Where in the World Are We?

A Performance Task

for students in grades 5 and/or 6

HELP WANTED – GPS REPAIR PERSON

NASA has received word that the Global Positioning System is malfunctioning. All over the world people are getting lost because their GPS devices are giving them the wrong information. The people at NASA are all completely busy planning a Mars mission, so they need to hire additional help to address the problem with the GPS. You have applied for the job, and now you are preparing for your interview. At your interview you will be expected to solve a simulated problem to show that you understand how the GPS system works. The simulated problem is, GPS receivers are producing incorrect locations. They are producing locations that are not even on earth! If you show enough understanding and skill, NASA may very well hire you for the position.

To prepare for your interview, work through the following explorations. Each one will help you understand a part of what you need to know in order to solve NASA's problem.

After you complete the explorations, you need to make a presentation in which you demonstrate to the NASA administrator that you have the necessary skills and knowledge for the job. In your presentation you must show that you have found the problem with the malfunctioning GPS. In your presentation you need to explain how a GPS receiver calculates distances to satellites. You must show that you know what error the system is making.

Your presentation must meet the following criteria in order for you to receive final consideration for the job:

- You must explain *correctly* how a GPS receiver calculates its distance to satellites.
- You must demonstrate *at least three correct calculations* of distance, selecting the appropriate data for these calculations from a data table.
- You must explain trilateration and how it is that the GPS system is designed to work.
- You must correctly diagnose the system in order to explain how it is that the system is producing locations that are not even on earth.

Good luck! I hope you get the job! NASA is supposed to be a *great* place to work!

Exploration #1: Finding your Place

How do satellites and your GPS receiver figure out where on earth you are? Before we can answer that question, let's answer a simpler question. Suppose you did not have a GPS receiver, and you were lost somewhere in the USA. But suppose that you could find out (somehow) exactly how far you were from a specific location. Would that help you out? For example, let's say that you knew you were 500 miles from Chicago, IL?

Would that tell you your exact location? No, you could be anywhere on a circle that is 500 miles away from Chicago. Using your compass, draw a circle centered on Chicago and at a distance of 500 miles from Chicago in every direction.

What if you *also* knew you were 600 miles from Dallas, TX?

Now do you know where you are? Draw a circle centered at Dallas and at a distance of 600 miles from Dallas all the way around. As you can see by the map, there are exactly two places that are *both* 500 miles from Chicago *and* 600 miles from Dallas. But which one is your location? You might be able to tell, if you know something about geography, but other than that, it would be guesswork. You need more information.

Suppose you *also* knew that you were 400 miles from Minneapolis, MN?

Draw a circle that is centered at Minneapolis and is a distance of 400 miles from Minneapolis in every direction.

As the map shows, there is *only one* location that is 500 miles from Chicago, *and* 600 miles from Dallas, *and* 400 miles from Minneapolis! You *must* be in Omaha, NE!

So – with one known distance, you could have been anywhere on a circle. With two known distances, you could have been on either of two places where the two circles intersect, but with three known distances, you can only be in the one place where all three circles intersect. This is a lot like what the satellites used in the GPS system do. They are used to give you at least three known distances from your current location. Instead of giving you measurements from cities to your location, they give you distances from themselves, satellites in orbit above the earth, to your location. Your GPS receiver just measures those distances. As long as your GPS receiver can measure the distance to three different satellites, it can find your location! Oh, there is one difference between our map with circles and what GPS satellites do. Since GPS satellites are in space, they are at the center of a *sphere* instead of being at the center of a circle. (A sphere is a 3-D shape, and a circle is only a 2-D shape. A sphere is a lot like a ball.) Where two spheres intersect, there are not just two points, there is a whole circle. Where three spheres intersect, there is not just one point, there are two. However, one of the two points will always be on earth, and the other will be in space. The GPS receiver has a database that includes all the locations on earth. It uses that database to look up the two locations where the three spheres intersect, then it just ignores the location that is in space.

There is a fancy word for this process you just learned about. That word is *trilateration*. Say it: try – la (sounds like the a in hat) – ter – a – shun.

Exploration #2: Learning about Distance, Time, and Rate in an e-Example

Before we can begin to work on the GPS satellite, we need to make sure we understand a few things about speed and distance. We'll use some make-believe runners to get the necessary background understandings.

Go to the e-Example applet http://www.nctm.org/standards/content.aspx?id=25037

1. Click on the male runner to switch his direction. Both runners must be facing to the right.
2. Drag both runners to the left side of the track. They must both be starting out at the same location.
3. Click the "play" button and watch the runners cross the screen. As they cross the screen, study the graph that is being produced. What do you notice? If you need to, play it several times so that you can see how the graph relates to the action.
4. To deepen your understanding of the relationship between the graph and the action, try changing the "step size" and re-running the action. Try it with larger step sizes, and then try it with smaller step sizes. Predict how the differences in step sizes will affect the graph before you hit the "play" button.
5. How does the graph change with larger step sizes?
6. How does the graph change with smaller step sizes?
7. Now change the step size for only ONE of the runners. Give one of the runners a step size of 2 and the other runner a step size of 1. Predict how this change will affect the outcome, both in terms of the action and of the graph. After you make your prediction, press "Play." Were your predictions correct?

On a table or a spreadsheet, keep track of several explorations in which you vary only the step size of one of the runners. Here is an example of an exploration where a person kept the red runner's step size at 2 while increasing the step size of the blue runner. You may want to try this exploration, or you may want to try one of your own. Fill out this chart, or make and fill out one like it of your own.

	Step Size	Time	Distance
red	2		
blue	1		
red	2		
blue	3		
red	2		
blue	4		
red	2		
blue	5		
red	2		
blue	10		

8. After exploring the effect of changing the runner's step size, what conclusions can you draw about the relationship between step size, time, and distance?

9. What do you notice about the distance?

10. When step size is larger, what happens to the time?

11. When step size is smaller, what happens to the time?

12. Can you complete the following: When distance is kept the same and speed (step size) increases, the time ______________________. When distance is kept the same and speed (step size) decreases, the time ________________________.

* For step #12, if you said that time decreases when speed increases, you were correct! If you said that time increases when speed decreases, you were correct! But then, that just makes sense, doesn't it? Keep it in mind, however.

Now let's explore what happens when we keep step size (speed) the same but we vary the distance that the runners have to go. To do this, set the step size to 2 for both runners. Drag one runner all the way to the left, but drag the other runner only half-way to the left. The starting position for this runner should be the 50-mark.

1. Predict what will happen before you hit "play."
2. Was your prediction correct?
3. Even though the distance for both runners is shown as 100, did they both run 100 steps? You would need to subtract their starting position from 100 to find out how many steps they actually went. How far did the two runners actually go?
4. Explore what happens when the distances vary. Here are some starting points for you to try:

	Step Size	Time	Starting Point	Distance (100-Starting Point)
red	2		0	100
blue	2		50	50
red	2		0	100
blue	2		40	60
red	2		0	100
blue	2		30	70
red	2		0	100
blue	2		20	80
red	2		0	100
blue	2		10	90

5. This table has already calculated the distance by subtracting the starting point from 100. You should fill in the times that the runners ran in order to complete the table.

6. What do you notice about the relationship between the time and the distance when the speed is kept the same? Make sure you are examining the *distance* and not the starting point.
7. Can you complete the following: When speed is kept the same and distance increases, the time ______________________. When speed is kept the same and distance decreases, the time ________________________.

* For step #7 above, if you said that time increases when distance increases, you were correct! If you said that time decreases when distance decreases, you were correct! But again, that just makes sense, doesn't it?

At each of the * points above, you were noticing some things that just make sense. But how about the math? Did you notice that there is a way to use math to figure out *exactly how much* the speed or distance or time will increase or decrease? Look at your tables again. Look at the speed column, the time column, and the distance column. Did you notice that you can *always* multiply the speed by the time to get the distance? Knowing this can help you make perfect predictions about the runners' times and distances every time. Try it and see. Experiment with many different speeds and distances. See if you can predict perfectly every time.

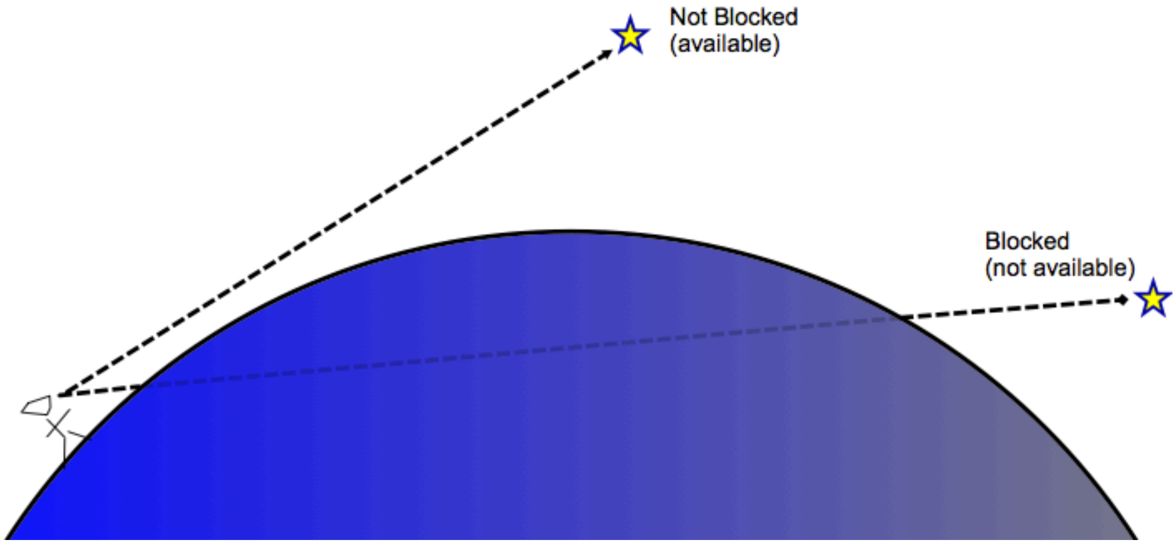
Mathematicians and scientists use a formula to use what you've just learned. That formula is $r \times t = d$. In the formula, r stands for rate (speed), t stands for time, and d stands for distance. So, the formula just means, rate times time equals distance. That's exactly what you just figured out!

Exploration #3 – Satellites, Clocks, and Calculations

So, how does $r \times t = d$ and trilateration help you understand what's wrong with the worldwide GPS system? These understandings are building blocks, but you need one more piece of knowledge before you can figure out what's wrong with the GPS. You need to understand how satellites and your GPS receiver work together to find distances.

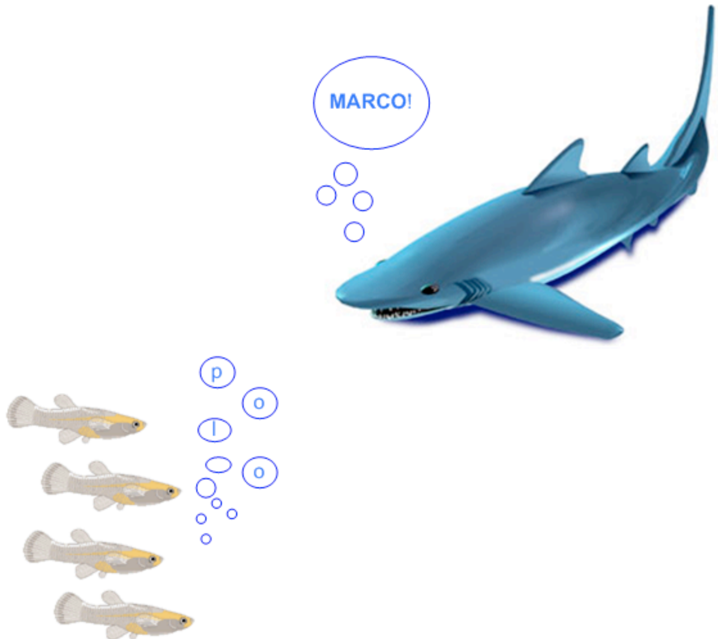
The Satellites

There are over 24 satellites at work at any given moment in the GPS system. All of these satellites are in orbit around the earth, and they are spread out so that there are at least four of them available to your GPS receiver at any given point in time. In order to be "available" to your GPS receiver, there needs to be a straight line between your receiver and a satellite, with nothing in the way to block a signal. If you are in a building, the building blocks the signal. If a satellite is on the other side of the earth, or even just beyond the horizon, the earth itself blocks the signal.



So, as long as there is a straight line of open space between your receiver and at least four satellites, your receiver can figure out your position by measuring the distances to the satellites and using trilateration. But how? How does your receiver measure those distances?

Have you ever played Marco Polo in the summertime?



If you are "Marco" you can start to figure out where the other players are by listening to their "Polo's." Your GPS receiver doesn't have to say "Marco," however. The satellites are constantly saying "Polo" without having to be asked. Well, they don't exactly say, "Polo," but you get the idea. Each satellite is constantly sending out packets of information about itself. Your GPS receiver *receives* that information from all available satellites. The packets of information include some identification information so that your receiver can figure out which satellite sent the information.

The Clocks

On board each satellite is an extremely accurate clock. The reason for the clock is so that each GPS satellite can send out its information packet at exactly the same moment. The signals all travel at exactly the same speed. If the signals all start out from all the satellites at the exact same moment, then the signal from the closest satellite will get to your receiver first. Then, a *really, really* short time later, the signal from the next closest satellite will arrive. Then, the signal from the next closest satellite will arrive. The signals travel *really, really* fast[1], and there isn't much time between when they arrive, but your GPS receiver can measure that amount of time.

[1] 186,000 miles per second!

Putting it all Together

This is where you need to put together the things you learned in Exploration #1 and Exploration #2. How does the GPS receiver calculate distances to satellites in order to use trilateration? Remember $r \times t = d$? The rate at which the signals travel is exactly the same for all the satellites. They all start out at the same moment, and the GPS is able to measure how long it took for the signals to arrive. Each signal has a different time because each satellite is a different distance away. So, the three distances are simply the rate times the three different times. Voila!

You may want to go back to the rate, time, distance applet (http://www.nctm.org/standards/content.aspx?id=25037) again and think of it in a new way. Instead of runners, think of it as a way of representing the GPS satellites. When you start the two runners in different positions, it is like having two satellites that are two different distances away. When you press "play," it is like the signals being sent out at exactly the same moment. You can see that the closer signal gets to the end (your GPS receiver) first. You can watch as the second signal travels further and takes longer. You can see that the two times are recorded. You can multiply the rate times the time to get the distance. This is what happens inside your GPS receiver.

NASA's Data

The following data table is from one of the GPS receivers that has been leading people astray. The data table shows you all the information from all 24 satellites at a specific moment in time. Can you find out what is wrong?

Satellite Identification Information	Rate	Time (in seconds) A value of 0 indicates a satellite that is not available.	Distance (in miles)
Satellite #1	186K miles/second	0	xxxxx
Satellite #2	186K miles/second	0	xxxxx
Satellite #3	186K miles/second	0.002	3720
Satellite #4	186K miles/second	0	xxxxx
Satellite #5	186K miles/second	0	xxxxx
Satellite #6	186K miles/second	0	xxxxx
Satellite #7	186K miles/second	0.008	14880
Satellite #8	186K miles/second	0	xxxxx
Satellite #9	186K miles/second	0	xxxxx
Satellite #10	186K miles/second	0	xxxxx
Satellite #11	186K miles/second	0	xxxxx
Satellite #12	186K miles/second	0	xxxxx
Satellite #13	186K miles/second	0.007	13020
Satellite #14	186K miles/second	0.003	5580
Satellite #15	186K miles/second	0	xxxxx
Satellite #16	186K miles/second	0.003	5580
Satellite #17	186K miles/second	0.004	7440
Satellite #18	186K miles/second	0	xxxxx
Satellite #19	186K miles/second	0.001	1860
Satellite #20	186K miles/second	0	xxxxx
Satellite #21	186K miles/second	0.009	16740
Satellite #22	186K miles/second	0	xxxxx
Satellite #23	186K miles/second	0	xxxxx
Satellite #24	186K miles/second	0	xxxxx

Your Solution

On the following lines, write your explanation of what you think is wrong with the data that the GPS receivers are receiving. Make sure you comment on whether there is a single satellite that seems to be malfunctioning or if they all seem to be having a problem. Speculate on whether the problem is more likely to be originating in the satellite(s) or in the receiver(s). If the problem is a single satellite, tell NASA which one needs to be fixed. If the problem seems more likely to be in the receivers, then NASA needs to find out if the faulty receivers are all from a single manufacturer. If so, then that manufacturer may have a faulty database that they are programming into the receivers. Explain to NASA what you think the problem might be, and give them some suggestions for what they might do next.

In your explanation, be sure to include the following. These are the things NASA is looking for to decide whether or not you will get the job:

- You must explain correctly how a GPS receiver calculates its distance to satellites.
- You must demonstrate at least three correct calculations of distance, selecting the appropriate data for these calculations from a data table.
- You must explain trilateration and how it is that the GPS is designed to work.

Where in the World are We?

A Performance Task for Students in Grades Five and/or Six

Teachers' Edition

CCSS Citations

***6.RP.3.b** – Understand ratio concepts and use ratio reasoning to solve problems.*

3. Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

    b. Solve unit rate problems including those involving unit pricing and constant speed.?

***6.EE.9** - Represent and analyze quantitative relationships between dependent and independent variables.*

9. Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation $d = 65t$ to represent the relationship between distance and time.

ISTE Citations

2. Communication and Collaboration

Students use digital media and environments to communicate and work collaboratively, including at a distance, to support individual learning and contribute to the learning of others.

4. Critical Thinking, Problem Solving, and Decision Making

Students use critical thinking skills to plan and conduct research, manage projects, solve problems, and make informed decisions using appropriate digital tools and resources.

Enduring Understanding(s)

The deductive logic of trilateration enables navigators or a technological system to determine a precise location on earth.

The relationship between rate, time, and distance, as expressed in the formula $R \times T = D$, is a powerful concept, enabling us to find an unknown distance when rate and time are known.

Essential Questions

How does trilateration enable us to determine with certainty a precise location on earth?

How are rate, time and distance related? How can we make use of that relationship to determine an unknown, given two components of that relationship?

Learning Outcomes

Students will be able to explain the relationship between rate, time, and distance.

Students will be able to solve for d in the formula $r \times t = d$, when provided with r and t.

Students will be able to explain the logic of trilateration in its use in a GPS location system.

Students will be able to apply mathematical reasoning and analysis to locate mathematical errors.

Materials

- Class set of compasses
- Access to internet for http://www.nctm.org/standards/content.aspx?id=25037
- Access to spreadsheet software

GRASPS

Goal: To get hired by NASA as a GPS repair person. In order to be hired, you need to find out what is wrong with a malfunctioning GPS satellite to show that you have the necessary skills for the job.

Role: You are a job applicant for the position of GPS repair-person.

Audience: The NASA administrator in charge of filling the vacancy for GPS repair person.

Situation: NASA has received word that the Global Positioning System is malfunctioning. The people at NASA are all completely busy planning a Mars mission, so they need to hire additional help to address the problem with the GPS. You have applied for the job, and now you are preparing for your interview. At your interview you will be expected to solve a simulated problem to show that you understand how the GPS works. The simulated problem is, GPS receivers are producing incorrect locations. They are producing locations that are not even on earth! If you show enough understanding and skill, NASA may very well hire you for the position.

Product, Performance, or Purpose: You need to make a presentation in which you demonstrate to the NASA administrator that you have the necessary skills and knowledge for the job. In your presentation you must show that you have found the problem with the malfunctioning GPS. In your presentation you need to explain how a GPS receiver calculates distances to satellites. You must show that you know what error the system is making.

Standards/Criteria for Success: Your presentation must meet the following criteria in order for you to receive final consideration for the job:

- You must explain correctly how a GPS receiver calculates its distance to satellites.
- You must demonstrate at least three correct calculations of distance, selecting the appropriate data for these calculations from a data table.
- You must explain trilateration and how it is that the GPS is designed to work.
- You must correctly diagnose the system in order to explain how it is that the system is producing locations that are not even on earth.

Resources

NCTM e-example Simulation

http://www.nctm.org/standards/content.aspx?id=25037