<u>Which Center is the Best?</u> – 2012 NCTM Conference, Philadelphia

Karen Hyers
Tartan High School, Oakdale, MN
khyers@isd622.org

Kristin Johnson
St. Louis Park High School, St. Louis Park, MN
johnson.kristin@slpschools.org

Common Core Connections:

HS.G-CO.12 Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.

HS.G-C.3 Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.

Circumcenter is the intersection of the perpendicular bisectors of the sides. This point is the center of the circle (called the circumcircle) that circumscribes the triangle.

- <u>Paper folding:</u> Draw ΔABC. Fold A onto B, crease. Fold A onto C, crease. Fold B onto C, crease.
- <u>Compass – straightedge:</u> Draw ΔABC. Use a compass setting greater than half the length of $\overline{AB}$. Place the point at A and make an arc crossing $\overline{AB}$. Without changing the setting, place the point at B and repeat. The arcs should intersect at two points. Using a straightedge, draw a segment through the intersection points. This is the perpendicular bisector of $\overline{AB}$. Repeat on $\overline{BC}$ and $\overline{AC}$.
- <u>TI-Nspire activity</u> #11301 – *Concurrency and the Circumcenter*
- <u>TI-83+ or TI-84+ Family activity</u> #6862 – *Exploring the Circumcenter of a Triangle*

Incenter is the intersection of the angle bisectors. This point is the center of the circle (called the incircle) that is inscribed in the triangle.

- <u>Paper folding:</u> Draw ΔABC. Fold $\overrightarrow{AB}$ onto $\overrightarrow{AC}$, crease. Fold $\overrightarrow{BC}$ onto $\overrightarrow{BA}$, crease. Fold $\overrightarrow{CA}$ onto $\overrightarrow{CB}$, crease.
- <u>Compass – straightedge</u>: Draw ΔABC. Place the compass point at A. Draw an arc crossing $\overline{AB}$ and $\overline{AC}$. From the points where the arc crosses each side, use the same setting to draw intersecting arcs inside the triangle. Draw a ray from A through this intersection point using a straightedge. This is the bisector of angle A. Repeat at vertices B and C.
- <u>TI-Nspire activity</u> #11359 – *Hanging with the Incenter*
- <u>TI-83+ or TI-84+ Family activity</u> #6861 – *Incenter of a Triangle*

Centroid is the intersection of the medians. This point is the center of gravity (balancing point) of the triangle if it is made of a uniform material.

- <u>Paper folding</u>: Draw ΔABC. Fold A onto B, make a small crease at the midpoint of $\overline{AB}$. Fold the triangle to form a crease through that midpoint and C. Repeat the steps for A onto C, then B onto C.
- <u>Compass – straightedge</u>: Draw ΔABC. Construct the perpendicular bisector of $\overline{AB}$ (as above) to find the midpoint of $\overline{AB}$. Using a straightedge, draw the segment through this midpoint and the opposite vertex C. This segment is the median through C. Repeat on $\overline{BC}$ and $\overline{AC}$.
- <u>TI-Nspire activity</u> #11403 – *Balancing Point*
- <u>TI-83+ or TI-84+ Family activity</u> #8001 – *NUMB3RS-Season 3-"Burn Rate"-Regular Polygon Centroids*

Orthocenter is the intersection of the altitudes.

- <u>Paper folding</u>: Draw ΔABC. Move A along $\overleftrightarrow{AB}$ until the fold will coincide with C, crease. Repeat for B along $\overleftrightarrow{BC}$ and C along $\overleftrightarrow{CA}$.
- <u>Compass – straightedge</u>: Draw ΔABC. Place the compass point at A and draw an arc that intersects $\overleftrightarrow{BC}$ in two points, B' and C'. (Note: you may need to extend segment *BC* for the intersections to occur.) Construct the perpendicular bisector of $\overline{B'C'}$ (as above). This segment is the altitude through A. Repeat for vertices B and C.
- <u>TI-Nspire activity</u> #11484 – *Hey Ortho, What's your Altitude?*
- <u>TI-84 Plus family activity</u> *# 6863 – Exploring the Orthocenter of a Triangle*

Further Problems for Exploration:

- Hospital Locator: http://illuminations.nctm.org/LessonDetail.aspx?id=L661
  - Three cities need a large hospital. You are hired to determine the best location for this facility.
- Activities for Students: Nelson, C and Williams, N. (2007) Sprinklers and Amusements Parks: What Do They Have to Do with Geometry? *Mathematics Teacher, 100, no 6*: 440.
  - The Parks Department is installing a circular sprinkler to water the lawn of a triangular park. Where should the sprinkler be placed to water as much lawn as possible without spraying the sidewalks?
  - You want to open a soft drink stand that is equidistant from the three most popular rides at the amusement park. Where should you be located?
- TI-83+ or TI-84+ Family activity: *NUMB3RS* Activity: Irregular Polygon Centroids Episode: "Burn Rate". Geo-profiling is an investigative technique used by law enforcement that uses the locations of connected crimes to determine the most probable area of offender residence. Suppose Don has determined that the locations are (0, 0), (6, 0) and (3, 5.2) when laid out on a map. Plot the points and find the location of the centroid by finding the intersection of the three medians. The activity online continues by finding the weighted centroid for non-regular polygons with an extension of finding the centroid of a smooth curve.
- Euler Line: http://illuminations.nctm.org/ActivityDetail.aspx?id=51 The Euler Line passes through the orthocenter, circumcenter and the centroid. What are its properties?
- Nine-Point Circle: In an arbitrary triangle, the circle that passes through the 3 midpoints of the sides, the 3 feet of the altitudes and the 3 points which are the midpoints of the segments from each vertex of the triangle to the orthocenter, is known as the nine-point circle. (aka – Euler's circle or the Feuerbach circle)
  - The radius of a triangle's circumcircle is twice the radius of that triangle's nine-point circle.
  - The center of any nine-point circle (the nine-point center) lies on the corresponding triangle's Euler line, at the midpoint between that triangle's orthocenter and circumcenter.
  - The nine-point center lies at the centroid of four points comprising the triangle's three vertices and its orthocenter.
- Napoleon Points: In ΔABC, form three equilateral triangles pointing away from the sides AB, AC, and BC. Construct a line through point A and the center of the triangle formed on BC. Repeat for the other 2 vertices and corresponding opposite equilateral triangles. The 3 lines meet at the first Napoleon point.
  - If the three equilateral triangles point inward instead of away from triangle ABC, the three lines meet in the *second Napoleon point*.
- Can you construct an incenter, circumcenter, and centroid of a quadrilateral? Polygon? Which ones exist? How does the shape of the quadrilateral/polygon make a difference?
  - Start with a circle. Put 4 points on the circle. Make 2 quadrilaterals; one connecting on the inside (cyclic quadrilateral) and one using the points as tangency points. What happens when you construct the perpendicular diagonals? Angle bisectors? How does this relate to the center of the circle?
- Van Aubel's theorem: For an arbitrary quadrilateral, construct a square on each side. The two line segments between the centers of opposite squares are of equal lengths and perpendicular. In other words, the center points of the four squares form the vertices of an orthodiagonal quadrilateral.
- Cevian Triangle: A **cevian** is any segment in a triangle with one endpoint on a vertex and the other endpoint on the opposite side. Medians, altitudes, and angle bisectors are special cases of cevians. For a given point P in the plane of a triangle ABC, the feet of the cevians through P form a triangle $P_aP_bP_c$ known as the *cevian triangle* of P with respect to the triangle ABC. By construction, triangles ABC and $P_aP_bP_c$ are perspective from point P. By Desargues' theorem, they are also perspective from a line.
- Gergonne Point: Construct a triangle and its incircle. Construct segments from the points of tangency of the incircle to the opposite vertices. This point of concurrency is called the Gergonne Point.
- Nagel Point: In any triangle *ABC*, the point where the lines connecting a vertex to the opposite sides tangency point with the excircle is called the triangle's Nagel point.
  - The Nagel point can also be found by the intersection point of the lines formed by connecting a triangle vertex to the point half way around the perimeter of the triangle, starting from each vertex.