Ready or not, here come the Common Core State Standards:

Focus on Middle School Mathematics

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"[CCSSM] are more aggressive in their timelines for teaching particular concepts … The standards at the middle grades include significant amounts of statistics and early algebra." (J. Confrey & E. Krupa, 2011)

"CCSSM represent significant curricular acceleration in grades K–8 … much Algebra 1, Geometry, and Statistics [content is included in] the middle grades." (B. Findell, 2012).

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- What are students expected to know upon entering middle school?
- What topics are new to Grade 6, 7 or 8?
- Are there new/different emphasis in the middle grades of CCSSM?
- Is the progression of particular mathematical topics different?

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Outline of Session

Themes/Shifts Across K-12

Emphasis in Grades 6-8 of CCSSM:
   Algebra
   Statistics and Probability
   Geometry and Measurement

Implications

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General Themes Across K-12 CCSSM

- Attention to mathematical practices/ processes
- Focus on conceptual development/ understanding

Little attention to or acknowledgement of technology as a tool for doing or learning mathematics

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The Mathematical Practices are NOT new

Processes (NCTM, 2000)

-Problem Solving
-Reasoning
-Connections
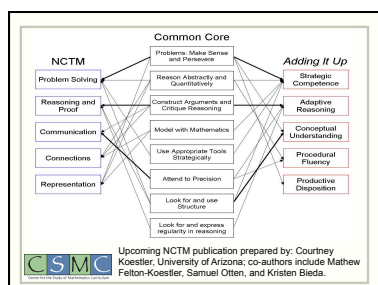-Communication
-Representation

Proficiencies (NRC, 2001)

-Strategic Competence
-Adaptive Reasoning
-Conceptual Understanding
-Procedural Fluency
-Productive Disposition

Practices (CCSSM, 2010)

-Make sense of problems and persevere in problem solving
-Reason abstractly and quantitatively
-Express regularity in repeated reasoning
-Construct viable arguments
-Look for and use structure
-Use tools strategically
-Attend to precision
-Model mathematics

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Common Core

NCTM

Problem Solving

Reasoning and Proof

Communication

Connections

Representation

Problems: Make Sense and Persevere

Reason Abstractly and Quantitatively

Construct Arguments and Critique Reasoning

Model with Mathematics

Use Appropriate Tools Strategically

Attend to Precision

Look for and use Structure

Look for and express regularity in reasoning

Adding It Up

Strategic Competence

Adaptive Reasoning

Conceptual Understanding

Procedural Fluency

Productive Disposition

Upcoming NCTM publication prepared by: Courtney Koestler, University of Arizona; co-authors include Mathew Felton-Koestler, Samuel Otten, and Kristen Bieda.

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Focus on Conceptual Development

*Gr. 5: Recognize volume as an attribute of solid figures and **understand concepts** of volume measurement.*

*Gr. 6: **Understand the concept** of a ratio and use ratio language to describe a ratio relationship between two quantities.*

*Gr. 7: **Understand** that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related.*

*Gr. 8: **Understand the connections** between proportional relationships, lines, and linear equations.*

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Curriculum Shifts (K-12)

- Some content moved to earlier grades (whole number and fraction computation)
- Some content delayed to later grades (statistics)
- Some shifts in emphasis (transformational geometry)
- Some models introduced earlier and with more emphasis (number line)

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**Curriculum Shifts:
Number & Operation**

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**Example:
Computation with Fractions**

Gr. 3. Develop an understanding of fractions as numbers.
Gr. 4. Extend understanding of fraction equivalence and ordering.
Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.
Understand decimal notation for fractions, and compare decimal fractions.
Gr. 5. Use equivalent fractions as a strategy **to add and subtract fractions**.
Apply and extend previous understandings of multiplication and division to **multiply and divide fractions**.
Gr. 6. Apply and extend previous understandings of multiplication and division to **divide fractions by fractions**.

C S M C

When do states expect students to proficiently add, subtract, multiply and divide fractions?*

	Addition and Subtraction of fractions	Multiplication of fractions	Division of fractions
4th grade	1 state		
5th grade	15 states (CCSSM)	2 states (CCSSM)	1 state
6th grade	20 states	25 states	24 states (CCSSM)
7th grade	6 states	13 states	14 states
8th grade		1 state	1 state

*For this summary, we identified the grade at which the standard indicated students were expected to compute fluently with common and uncommon denominators when adding and subtracting fractions.

C S M C

Curriculum Shifts Related to Algebra

C S M C

What Is the Same?

Solve one-variable equations
– Grade 6
$x + 8 = 27$ or $3x = 21$

– Grade 7
$2x + 11 = 23$ or $3(x + 2) = 21$

C S M C

What is the Same but with More Emphasis?

Concept of variable
A number in a generalized pattern ($r + s = s + r$)
A specific value ($14 = 3x - 1$)
A quantity that varies in relation to others ($y = 3x$)
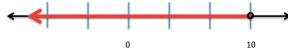A parameter ($y = mx$)
An arbitrary or abstract placeholder (Factor $t^2 + 3t$)

C S M C

What is Different?

– Grade 6
$10 > x$ or $10 < x$

0   10

– Grade 7:
$3(x + 2) > 21$

C S M C

What is Different?

– Grade 6: Evaluate expressions

$A = bh$

If the base of a rectangle is 4 units, and the height is 6.5 units, what is the area?

C S M C

What is Different?

– Grade 6: Read and write expressions

Write the expression that represents 'the product of b and 7, increased by 2.'

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What is Different?

–Grade 7: Use equivalent expressions in a mathematical sense and to model a context

Simplify: $3(2y + 4)$

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What is Different?

–Grade 8: Solve systems of linear equations
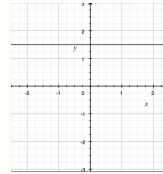
Solve this system of equations by an algebraic method.

$$2x + y = 10$$
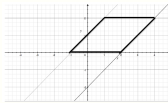$$y = 4$$

Compare your solution method to solving by graphing.

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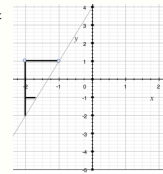## What is Different?

Grade 6: Find distance between points

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What is Different?

– Grade 7: Graph polygons on coordinate plane; Link ordered pairs to proportional relationships

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What is Different?

– Grade 8: Interpret unit rate as slope; Use similar triangles to explain slope; analyze graphs of functions (linear and non-linear)

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Curriculum Shifts Related to Statistics & Probability

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Statistics and Probability Clusters in Grades 6-8

Grade 6
• *Develop understanding of statistical variability.*
• *Summarize and describe distributions.*

Grade 7
• *Use random sampling to draw inferences about a population*
• *Draw informal comparative inferences about two populations.*
• *Investigate chance processes and develop, use, and evaluate probability models.*

Grade 8
• *Investigate patterns of association in bivariate data.*

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What Is the Same?

–Grade 6

Summarize numerical data sets in relation to their context, such as by:

a. Reporting the number of observations.

b. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement.

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What Is Different?

–Grade 6

*c. Giving quantitative measures of center (median and/or mean) and variability **(interquartile range and/or mean absolute deviation**), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.*

*d. **Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.***

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What Is the Same?

*Find probabilities of compound events using **organized lists, tables, tree diagrams**, and simulation.*

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What Is Different?

Design and use a simulation to generate frequencies for compound events.

For example, use random digits as a simulation tool to approximate the answer to the question: If 40 percent of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?

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	Numerical Data	Categorical Data
Numerical Data	Correlation – Line of best fit (Grade 8)	Comparing two populations (Grade 7)
Categorical Data	Comparing two populations (Grade 7)	Association – Frequencies (Grade 8)

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What Is the Same but with Different Emphasis?

–Grade 8

Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept.

• Working with scatter plots and linear models is not unusual.
• Interpreting the slope of linear models in context is a new emphasis.

Graph created using CPMP Tools

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What is Different?

–Grade 8

Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table.

		Do you have a curfew?	
		Yes	No
Do you have assigned chores?	Yes	40	10
	No	10	40

The table above illustrates the results of a survey of 100 students. Is there evidence that those who have a curfew also tend to have chores?

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Curriculum Shifts Related to Geometry and Measurement

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Focus in middle school geometry

Grade 6: Solve problems involving area, surface area, and volume

Grade 7:
- Draw, construct, and describe geometrical figures
- Solve problems involving angle measure, area, surface area, and volume

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Geometry/Measurement

Grade 8
- Congruence and similarity
- Pythagorean Theorem
- Solving problems involving volumes of cylinders, cones, and spheres

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What is the same?

Area, surface area, and volume

- Calculate area of triangles, special quadrilaterals, circles, and polygons composing or decomposing into other shapes
- Calculate surface area and volumes of 3-D figures including rectangular prisms, cones, cylinders, and spheres

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What is the Same but with More Emphasis?
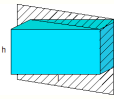
Pythagorean Theorem

- Explain a proof of Pythagorean and its converse
- Use the theorem to find side lengths in right triangle in 3D and distance between two points in a coordinate system (Grade 8)

$$c^2 = x^2 + y^2 \qquad s^2 = c^2 + z^2$$
$$\therefore \quad s^2 = x^2 + y^2 + z^2$$
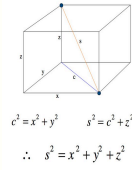
What is Different?
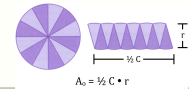
Connections of 2D and 3D objects

- Describe 2D figures that result from slicing 3D figures (Grade 7)

What is Different?

Draw, construction, reasoning

- Draw geometric shapes with given conditions (focusing on triangles)
- Give informal derivation of the relationship between circumference and area of a circle (Grade 7)

$A_o = \frac{1}{2} C \cdot r$

What is Different?

Transformations

- Verify experimentally properties of rigid transformations
- Describe the effects of transformations on 2-D figures using coordinates
- Understand congruence and similarity through transformations (Grade 8)

Implications for Transitioning to CCSS

Curriculum

- How will existing materials be adapted?
- How will new materials be evaluated for adoption?
- How will topics be sequenced and how much time will be allotted for each?

Implications for Transitioning to CCSS

Assessment

- How will teachers determine students' preparedness for grade-level CCSS?
- How will schools monitor student progress?
- How will teachers link classroom assessment with summative assessments?
- How will student assessment influence the way in which teachers are evaluated?

Implications for Transitioning to CCSS

Instruction

- How will schools and teachers determine a match (or not) between current instructional techniques and those needed for CCSS?
- How will mismatches between existing curriculum materials' instructional approach and CCSS be negotiated?
- What types of professional development will be helpful to teachers?

It's about INSTRUCTION (Teaching)

The single greatest determinant of learning is not socioeconomic factors or funding levels. It is instruction (teaching). Acknowledgment of this fact continues to elude us.

Schmoker, M . (2006). *Results now: How we can achieve unprecedented improvements in teaching and learning.* Alexandria, VA: Association for Supervision and Curriculum Development.

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