MODELS FOR POSITIVE AND NEGATIVE RATIONAL NUMBER OPERATIONS

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RATIONALE

- The ability to think about rational numbers is an important concept for not only school mathematics, but for participation in our society. We rely on our understanding of this concept and our ability to manipulate these numbers for personal, economic and political proficiency.

- There are challenges in introducing negative numbers and frequently students overgeneralize their experiences with positive numbers.

INSTRUCTIONAL STRATEGIES

- Some of the challenges with respect to operations with integers is a result of its abstract nature and our inability to directly model some operations with real life examples.

- So what do we do?

INSTRUCTIONAL GOALS

- **Procedural knowledge – "ability to execute action sequences to solve problems"**

- **Procedural fluency – "skill in carrying out procedures flexibly, accurately, efficiently, and appropriately"**

- **Conceptual understanding – "comprehension of mathematical concepts, operations, and relations"**

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GUIDING QUESTIONS

- How can we use different models to help students develop procedural knowledge, procedural fluency, and conceptual understanding?

- What are the challenges of understanding different models?

- What are some alternatives strategies students may use with different models?

REPRESENTATIONS AND MODELS

- Physical objects

- Number line

- Real-life contexts

CHIP MODEL

- Using physical objects to represent positive and negative numbers, i.e. black chips as positive numbers and white chips as negative numbers.

  - Addition

5 + 2	-5 + 2
5 + -2	-5 + -2

  - Subtraction

5 – 2	-5 – 2
5 – -2	-5 – -2

GUIDING QUESTIONS

- How can we use the chip model to help students develop procedural knowledge, procedural fluency, and conceptual understanding?

- What are the challenges of understanding the chip model?

- What are some alternatives strategies students may use with the chip model?

CHIP MODEL

- **Problem: 5 – 2**

Whenever I'm doing subtraction, I'm thinking any number after the subtraction sign is a pair. What we call a zero pair. A set of zero pairs. A negative and a positive. So, let's illustrate that, and let's see. Start out with five pluses. Because we have two over here, we're going to set up two sets of pairs—one positive, one negative. What does the subtraction tell me? Subtract the two positives. Now, I've gotten rid of my two positives. So what do I do next? I create my pairs. I'm getting rid of my pairs. And I have three positives left.

TEMPERATURE

- The greatest one-day temperature change in world records occurred at Browning, Montana from January 23–24 in 1916. The temperature fell from 44°F to -56°F in less than 24 hours.

  - By how many degrees did the temperature change in that period?
  - How could you express the calculation of temperature change and the resulting temperature with a number sentence?

CMP, *Accentuate the Negative*
ACE Question

MOTION MODEL

- Write number sentences that express your answers to these questions. Use positive numbers for running speeds to the right and negative numbers for running speeds to the left. Use positive numbers for time in the future and negative numbers for time in the past. Each runner runs at a constant speed.

  - Hahn passes the 0 point running 5 meters per second to the right. Where is he 10 seconds later?

  - Aurelia passes the 0 point running to the left at 6 meters per second. Where is she 8 seconds later?

  - Pascal passes the 0 point running to the right at 3 meters per second. Where was he 6 seconds earlier?

  - Tori passes the 0 point running to the left at 5 meters per second. Where was she 7 seconds earlier?

MOTION MODEL

- Write division sentences that express your answers to the questions below.
  - Dwayne goes from 0 to 15 in 5 seconds. At what rate (distance per second) does he run?
  - Aurelia reaches -12 only 3 seconds after passing 0. At what rate does she run to the left.
  - Pascal passes 0 running to the right at the rate of 5 meters per second. When did he leave the point -50?
  - Tori wants to reach the point -40 running to the left at 8 meters per second. How long will it take her from the time she passes 0?

CMP, *Accentuate the Negative*

GUIDING QUESTIONS

- How can we use the number line and real life contexts to help students develop procedural knowledge, procedural fluency, and conceptual understanding?

- What are the challenges of understanding the number line and these real life contexts?

- What are some alternatives strategies students may use with these real life contexts?

DEBT AND CREDIT

- **Which of the following students made bad decisions about their finances?**
  - Ann: She took away an asset of +$200 from her net worth.
  - Bradley: He added an asset of +$3000 to his net worth.
  - Devon: He added a debt of -$650 to his net worth.
  - Ernie: He took away a debt of -$5400 from his net worth.

Stephan, M. (2009). What are you worth?

MTMS, 15(1), 19–23.

GUIDING QUESTIONS

- How can we use debt and credit to help students develop procedural knowledge, procedural fluency, and conceptual understanding?

- What are the challenges of understanding the debt and credit context?

- What are some alternatives strategies students may use to understand debt and credit?

CONCLUSION

- As we help our students learn about and understand negative numbers and operations with positive and negative rational numbers, it is important that we help them gain procedural knowledge, procedural fluency and conceptual knowledge.

- Planning together – collaboration, building mathematical knowledge, reflecting on pedagogy.

RESOURCES

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THANK YOU!