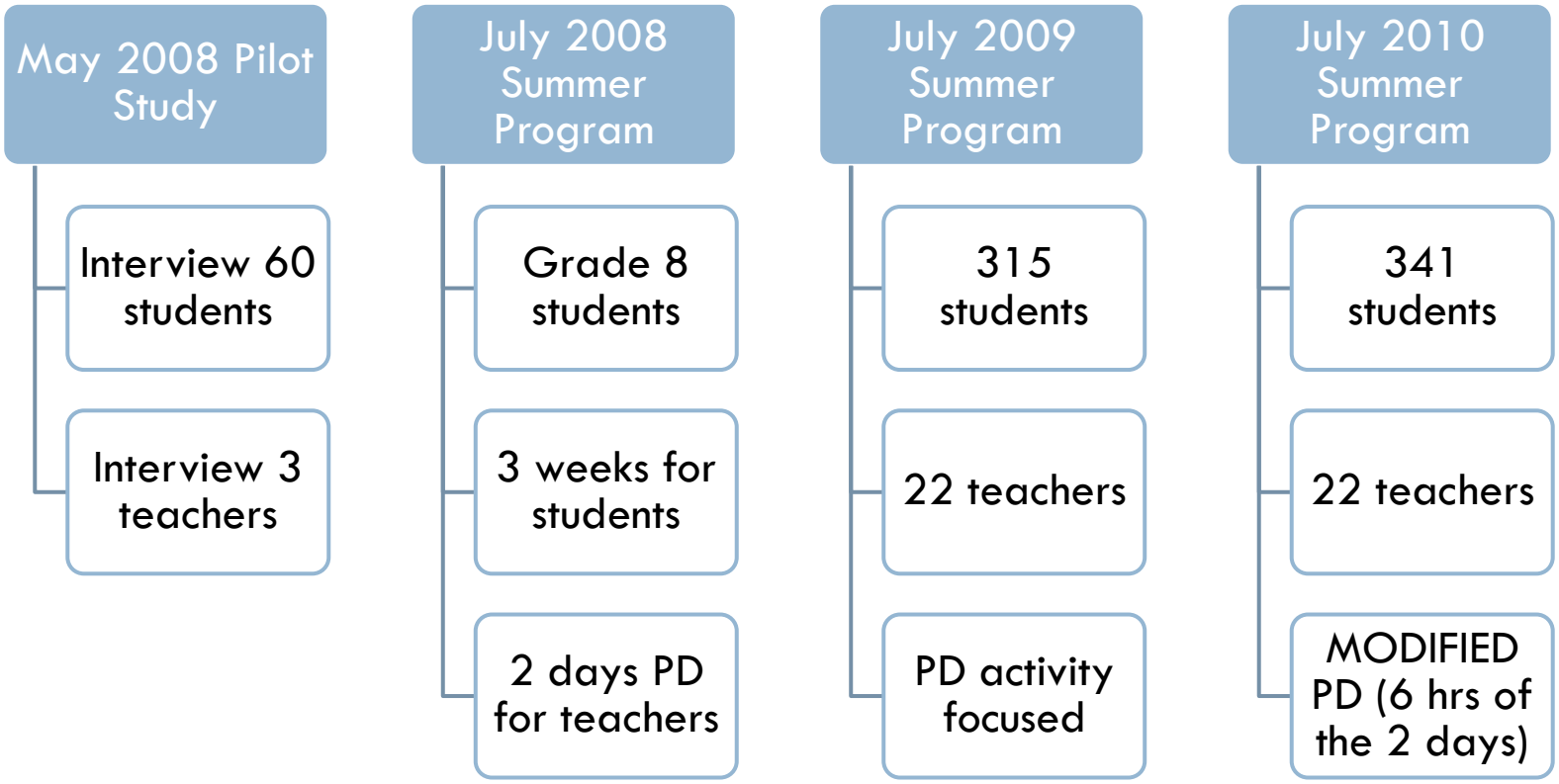
Developing and testing a theory for improving teacher and student understanding of integers

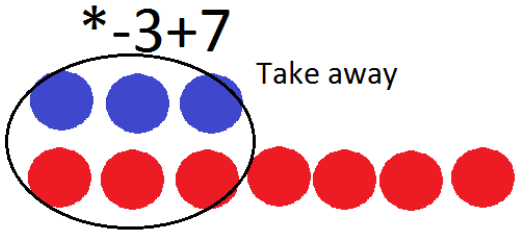


Why integer operations?

- Foundational Mathematic Concept
- Applications in STEM fields
- Impacts accuracy of solution to many problems
- Mathematics Ed community hasn't found an instructional model that works

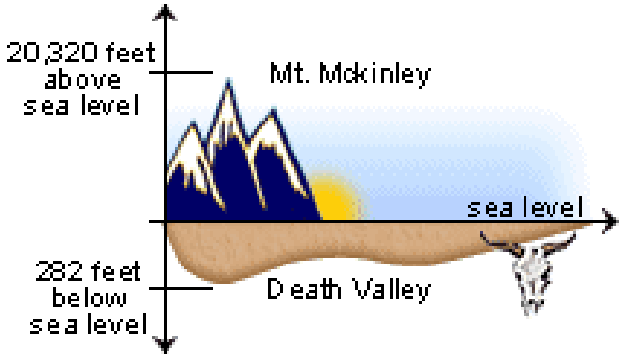
Models for Teaching Integer Operations

Annihilation model



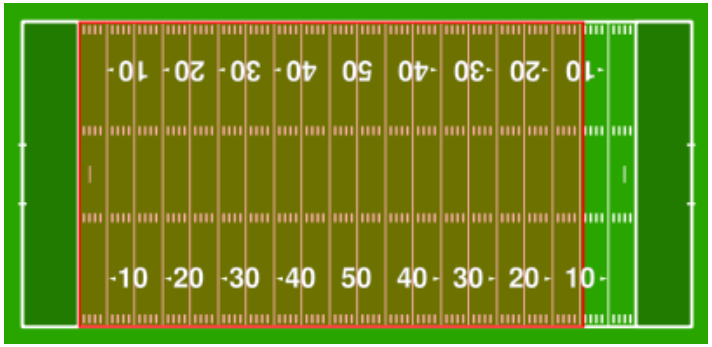
Number line

Elevation or Elevator



Metaphors

Real World Application



Pilot Study- May 2008

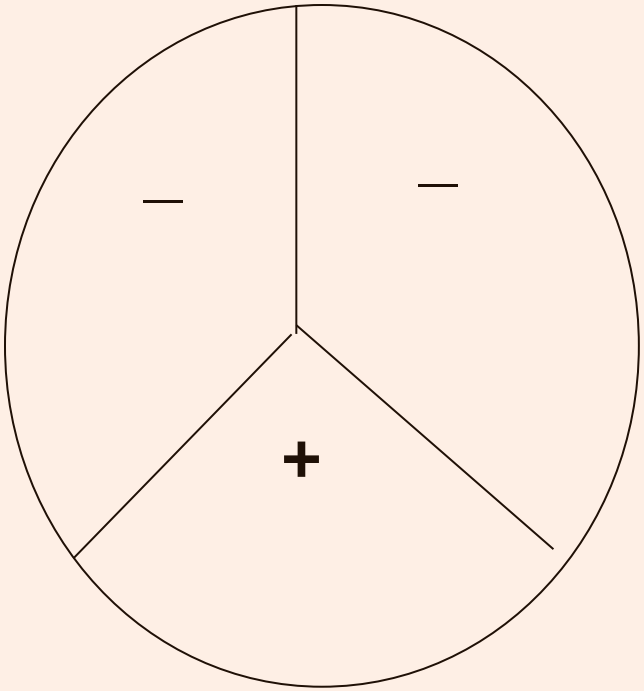
- Structured interviews
- Students in grade 7, 9, 11
- Given an integer operation expression, how would you solve it?

Overall Percent Accuracy of Integer Arithmetic

Accuracy	Grade 7 (*n* = 21)	Grade 9 (*n* = 24)	Grade 11 (*n* = 20)	Total (*n* = 65)
-5 + 8 = 3	62	100	60	74
-3 + -6 = -9	57	79	45	60
2 – 7 = -5	19	63	45	42
-3 – 5 = -8	19	25	35	26
-4 x 5 = -20	76	75	75	75
Total	47	68	55	57

Grade 7 Student

$-5 + 8 = ?$



7th grade student response:
"*I used this*" (points to Pie Man)

"*Negative and positive, I went like this*" (Student covered the negative and positive signs with two fingers)

"*and then you have a negative*"

Pie Man

Theoretical Framework

Pedagogical Content Knowledge (PCK)

- Mathematics
- Representations
- Student's thinking
- Decision Making
- Clarifying examples and counter examples

Argumentation

- Reasoning
- Justifying their thinking
- Making claims and warrants
- Classroom Norms

PROFESSIONAL DEVELOPMENT
July 2010 (6hrs)

- Real world contexts
- Number line vector representation
- Student misconceptions
- Promoting productive classroom discourse

IMPROVED TEACHER PCK
July 2010 (3 wks)

- Implement activities with real world connections
- Implement number line vector model activities
- Facilitate and encourage classroom discourse and argumentation

IMPROVED STUDENT UNDERSTANDING
July 2010 (3 wks)

- Model integer operations number line
- Make connections between different models
- Use argumentation to make claims and warrants for a particular solution and solution strategy

Theory of Change

Summer Program

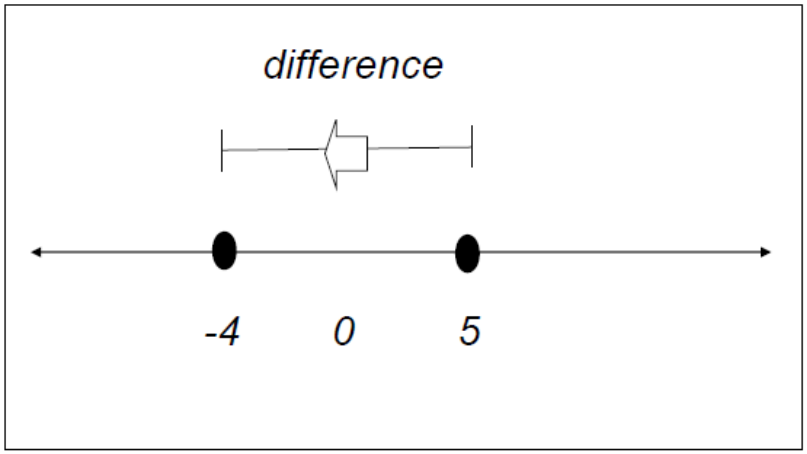
- Grade 8 Students who had not passed the state assessment in mathematics
- Requirement to be promoted to grade 9
- 14 days, 9:15-1:45pm
- Program started in 2008
- Curriculum focus:
  - Generalizing Patterns using Algebra
  - Positive and Negative Numbers

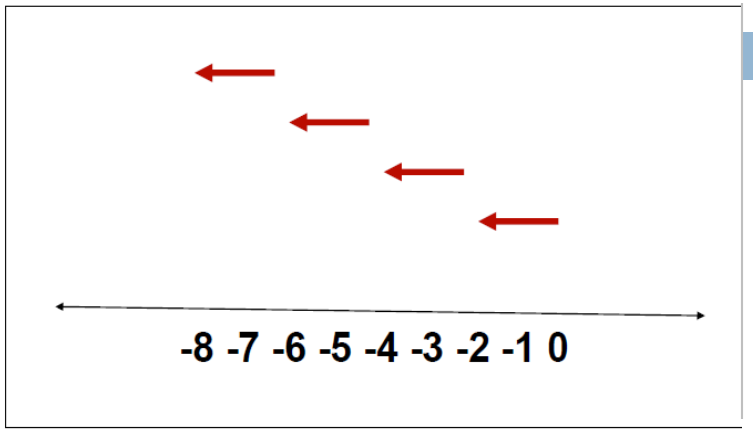
Curriculum

- *America's Choice Navigator*
  - *Generalizing Patterns*
  - *Positive and Negative Numbers*

- 60 minute lessons and activities
  - Misconceptions
  - Student discourse

Subtraction and Multiplication





$^{-}4 - 5 = -9$

$4 \times (^{-}2) = {}^{-}8$

- Purpose of negative numbers
- Comprehensive
- Prepares students for higher math and science

Summer Program 2009

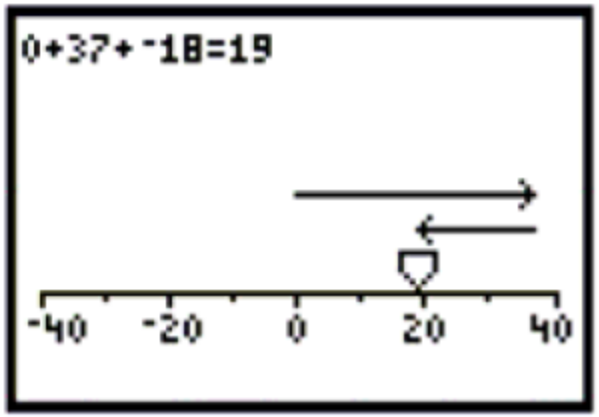
Topics:	Pretest (*n* = 206)	Posttest (*n* = 242)	Growth (*n* = 177)
Positive & Negative Numbers	43%	49%	+6%
Patterns	40%	50%	+10%

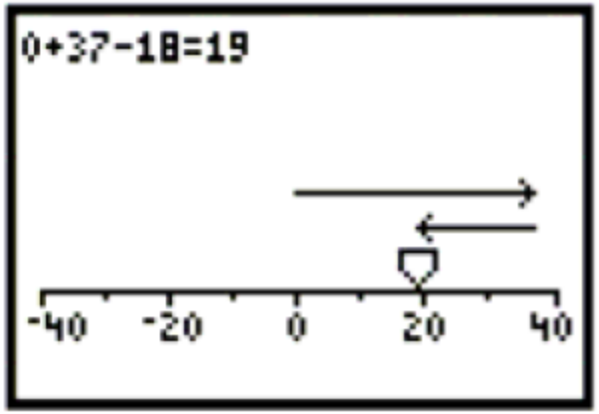
Modified Summer Program with a Focus on Conceptual Understanding and Argumentation

Argumentation (90%)

Teacher Talk (10%)

Addition of TI-73 Calculator NumLine Activities





Research Question #1

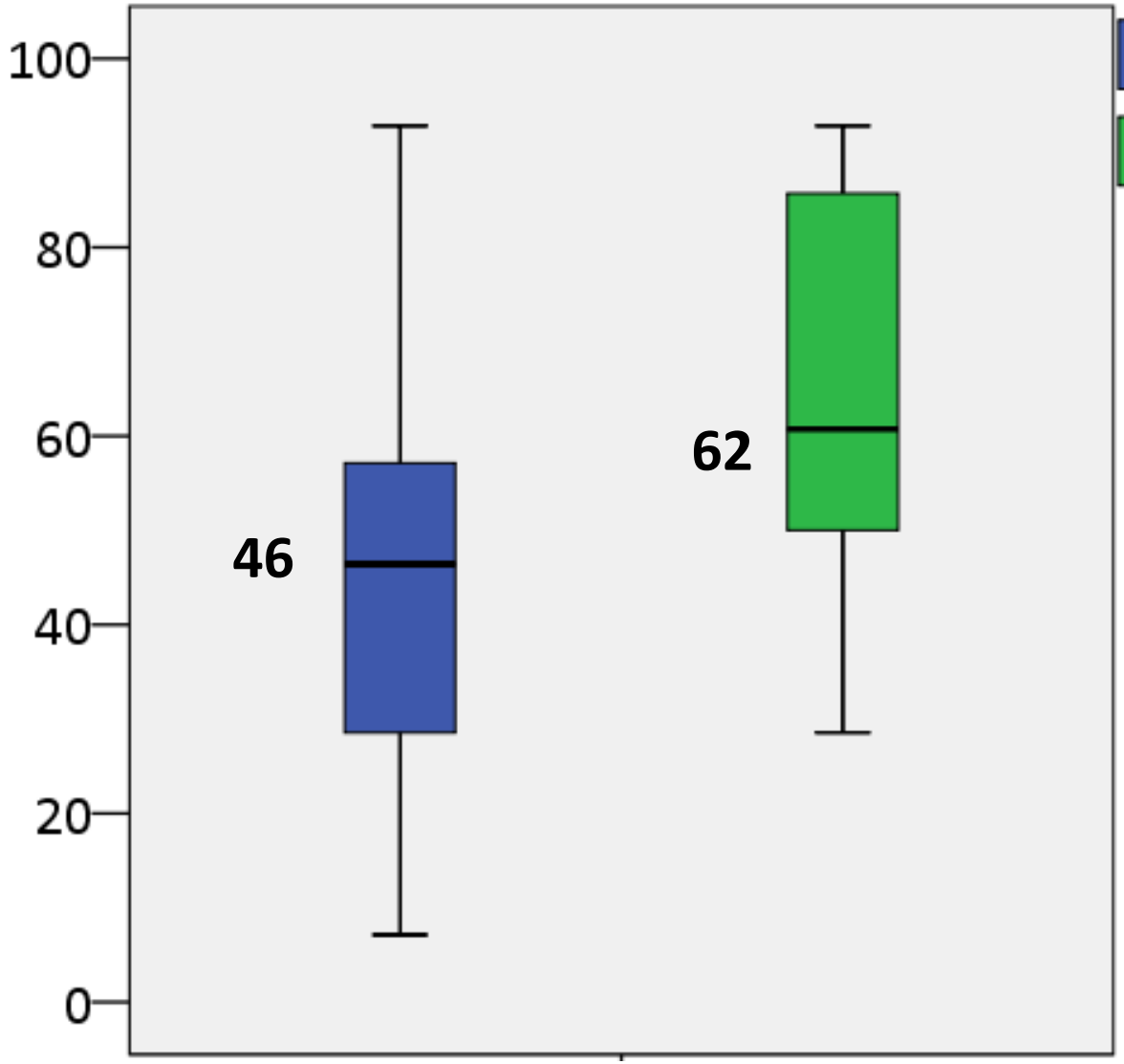
- What are the general patterns of teacher PCK related to integer operations?

Percent of Teachers Who Achieved Ratings

Questions: *(Note: 0 points for incorrect or no response, 1 point for partially correct response, 2 points for complete correct response)*	Pre-test Ratings (*n* = 18 teachers)		
	0	1	2
Explain the solution of 5 – (-8)?	50	33	17
Given -5 x (-8). Why does the answer have the sign it does?	72	17	11
(-6) + (+7) and 6 – (+7) read incorrectly	22	28	50
4 – 7 = 3, what is the misconception and what is a teaching strategy	6	44	50
Is 3 –5 the same as 3 + (-5)?	61	28	11
Prior experience with argumentation in class	33	28	39
Real world and domain applications	11	72	17

Research Question #2

- To what extent did PD change teacher PCK ?



Note: Difference is statistically significant at $p < .01$

Teacher Pedagogical Content Knowledge (*n* = 18)

Change in Teacher Understanding of Integer Operations

Question (N = 18 teachers)	Pretest Mean (*SD*)	Posttest Mean (*SD*)	Difference (SE)	*t*	*p*-value
Q1. 5 – (-8)	.67 (.77)	1.11 (.96)	.44 (.32)	1.41	.18
Q2. -5 x (-8)	.39 (.70)	1.00 (.97)	.61 (.26)	2.37	.03*
Q3. (-6) + (+7), 6 – (+7)	1.28 (.83)	1.83 (.38)	.56 (.17)	3.34	$p < .01$
Q4. 4 – 7 = 3	1.44 (.62)	1.61 (.61)	.17 (.12)	1.37	.19
Q5. 3 – 5, 3 + (-5)	.50 (.71)	.89 (.68)	.39 (.20)	1.94	.07
Q6. Prior use of Argumentation	1.06 (.87)	1.22 (.88)	.17 (.20)	.83	.42
Q7. Applications	1.06 (.54)	1.33 (.59)	.28 (.11)	2.55	.02*

Research Question #3

- Is there a statistically significant difference between Jumpstart 2010 and Jumpstart 2009 in student performance?

Hierarchical Linear Model (HLM) for 2009 vs. 2010 Comparison

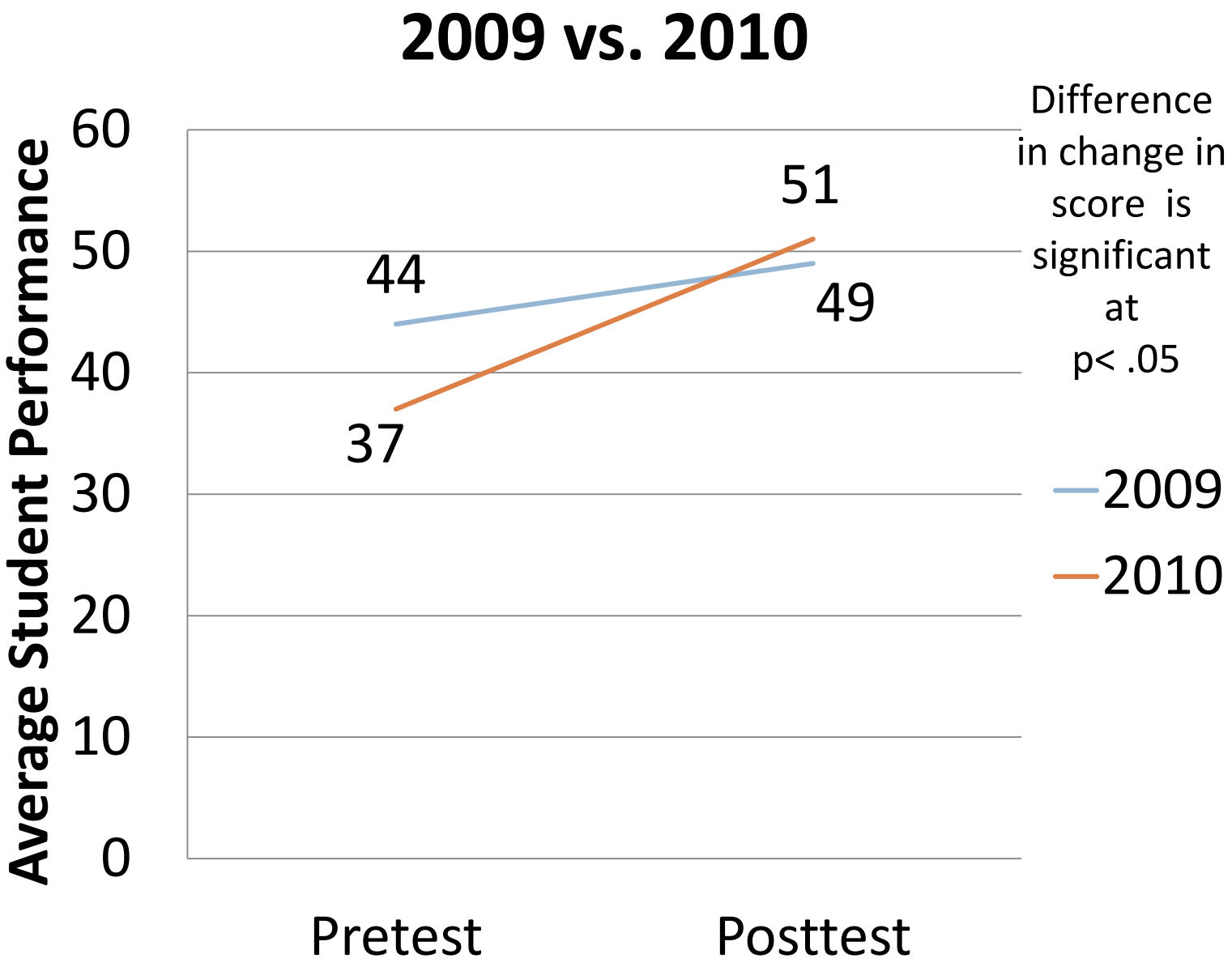
Level-1 Model

$$Y_{ij} = \beta_0 + r_{ij}$$

Level-2 Model

$$\beta_0 = \gamma_{00} + \gamma_{10} \text{ (YEAR) } u_{0j}$$

Y_{ij} was used to represent each outcome measure (pretest and posttest) and the change in score of students between pre- and posttest.


2009 vs. 2010
Average Student Performance
60
50
40
30
20
10
0
44
51
49
37
Pretest
Posttest
Difference in change in score is significant at p< .05
2009
2010

Percent Correct out of 100	2010 Mean (*n*=177) (*SD*)	2009 Mean (*n* = 177) (*SD*)	Difference (SE)	*t*	*p*-value
Pre-test	37 (17)	43 (19)	-7 (4)	-1.77	.08
Posttest	51 (15)	49 (21)	2 (4)	.41	.68
Improvement	14 (17)	06 (19)	8 (3)	2.20	.03*

*Statistically significant at the $p < .05$ level.

Research Question #4

- Do differences in teacher PCK explain more of the variance in student performance than years teaching experience?

HLM Analysis to Model Posttest Fully Conditional Model

Level-1 Model

$$Y_{ij} = \beta_0 + \beta_1 * (\text{Student Pretest}) + r_{ij}$$

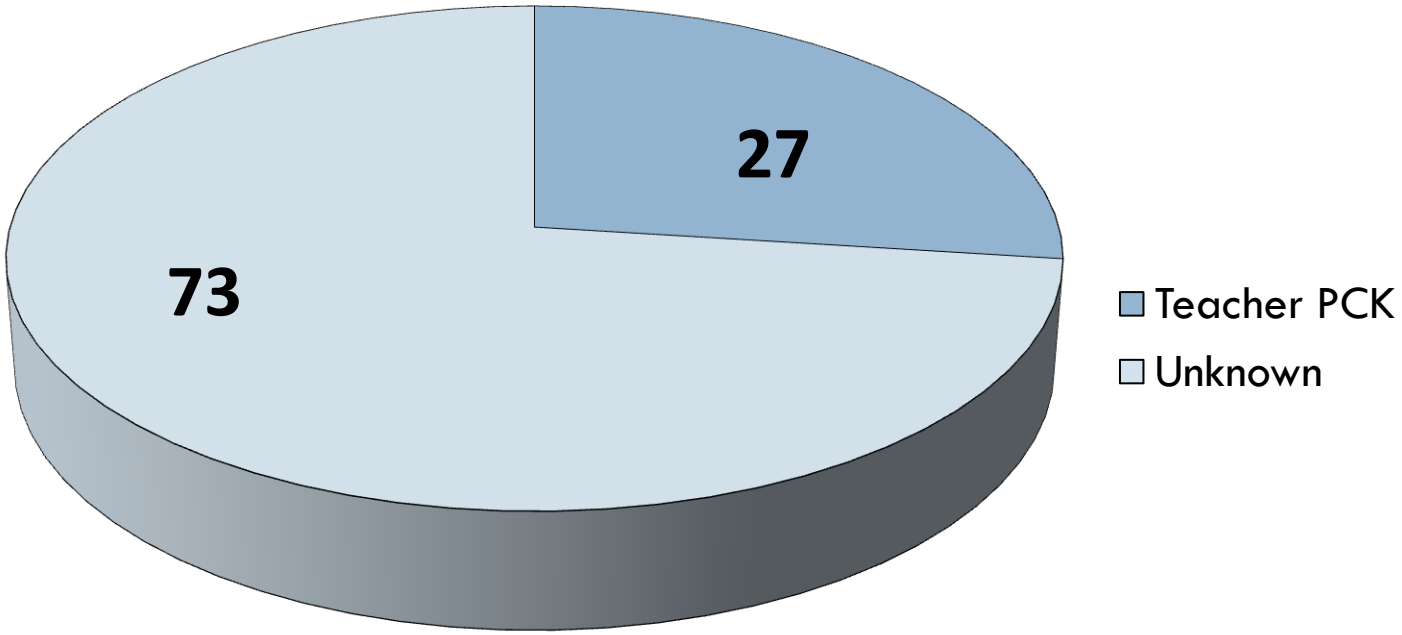
Level-2 Model

$$\beta_0 = \gamma_{00} + \gamma_{01} * (\text{Teacher Experience}) + \gamma_{02} * (\text{Teacher PCK Pretest}) + \gamma_{03} * (\text{Teacher PCK Posttest}) + u_{0j}$$

$$\beta_1 = \gamma_{10}$$

Y_{ij} is the posttest score of student *i* in class *j*

Percent of Level-2 (teacher) Variance in Student Knowledge



Controlling for prior student and teacher knowledge as well as teacher experience, teacher's pedagogical content knowledge (PCK) significantly predicted student posttest performance (p=.033) (1 pt increase in PCK, .22 increase in student performance)

Implications for Future Directions

- Implications for Equity for All Students
  - Focus on conceptual development not activities
  - Focus on argumentation
- PD for in-service and pre-service teachers
- Measuring PCK
- Supporting teachers in using argumentation in the classroom

Part 4: Questions & Discussion