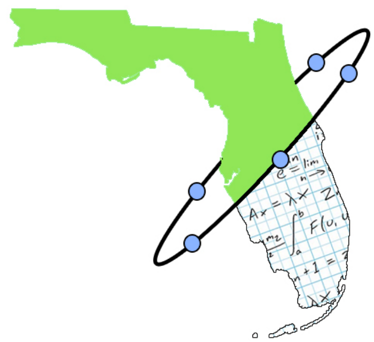
Visualizing Systems of Equations

GeoGebra



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Free mathematics software for learning and teaching

GeoGebra

www.geogebra.org

- ✓ Graphics, algebra and tables are connected and fully dynamic
- ✓ Easy-to-use interface, yet many powerful features
- ✓ Authoring tool to create interactive learning materials as web pages
- ✓ Available in many languages for millions of users around the world
- ✓ Free and open source software

System of Linear Equations

An 8-pound mixture of M&M's and raisins costs $18. If a lb. of M&M's costs $3, and a lb. of raisins costs $2, then how many pounds of each type are in the mixture?

x → lbs of M&M's

Y → lbs of raisins

$$\begin{cases} x + y = 8 \\ 3x + 2y = 18 \end{cases}$$

Gauss' Method of Elimination

If a linear system is changed to another by one of these operations:

(1) Swapping – an equation is swapped with another
(2) Rescaling - an equation has both sides multiplied by a nonzero constant
(3) Row combination - an equation is replaced by the sum of itself and a multiple of another

then the two systems have the same set of solutions.

Restrictions to the Method

- Multiplying a row by 0
  - ✓that can change the solution set of the system.
- Adding a multiple of a row to itself
  - ✓adding −1 times the row to itself has the effect of multiplying the row by 0.
- Swapping a row with itself
  - ✓it's pointless.

Example

$$\begin{cases} x + y = 8 \\ 3x + 2y = 18 \end{cases}$$

✓Multiply the first row by -3 and add to the second row.
✓Write the result as the new second row

$$\begin{array}{r} -3x - 3y = -24 \\ 3x + 2y = 18 \\ \hline -y = -6 \end{array}$$

$$\begin{cases} x + y = 8 \\ -y = -6 \end{cases}$$

Example (Cont)

$$\begin{cases} x + y = 8 \\ -y = -6 \end{cases}$$

✓Add the two rows to eliminate the ***y*** in the first row

✓Write the result as the new first row

$$\begin{cases} x = 2 \\ -y = -6 \end{cases}$$

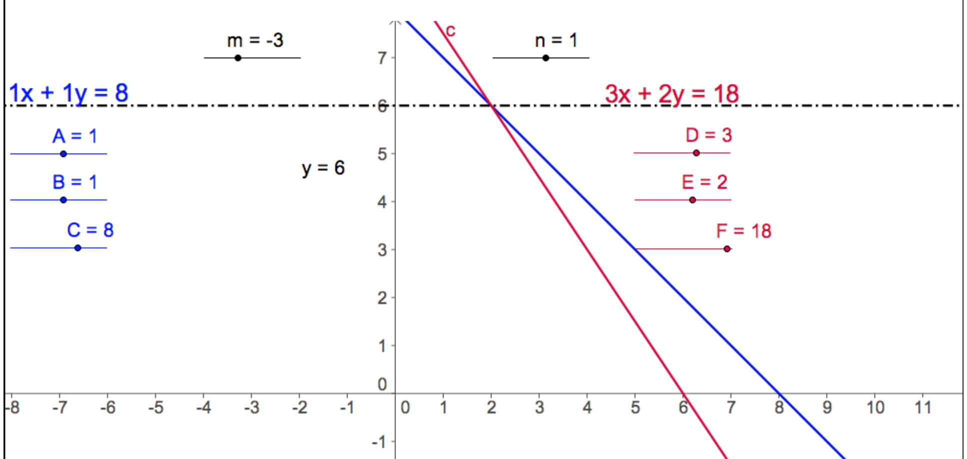
✓Multiply the second row by -1

$$\begin{cases} x + y = 8 \\ 3x + 2y = 18 \end{cases}$$

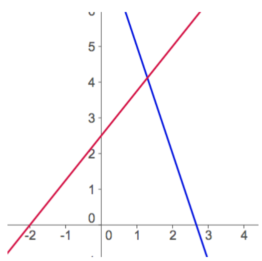
Changed to

$$\begin{cases} x = 2 \\ y = 6 \end{cases}$$

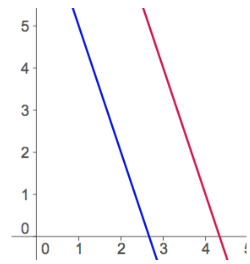
Geometric Interpretation



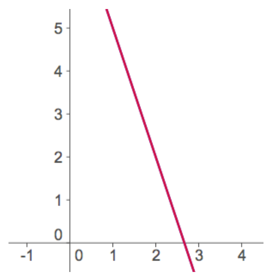
Possible Types of Solutions



Unique solution



No solution



Infinite solutions

General Behavior of Linear Combination

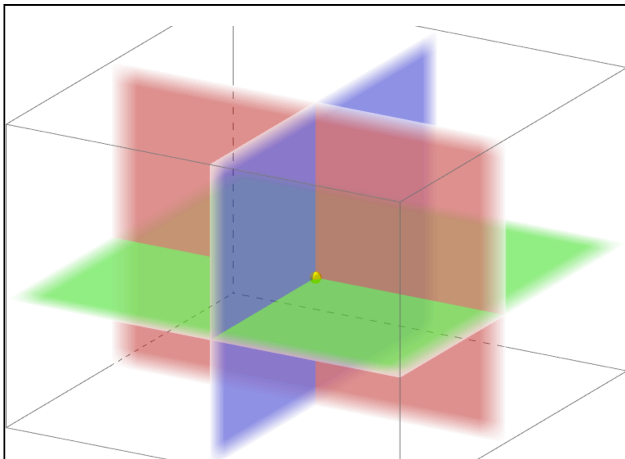
- If solution exists - the new line (row combination) passes through the point of intersection (solution).
- If no solution – the new line is parallel to the other lines
- If infinite solutions – the new line overlaps the other two.

In General…

$$\begin{cases} ax + by = c \\ dx + ey = f \end{cases} \qquad \left(\begin{array}{cc|c} a & b & c \\ d & e & f \end{array}\right)$$

Assuming a, b, c, d are not equal to 0

Unique solution if: $ae - bd \neq 0$

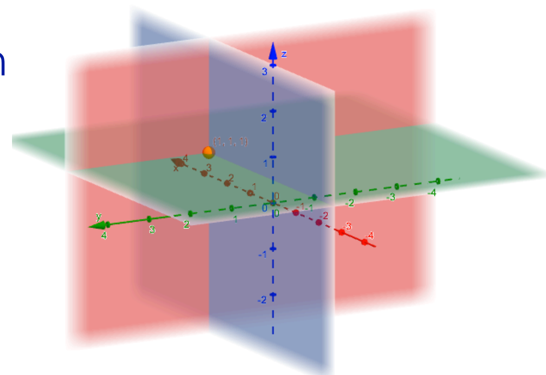


3 x 3 System of Equations

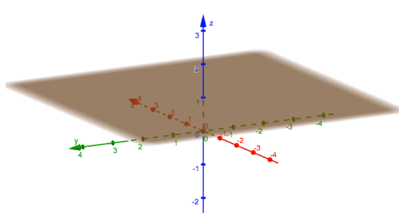
$$\begin{cases} A_1x + B_1y + C_1z = D_1 \\ A_2x + B_2y + C_2z = D_2 \\ A_3x + B_3y + C_3z = D_3 \end{cases} \qquad \left(\begin{array}{ccc|c} A_1 & B_1 & C_1 & D_1 \\ A_2 & B_2 & C_2 & D_2 \\ A_3 & B_3 & C_3 & D_3 \end{array}\right)$$

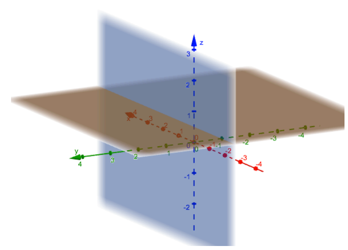
Possibilities with Systems of equations in 3 Variables

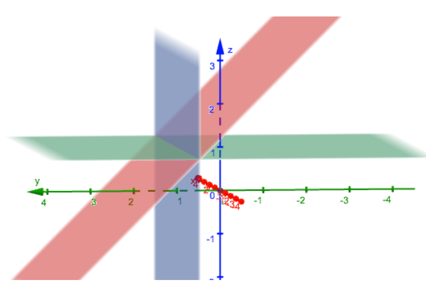
- Unique solution
  - A point



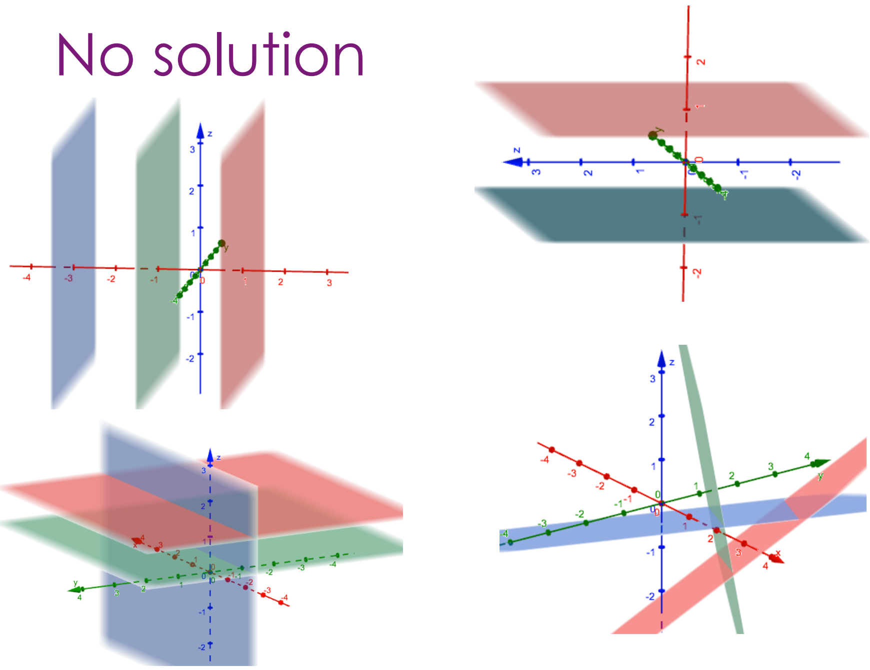
Infinite Solutions







No solution



Solving a System of Equations

$$\begin{cases} 1x + 1y + 2z = 8 \\ -1x - 2y + 3z = 1 \\ 3x - 7y + 4z = 10 \end{cases}$$

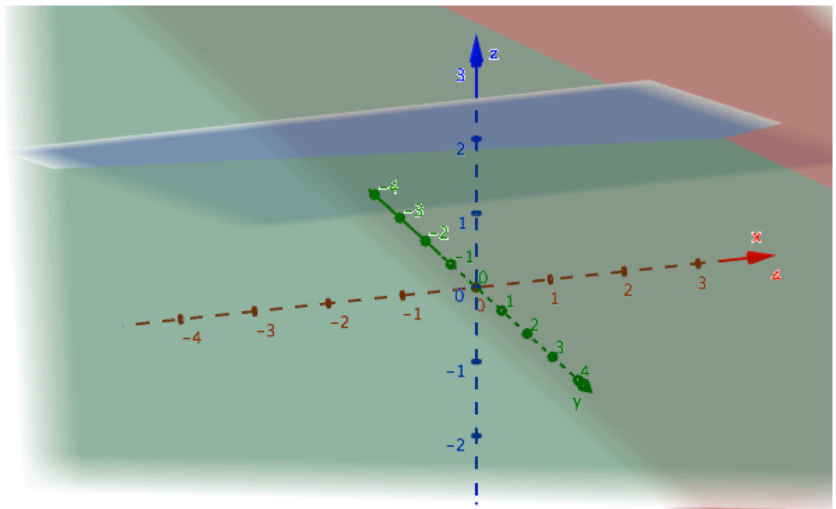
$$\left(\begin{array}{ccc|c} 1 & 1 & 2 & 8 \\ -1 & -2 & 3 & 1 \\ 3 & -7 & 4 & 10 \end{array}\right)$$

R1 + R2
Replace row 2

$$\left(\begin{array}{ccc|c} 1 & 1 & 2 & 8 \\ 0 & -1 & 5 & 9 \\ 3 & -7 & 4 & 10 \end{array}\right)$$

The new plane (blue) is parallel to the x-axis

-y + 5z = 9

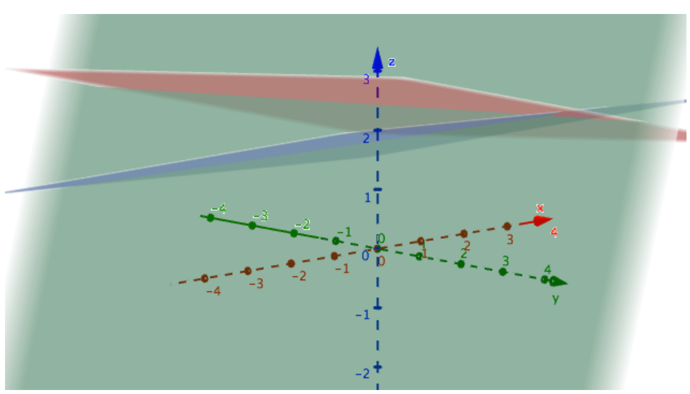


$$\left(\begin{array}{ccc|c} 1 & 1 & 2 & 8 \\ 0 & -1 & 5 & 9 \\ 3 & -7 & 4 & 10 \end{array}\right)$$

R1 + R2
Replace row 1

$$\left(\begin{array}{ccc|c} 1 & 0 & 7 & 17 \\ 0 & -1 & 5 & 9 \\ 3 & -7 & 4 & 10 \end{array}\right)$$

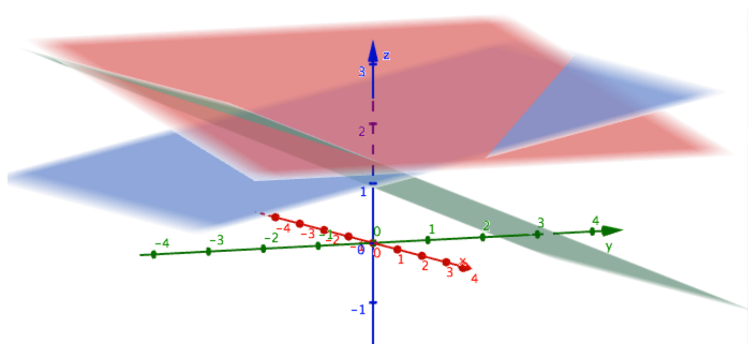
The new plane (red) is parallel to the y-axis



$$\left(\begin{array}{ccc|c} 1 & 0 & 7 & 17 \\ 0 & -1 & 5 & 9 \\ 3 & -7 & 4 & 10 \end{array}\right) \qquad \left(\begin{array}{ccc|c} 1 & 0 & 7 & 17 \\ 0 & -1 & 5 & 9 \\ 0 & 7 & 17 & 41 \end{array}\right)$$

R1*3 – R3
Replace row 3

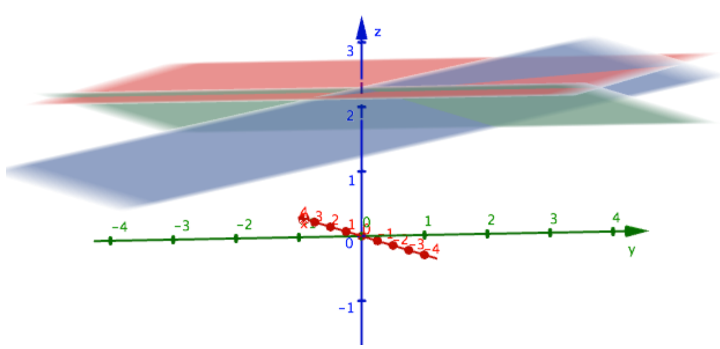
Green plane is now parallel to the x-axis



$$\left(\begin{array}{ccc|c} 1 & 0 & 7 & 17 \\ 0 & -1 & 5 & 9 \\ 0 & 7 & 17 & 41 \end{array}\right) \qquad \left(\begin{array}{ccc|c} 1 & 0 & 7 & 17 \\ 0 & -1 & 5 & 9 \\ 0 & 0 & 1 & 2 \end{array}\right)$$

(R2*7 + R3)/52
Replace row 3

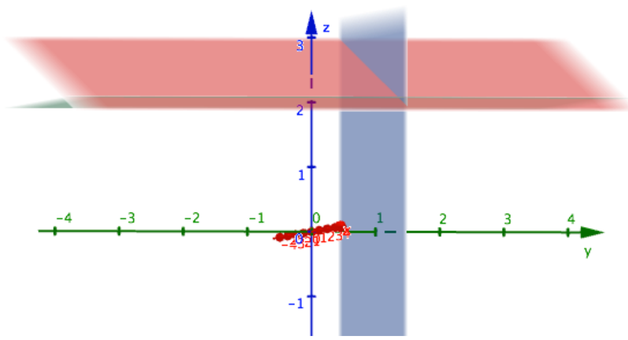
Green Plane is perpendicular to the z-axis



$$\left(\begin{array}{ccc|c} 1 & 0 & 7 & 17 \\ 0 & -1 & 5 & 9 \\ 0 & 0 & 1 & 2 \end{array}\right) \qquad \left(\begin{array}{ccc|c} 1 & 0 & 7 & 17 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{array}\right)$$

R3*5 – R2
Replace row 2

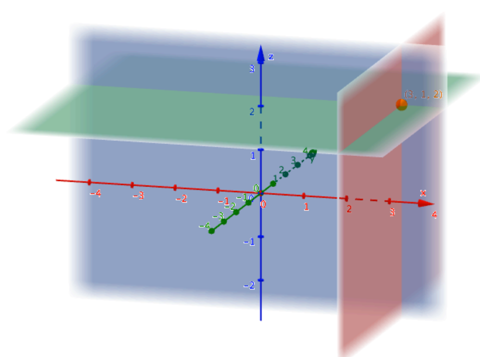
Blue plane is perpendicular to the y-axis



$$\left(\begin{array}{ccc|c} 1 & 0 & 7 & 17 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{array}\right) \qquad \left(\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{array}\right)$$

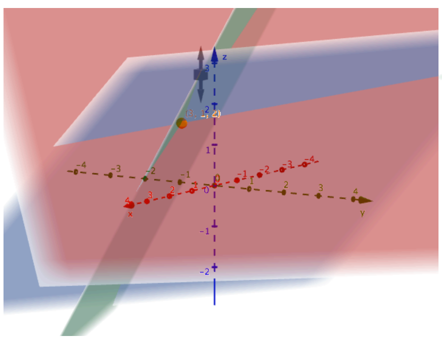
R1 - R3*7
Replace row 1

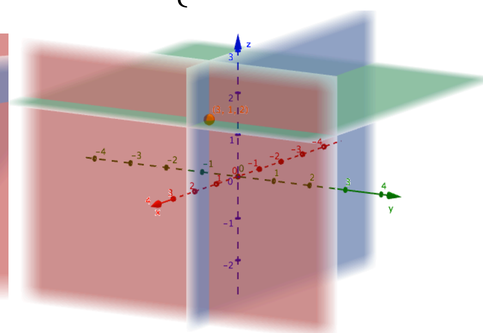
Red plane perpendicular to the x-axis



New Equivalent System

$$\begin{cases} 1x + 1y + 2z = 8 \\ -1x - 2y + 3z = 1 \\ 3x - 7y + 4z = 10 \end{cases} \quad \text{To} \quad \begin{cases} 1x = 3 \\ 1y = 1 \\ 1z = 2 \end{cases}$$





What is the solution?

$$\begin{cases} 2x + 3y - 4z = -11 \\ 5x + 5y + 5z = 6 \\ -6x - 9y + 12z = -14 \end{cases}$$

Two parallel planes intersected by a third plane

What is the solution?

$$\begin{cases} -2x + 3y + 5z = 2 \\ 4x - 6y - 10z = 8 \\ x - 1.5y - 2.5z = -3 \end{cases}$$

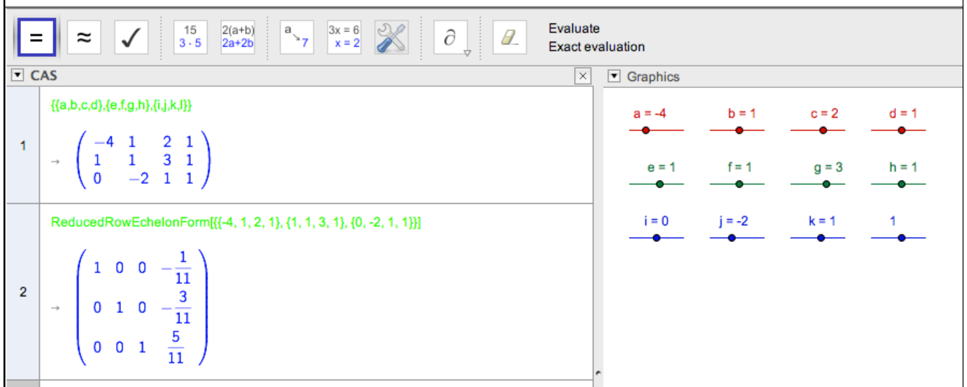
Three parallel planes

What is the solution?

$$\begin{cases} 3x + 2y - z = 10 \\ x + 4y + 2z = 3 \\ 4x - 24y - 20z = 4 \end{cases}$$

Three non-parallel planes that form a type of triangle

CAS in GeoGebra 4.2



Information

- Downloading GeoGebra 5.0
- Construction of ggb files

Uploaded in the Conference Online Planner and Conference App

Thank You!

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