steps
PROFESSIONAL DEVELOPMENT

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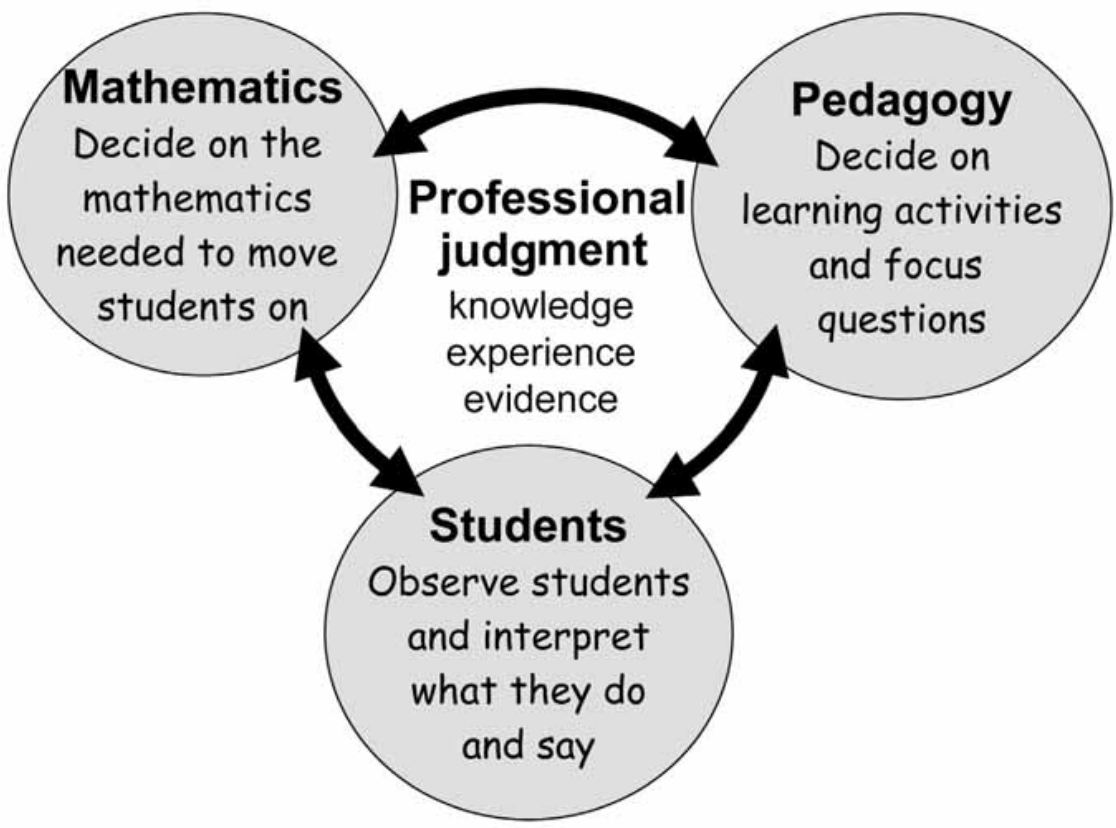
NCTM Conference Presentation

From Telling to Asking:

Using Focus Questions to Differentiate Instruction

Improving the mathematics outcomes of students

Planning Cycle



Diagnostic TASK

FOCUS
Understand Numbers
- Key Understanding 2

Emus/Rabbits/Sheep

Years/Grades 2–4

Emus may be replaced by any animal that lives in small groups, e.g. monkeys in two trees.

Purpose

To find out whether children can partition using materials, or with numbers

Equipment

Sheet of paper
Collections of countable objects if necessary

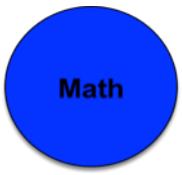
Producing work samples

Have the children create partitions of the same size collection in response to a story, e.g. read 'Edward the Emu' or 'Edwina The Emu' by Sheena Knowles, 'Peter Rabbit' series by Beatrix Potter.

Ask the child to show all the different ways they could put 12 emus/rabbits in two yards/cages. At first limit the child to paper and pen, if they struggle, then suggest that they use some materials to help.

Observe if the child:

- uses numbers alone by recalling basic facts
- uses numbers alone, moving 'one' from this number to that number
- uses numbers alone, counting on from one number to find the next
- draws lines, dots or other symbols and counts, and then record their partitioning
- uses materials to count and then record their partitioning.



Understanding the Mathematics

To partition a number:

- It is easier to see how many there are when collections are in special arrangements
- Thinking "part-part-whole" can help us to see "how many there are"
- The same number can be thought of in parts in different ways
- A number can also be thought of in more than two parts

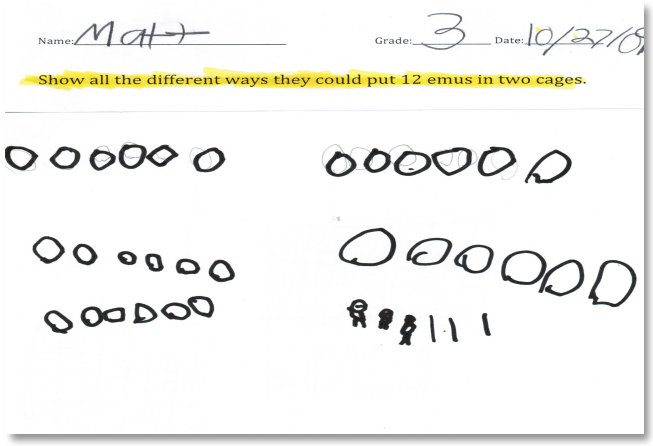
To represent a solution:

- Initially students should solve problems by acting them out**, modeling with materials and diagrams**
- Students can link various problem situations to the addition and subtraction symbols **with the aid of diagrams**
- Students deal with all the problem types. **They may need to "imagine diagrams" for larger numbers**
- Students generalize relationships between addition and subtraction in an **abstract way**

Diagnostic Map

Supporting Judgments about Student Learning

5-9 years old	7-11 years old	9-13 years old
•Students are learning to "see" that two looks different than three and connect this to the counting sequence. •Students should work on extending subitizing to collections of five, six, and beyond	Students learn to "interpret" small numbers as compositions of other numbers	Make sense that any whole number can be rewritten as the addition of other numbers

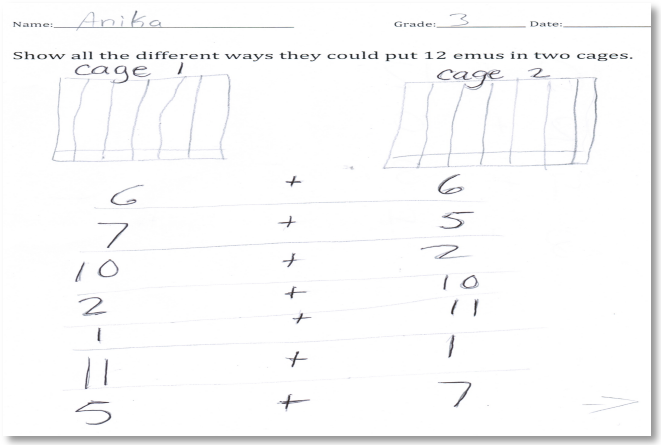


What does Matt need to learn next?

A FOCUS QUESTION for Matt…..
What would happen if we put 5 emus in one cage? How many emus would we have for the other cage?

What does Nimra need to learn next?

A FOCUS QUESTION for Nimra….
Could you represent your thinking without drawing emus?



What does Anika need to learn next?

A FOCUS QUESTION for Anika…
What would happen if we had three cages?

Focus Questions

Open-Ended Questions	Focused Mathematical Questions
How many ways can you put 12 emus in 2 cages?	What would happen if we put 5 in one cage? How many would go in the other cage?
	Could there be more than one way?
	How do you know you have all of the ways?
Show me how you arranged the emus?	Could you represent your thinking without pictures?
	Could you use numbers to represent your thinking?

Open-Ended Questions	Focused Mathematical Questions
Why did you put the sticks on your tape like that?	Does it matter if there are spaces between the popsticks?
	Does it matter if the spaces are not all the same size?
	Does it matter if the sticks overlap or are crooked?
What are the numbers on your tape for?	What would happen if you wrote the number in the space rather than at the end of the unit?
	Does it matter where the numbers go on your measuring tape?

Diagnostic TASK

FOCUS
Direct Measure
- Key Understanding 4
- Did You Know? p. 136

Broken Ruler

Years 3–7

Purpose

To reveal if the student:
- can use the marks on the ruler to measure in centimetres
- understands how the number on the scale relates to the units.

Materials

For small groups or the whole class, use the attached worksheet.

For individual interviews, use a broken piece of ruler and an object to measure.

Producing work samples

It is important that the students use the picture of the broken ruler and **do not** use their own ruler to work out the length of the leaf.

Use this task in an individual interview, or with a small group or whole class.

Individual interview
Provide a broken ruler and ask the student to measure an object that is shorter than the piece of ruler.

Small group or whole class
Read out the problem while the students follow on the sheet. Ask them to write a full explanation of how they worked it out. It may be necessary to do some follow-up interviews to clarify what individual children are thinking.

After the students have found an answer
Ask, *How did you work out the answer?*

Record what the students do and say.

Observe whether they count the spaces or the marks, or use the numbers and any other operation to work out the size of the leaf. Do they count the starting number as zero or as one?

Excerpts taken from First Steps in Mathematics
Department of Education and Training of Western Australia, 2007

Broken Ruler

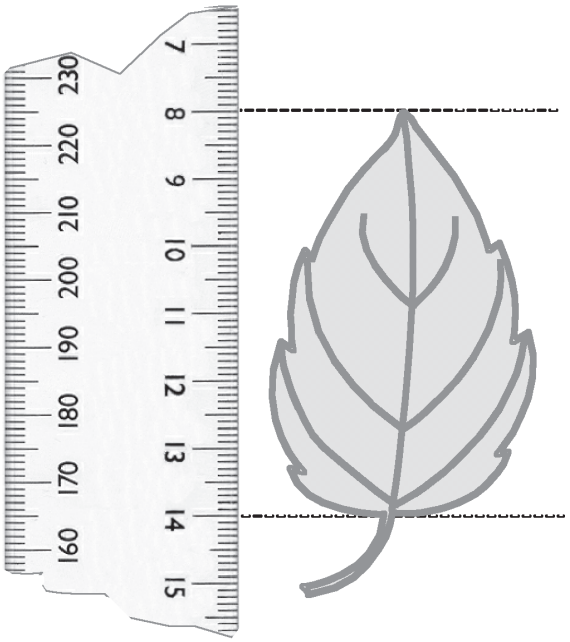
Name _____ Year _____ Date _____

(Do not use another ruler for this.)

Andrea wanted to measure the length of the leaf she had collected for Science.

All she could find was a broken ruler.

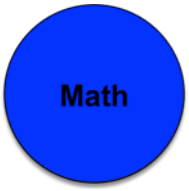
She lined up the ruler and the leaf like this.



Can you say how long the leaf is, using the broken ruler?

The leaf is _____ cm long.

Explain how you got this.



Understanding the Mathematics

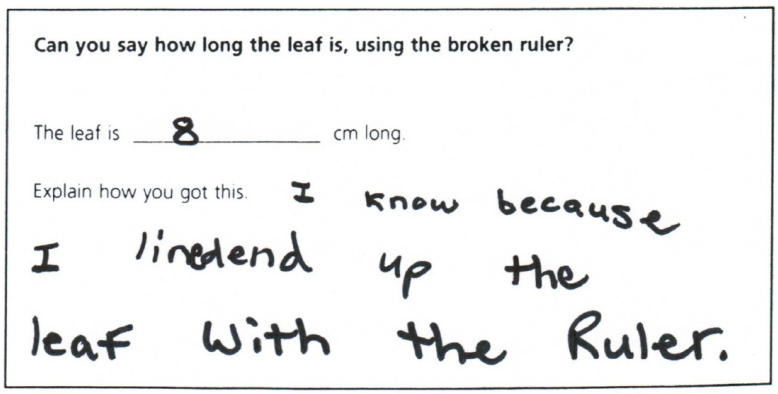
To link a conventional ruler with units of length, students need to:

- move from using multiple copies of a unit of length to work out 'how many fit' to
- the idea that you can use one copy of the unit and 'mark it off' along the thing to be measured to
- making a tape measure using a unit.

Diagnostic Map

Supporting Judgments about Student Learning

5-9 years old	7-11 years old	9-13 years old
Count units and call it measuring	While many have learned to use the marks on a conventional ruler to 'measure' lengths, often do not see the connection between this process and the repetition of units Connect the repetition of a unit with the numbers on a whole number calibrated scale	May interpret whole numbered marks on a calibrated scale as units but may not interpret the meaning of unlabeled graduations Interpret the unnumbered graduations on a familiar whole number scale



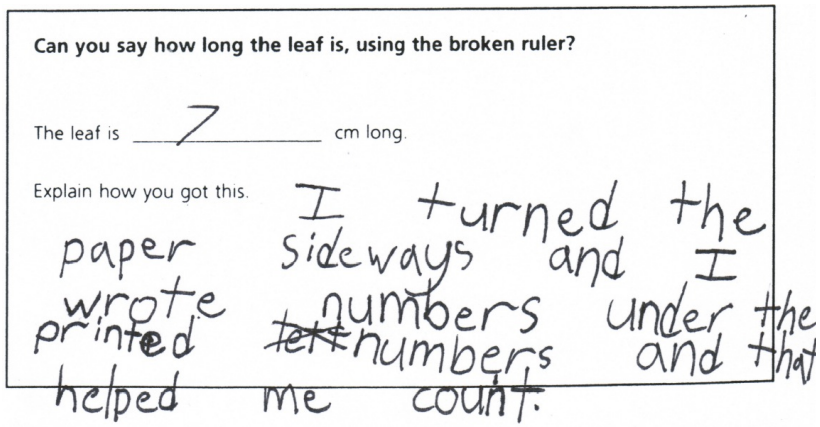
Can you say how long the leaf is, using the broken ruler?

The leaf is 8 cm long.

Explain how you got this. I know because I linedend up the leaf with the Ruler.

What does Alicia need to learn next?

A FOCUS QUESTION for Alicia… *What are we counting when we measure?*



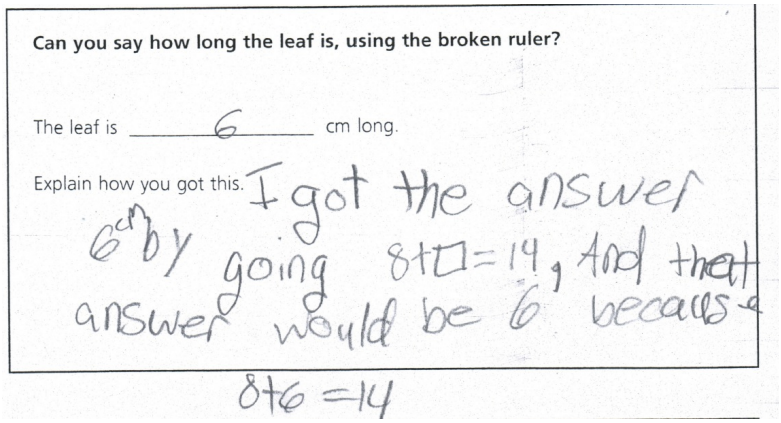
Can you say how long the leaf is, using the broken ruler?

The leaf is 7 cm long.

Explain how you got this. I turned the paper sideways and I wrote numbers under the printed ~~tek~~ numbers and that helped me count.

What does Connor need to learn next?

A FOCUS QUESTION for Connor… *Where does the first unit begin? Where does the unit end?*



Can you say how long the leaf is, using the broken ruler?

The leaf is 6 cm long.

Explain how you got this. I got the answer 6cm by going 8+□=14, And that answer would be 6 because

8+6=14

What does Amy need to learn next?

A FOCUS QUESTION for Amy… *Can you tell me what the long line between 11 and 12 means?*

SAMPLE LESSON 2

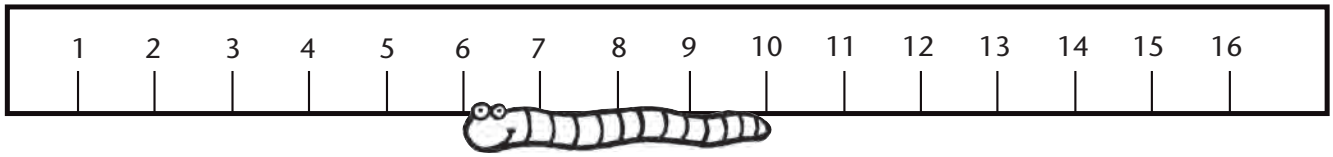
Sample Learning Activity: Middle—'Matchstick Tapes', page 128

Key Understanding 4: Calibrated scales can be used as a substitute for repeating units when measuring length, capacity, mass, angle and time.

Working Towards: Levels 2 and 3

Teacher's Purpose

My Year 4 students found the following drawing in an old maths book.



The probing questions included:

How did you work it out?

Show me what you counted.

Where did you start your count from?

They all agreed it was 5 centimetres long! Probing revealed that the students were counting the marks, not the spaces between the marks. None thought to find the difference between 6 centimetres and 10 centimetres. They had not connected the calibrations on their rulers with the way we use physical units during length measuring activities. I decided I needed to help them make the link.

Action and Reflection

I asked students to make a tape measure from paper tape using their own choice of a unit. I provided popsticks, straws, matchsticks, toothpicks, glue and paper tape.

They began gluing units along the tape, lining up their chosen units end-to-end without leaving spaces or overlaps. After a few minutes, Alex and Yenchee came and asked, 'Couldn't we just use a pen to mark where each popstick comes to?'

I was pleased to agree to this and thought it might be useful later in the lesson. When the tapes were dry, the students tried them out, measuring many objects in the room and around the school.

As a class, we then talked about the effectiveness of the measuring tapes. I was able to draw out the many disadvantages of having the actual units glued to the tape. Students made comments like, 'Using popsticks was good

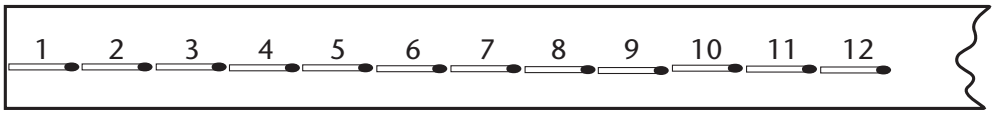
for straight things, but they wouldn't bend and the paper tore when we tried to measure around the tree' and 'Some of the matches fell off when we used ours.'

Then, Sharla said, 'It didn't really matter, because we could see where they'd been stuck on and we just counted where they'd been.'

This seemed the perfect opportunity to ask Alex and Yenchee to show how their tape was marked. Everyone thought this was much better than using the units themselves. I asked if anyone had difficulty counting the units as they were measuring.

Daniel said, 'Ours was matchsticks and Ben kept losing count, so, in the end, what we did was write the numbers next to the matchsticks so we didn't have to count.'

Several others had also done this, but I noticed that they had written the numbers next to the units.



I decided that this needed to be the focus of my next lesson.

Drawing out the disadvantages:

What problems did you have using your tape for measuring curved things?

Did you have the same problems when you were measuring straight things?

Did it matter that your popsticks fell off?

What did you do when your tape ripped?

Did that change the length of the popstick unit?

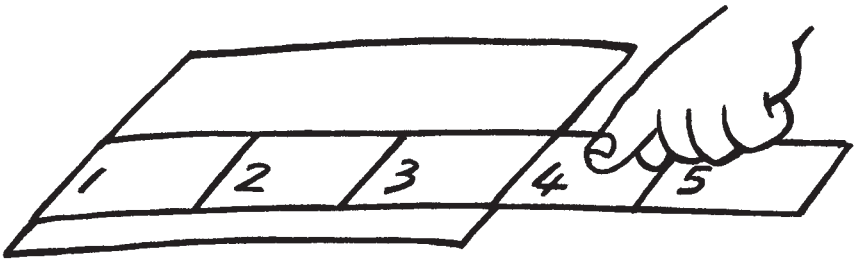
KU 4

Connection and Challenge

The following day, I asked everyone to use what they'd found out and create a new 'improved' measuring tape using an agreed unit. We decided to make a 'matchstick measuring tape'. All took care to accurately mark the length of each matchstick on their tape, showing they had understood how the markings represented the units. When it came to numbering their scales, however, many wrote the numbers next to the space where the match had been, not close to the end marks. Several students had written '1' at the very beginning of the tape. There were a few who numbered their tapes in the conventional way.

Because I wanted to help students see for themselves how their unconventional markings could cause difficulties, I asked students in pairs to use their tapes to measure the same objects and talk about any differences they found. To maximise the chance for conflicting results, I purposely paired students who used different numbering methods.

This also helped me to see how individuals interpreted their own tape markings. I overheard Angie complain to Mohammed that he had written four where the three should be. Mohammed argued, 'But that's for that match, that four is for that match' (pointing to the fourth space).



To facilitate the discussion I asked:

Show us how you used your tape to measure this book.

How did you know how big the book is?

Did the number help?

If the end of the book was not on a mark, how did you know which matchstick to finish your count on?

Can we say seven-and-a-half matchsticks long?

Angie was not satisfied and continued to insist it was misleading to do it that way. I called the class together at that stage and talked about everyone's experiences. I asked Angie to explain how her tape worked.

'I just line it up and look at the mark and the number is just there. See, it is three popsticks long.'

Drawing Out the Mathematical Idea

I asked other students to demonstrate how their tapes worked, including Mohammed. It soon became obvious to all the students that positioning the numbers at the right-hand end of each unit space (rather than within it or at the beginning of it) made it easier to keep track of the number of units and eventually everyone agreed this was the best strategy.

I was now confident that the students knew the value of marking off the length of their unit and using a number for each mark, but I wasn't sure that they had connected this with the way a standard ruler works. I decided that this would be the focus of tomorrow's lesson.

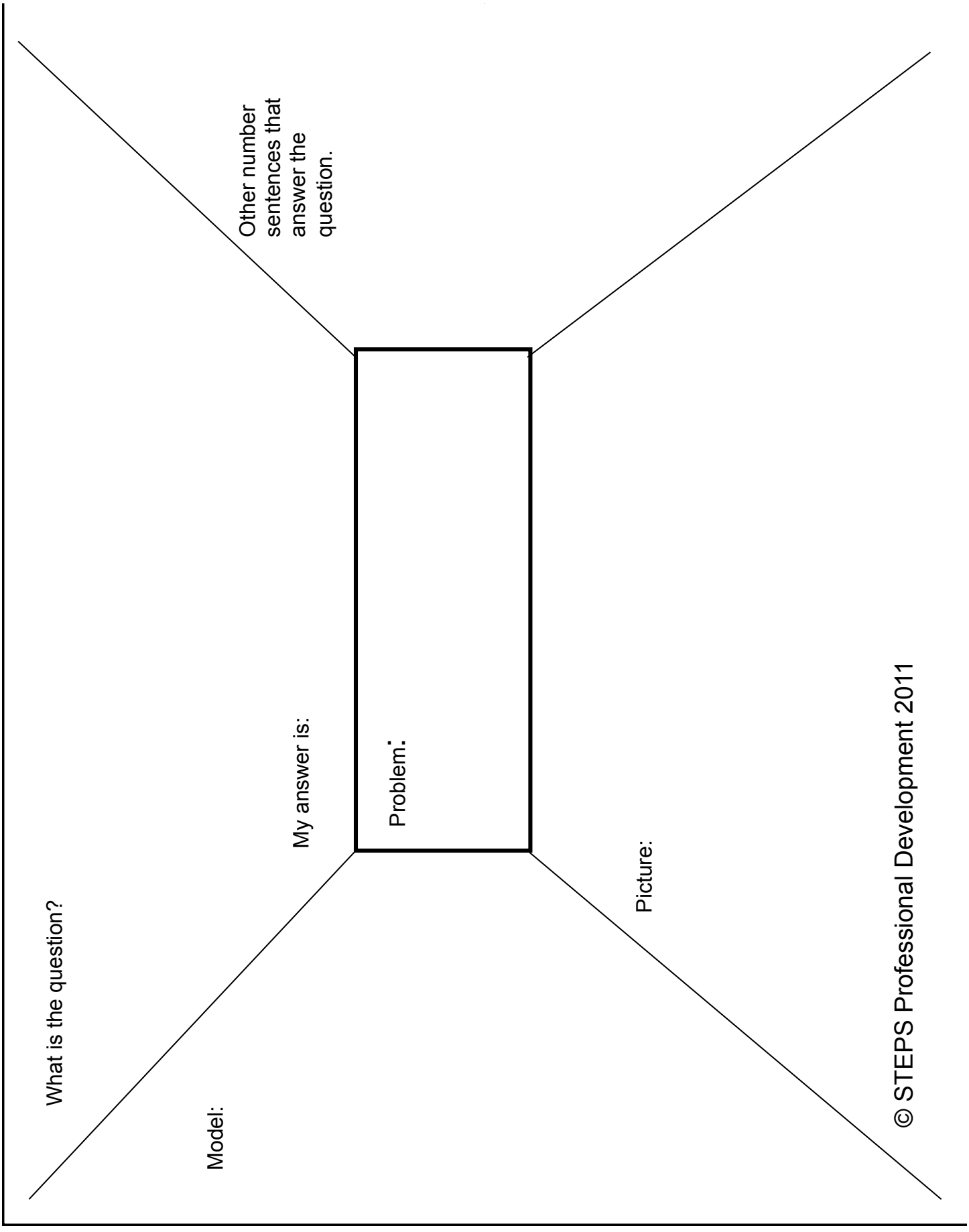
Did You Know?

A diagnostic activity for the middle and later years

Seven hundred students in upper primary and lower secondary schools were asked if they could tell how long the leaf is from the broken ruler. The illustration was accurate; that is, the distance between the centimetre marks was actually one centimetre on the diagram.

A small minority of students said 15 centimetres. Almost half in Year 5, one third in Year 6 and one quarter in Years 7 and 8 said the leaf was 7 centimetres long.

One wrote: *Starting from 9 cm (since it's a broken ruler), I counted 9 cm as 1 cm, 10 cm as 2 cm, 11 cm as ...*


What is the question?
Other number sentences that answer the question.
My answer is:
Problem:
Model:
Picture:
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