Welcome!
We encourage you to sit in groups

DIFFERENTIATING INSTRUCTION EFFECTIVELY BASED ON CLASSROOM ACTIVITIES AND ASSESSMENTS

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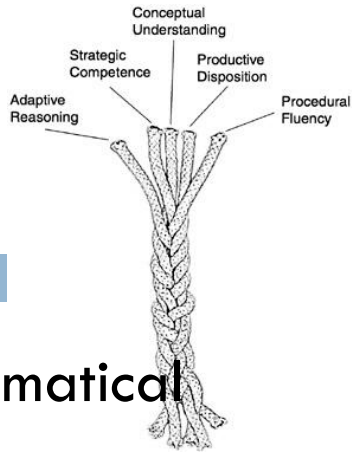
Naperville Community Unit
School District 203
Naperville, IL

What will you learn in this session?

- How learning progressions/trajectories can help improve instruction
- How to use class work more effectively
- How to differentiate instruction and find resources
- Comments? Feedback?

The Five Strands of Mathematics



- **Conceptual understanding:** Comprehension of mathematical concepts, operations, and relations
- **Procedural fluency:** Skill in carrying out procedures flexibly, accurately, efficiently, and appropriately
- **Strategic competence:** Ability to formulate, represent, and solve mathematical problems
- **Adaptive reasoning:** Capacity for logical thought, reflection, explanation, and justification
- **Productive disposition:** Habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy

(National Research Council, 2001)

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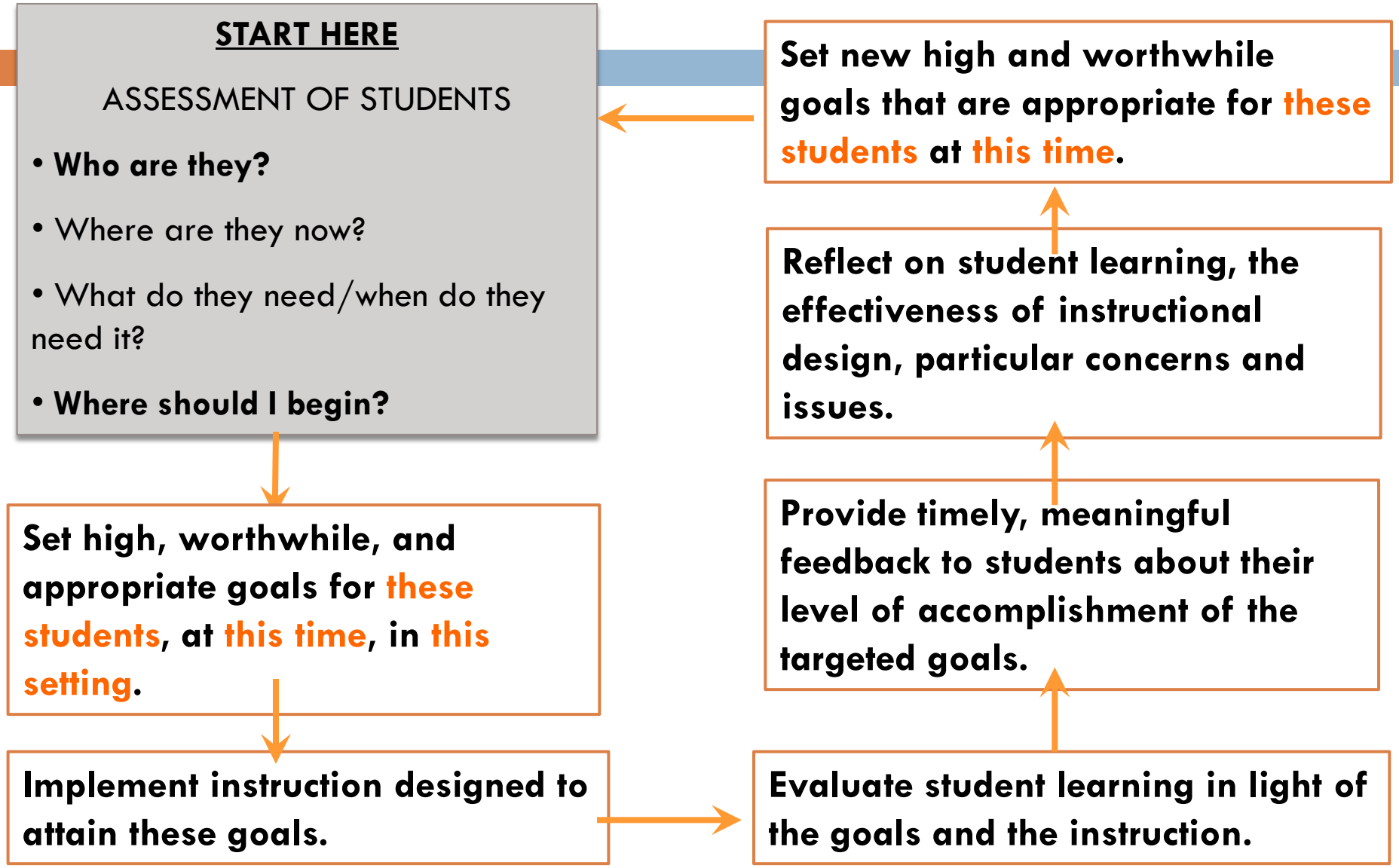
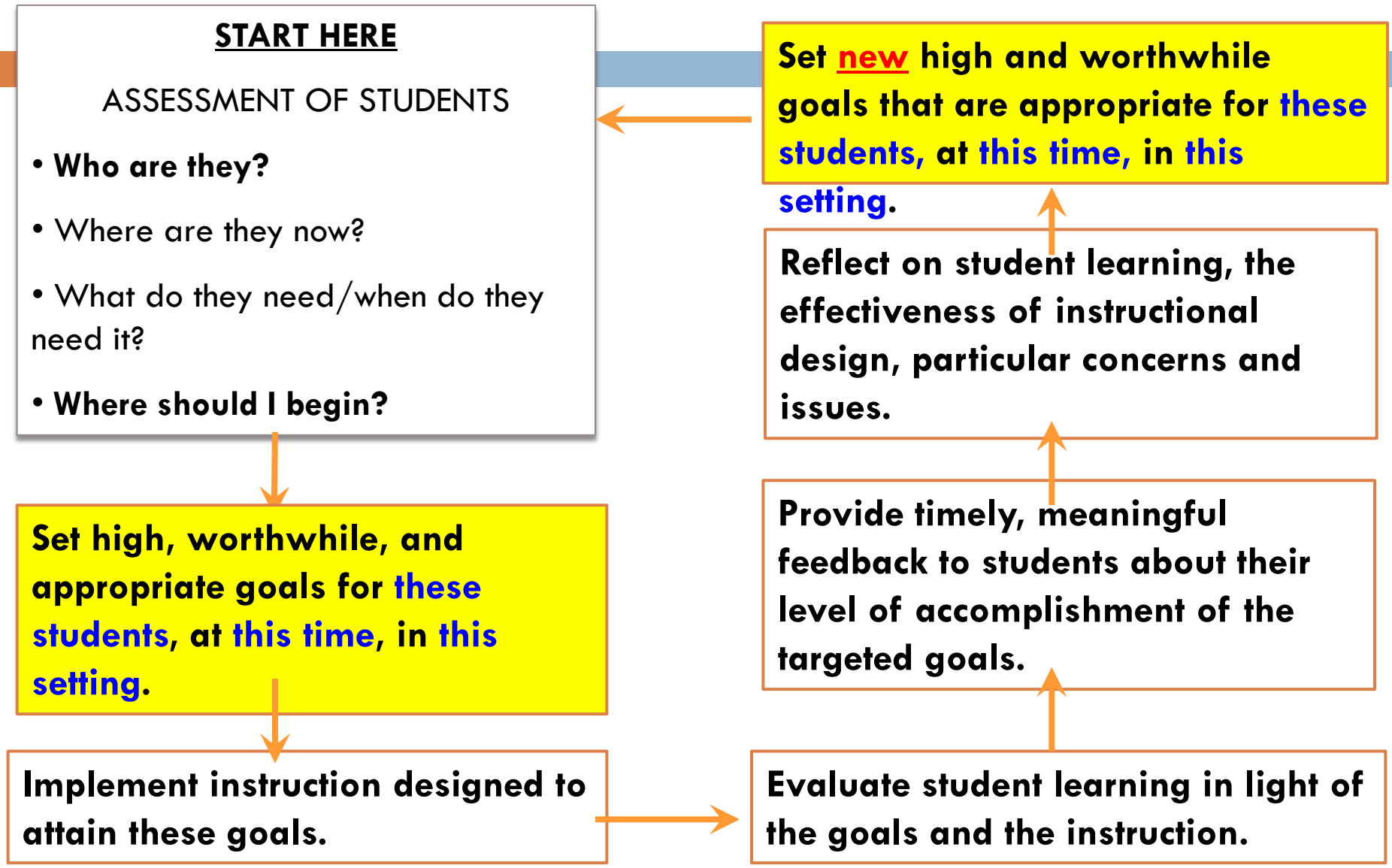
Cycles of Instruction: Enhanced Architecture of Accomplished Teaching



(Einhorn, 2002); www.nbrc.illinoisstate.edu

Cycles of Instruction



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What is a learning progression/trajectory?

- An empirically supported developmental progression of how students move through successive refinements from informal to complex ideas, taking into consideration needed instructional practices, tasks, and tools.

- Common Core State Standards
  - "Provide a consistent, clear understanding of what students are expected to learn"
  - Adopted by many states
  - Includes the *Standards* and the *Mathematical Practices*

(Confrey et al, 2010); www.corestandards.org

Why use a learning progression/trajectory?

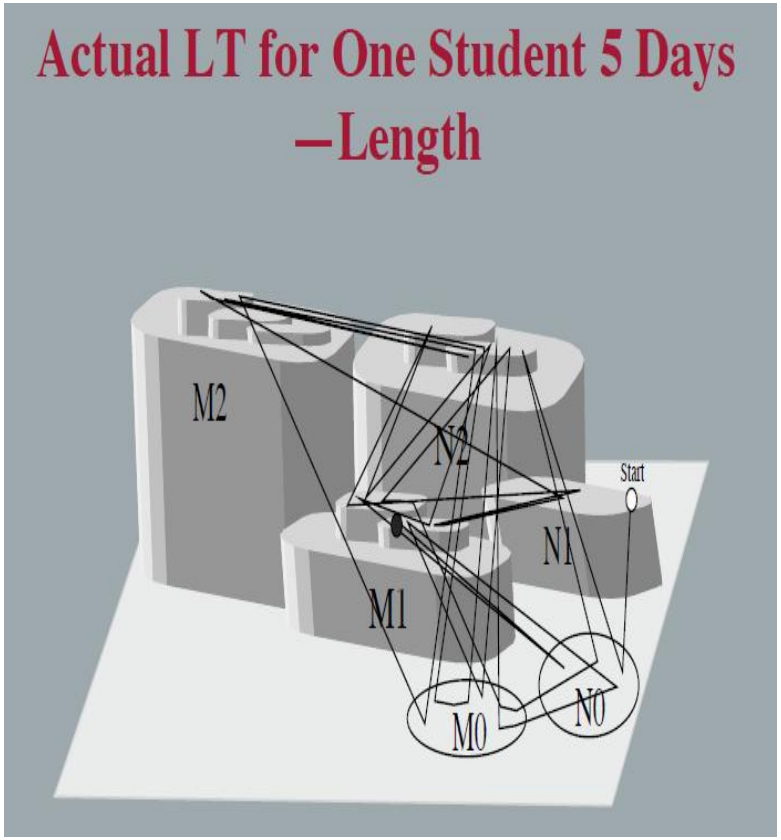
- Identify a content area and the goal level understanding
- Recognize that children's background knowledge are diverse and are a relevant starting point
- To predict what students can do today given what they did yesterday
- To recognize the meaning of students' errors
- To better serve students with diverse needs, by having a guideline to modify material

About Learning Trajectories/Progressions

Table 1. Development of Equal Partitioning and Unitizing

Objects	3 objects + 5 objects = 8 objects
Pure numbers	3 ones + 5 ones = 8 ones
Groups of objects	3 groups of 10 objects + 5 groups of 10 objects = 8 groups of 10 objects = 80
Groups of 10 ones are tens	3 tens + 5 tens = 8 tens
Equal lengths are units	3 inches + 5 inches = 8 inches
A length can be equipartitioned into equal sized units.	: ¼ + ¼ + ¼ + ¼ = 1
A part of 1 inch, ¼ inch, can be counted, added, etc. as a unit	3 (¼ inches) + 5 (¼ inches) = 8 (¼ inches)
Unit fractions as pure numbers can be counted, added and multiplied	3(¼) + 5(¼) = 8(¼) = 8/4
Expressions with letters can be read as uncalculated numbers	3(x + 1) + 5(x+1) = 8(x+1)

(Daro, 2011)



(Battista, 2010)

Learning Progressions on the Web

- Learning Progression Frameworks Designed for Use with the CCSS in Mathematics K-12, Karin K Hess:
  - Http://www.nciea.org/publications/Math_LPF_KH11.pdf
- Progressions Documents for the Common Core Math Standards
  - http://ime.math.arizona.edu/progressions/#products
- Learning Progressions connected to the CCSS in poster format ($20 each)
  - http://www.wirelessgeneration.com/posters

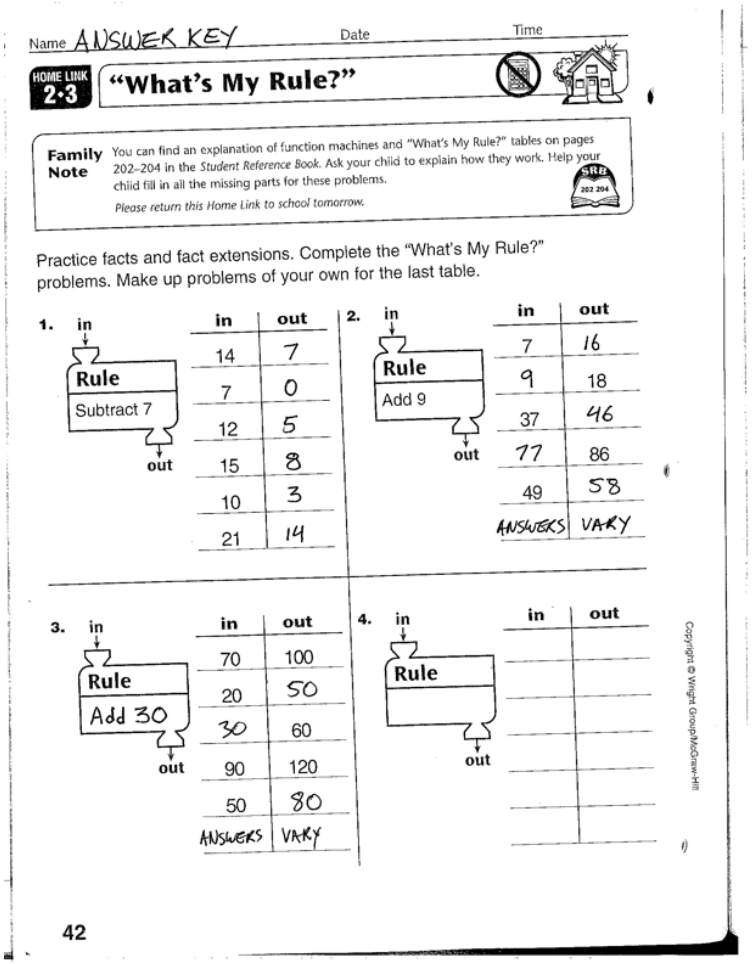
- How learning progressions/trajectories can help improve instruction
- How to use class work more effectively
- How to differentiate instruction and find resources
- Comments? Feedback?

Example A: Everyday Mathematics (EM) "What's My Rule?" Grade 3, Unit 2

Evaluating a student's response

Example A

"What's My Rule?":
Function Machines; Input/Output Machines; Tables



Name ANSWER KEY Date Time

HOME LINK 2·3 **"What's My Rule?"**

Family Note You can find an explanation of function machines and "What's My Rule?" tables on pages 202–204 in the *Student Reference Book*. Ask your child to explain how they work. Help your child fill in all the missing parts for these problems.

Please return this Home Link to school tomorrow.

SRB 202 204

Practice facts and fact extensions. Complete the "What's My Rule?" problems. Make up problems of your own for the last table.

1. in → Rule: Subtract 7 → out

in	out
14	7
7	0
12	5
15	8
10	3
21	14

2. in → Rule: Add 9 → out

in	out
7	16
9	18
37	46
77	86
49	58
ANSWERS	VARY

3. in → Rule: Add 30 → out

in	out
70	100
20	50
30	60
90	120
50	80
ANSWERS	VARY

4. in → Rule: ____ → out

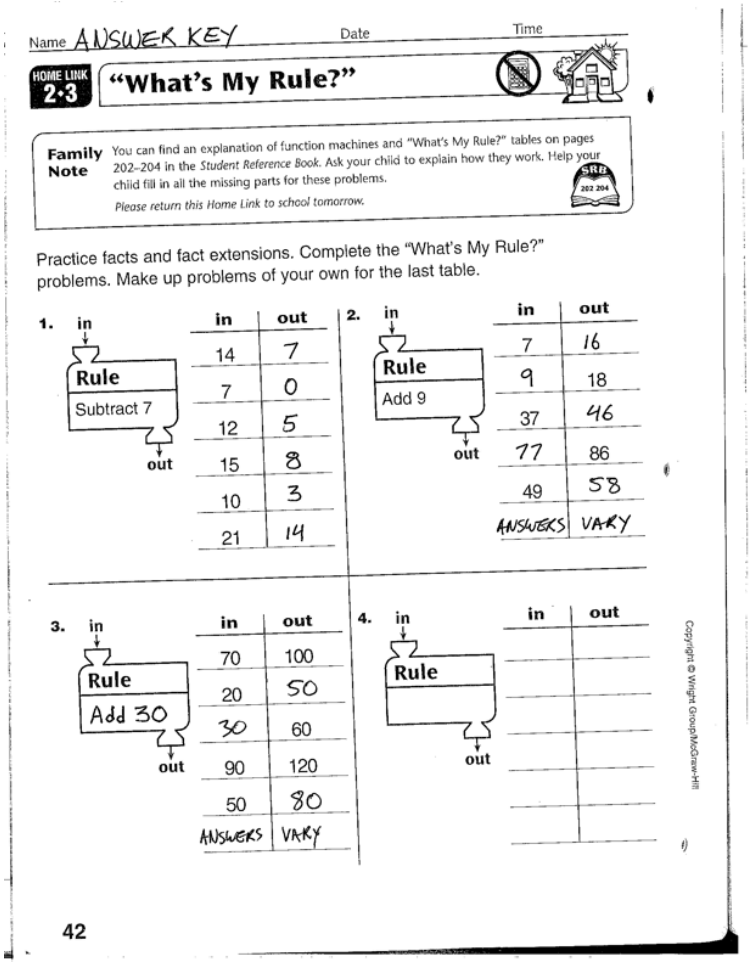
in	out

42

- What could this activity tell you about **student thinking and understanding?**
  - How might students **think** about this activity? **Respond?**
  - How could you **interpret** students' responses?

Example A

"What's My Rule?": Curricular Goals and Key Concepts and Skills



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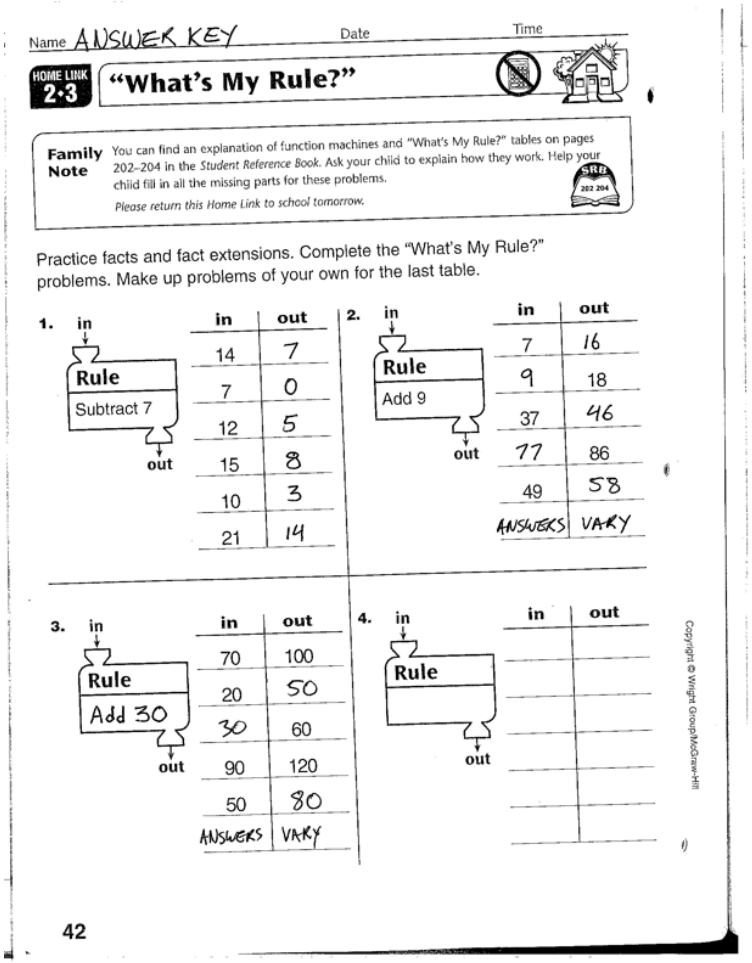
- Patterns functions and algebra
  - Describe rules for patterns and use them to solve problems
  - Describe rules and patterns in "What's My Rule?" tables; use them to solve addition and subtraction problems
  - Use basic facts to compute extended facts

"What's My Rule?"



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- What types of errors would you expect?
- What will we learn when the answer is correct? Incorrect? Partially correct?

Samples of Student work

- Any surprises?
- Anything you want to add?

Earlier, we asked:

- "What types of errors would you expect?"
  - Were any of your predictions confirmed?
- "What will we learn when the answer is correct? Incorrect? Partially correct?"
  - Do you still agree with your answers?

START HERE

ASSESSMENT OF STUDENTS

- **Who are they?**
- Where are they now?
- What do they need/when do they need it?
- **Where should I begin?**

Set high, worthwhile, and appropriate goals for these students, at this time, in this setting.

Implement instruction designed to attain these goals.

Evaluate student learning in light of the goals and the instruction.

Provide timely, meaningful feedback to students about their level of accomplishment of the targeted goals.

Reflect on student learning, the effectiveness of instructional design, particular concerns and issues.

Set new high and worthwhile goals that are appropriate for these students at this time.

(Einhorn, 2002); www.nbrc.illinoisstate.edu

- How learning progressions/trajectories can help improve instruction
- How to use class work more effectively
- How to differentiate instruction and find resources
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Differentiating Instruction

- Evaluating student understanding from the student work
- Make inferences about what students know and can/will do
- Differentiate instruction based those inferences

Differentiating Instruction: Feedback

- **Feedback is a form of differentiation**
- Provide immediate feedback
  - Connected to moving learning forward
  - Feedback can be from teacher, **self-assessment**, and/or classmate feedback
  - Give feedback in ways that students learn more
    - i.e. Feedback is based on a plan/expectation/theory of how student will progress – student should have a sense of where they are headed

(Black & Wiliam, 1998)

Example A

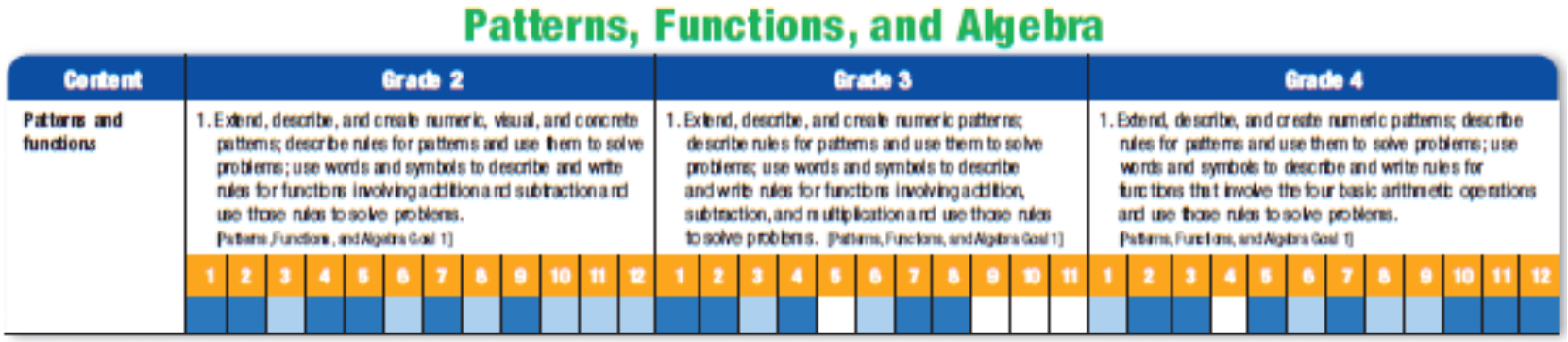
Differentiating Instruction: Modifying the activity

- Using curricular materials to modify the activity
  - Finding materials within the curriculum to address the needs of your students
  - Using the curricular materials to vary the content, the process, or the product of what already exists
- Using supplemental materials

Example A

Differentiating Instruction: Using Curricular Materials

- Find materials within the curriculum
  - Reading the teacher's materials
  - Get materials from prior and subsequent grades



Patterns, Functions, and Algebra

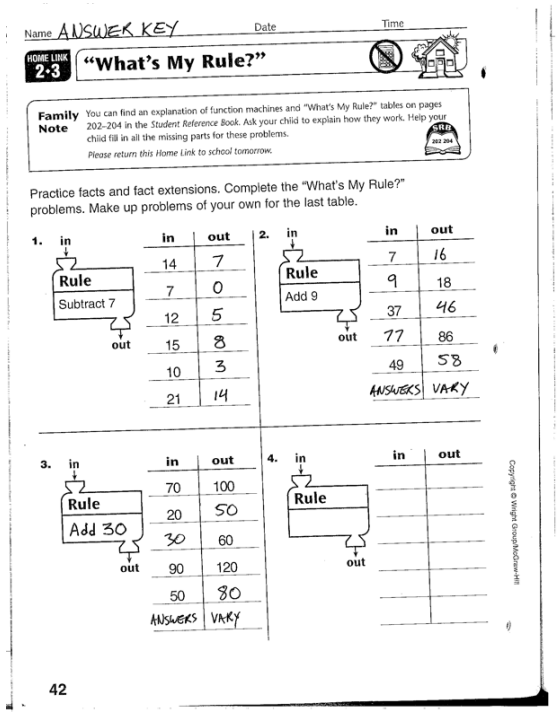
Content	Grade 2	Grade 3	Grade 4
Patterns and functions	1. Extend, describe, and create numeric, visual, and concrete patterns; describe rules for patterns and use them to solve problems; use words and symbols to describe and write rules for functions involving addition and subtraction and use those rules to solve problems. [Patterns, Functions, and Algebra Goal 1]	1. Extend, describe, and create numeric patterns; describe rules for patterns and use them to solve problems; use words and symbols to describe and write rules for functions involving addition, subtraction, and multiplication and use those rules to solve problems. [Patterns, Functions, and Algebra Goal 1]	1. Extend, describe, and create numeric patterns; describe rules for patterns and use them to solve problems; use words and symbols to describe and write rules for functions that involve the four basic arithmetic operations and use those rules to solve problems. [Patterns, Functions, and Algebra Goal 1]

Everyday Mathematics Gr 3

Example A

Modifying the Activity: varying the content, the process, or the product

- **Identify student understanding of key concepts and skills**



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- Applying a rule in the rule box
- Directionality
- Inverse operations (working backwards)
- Addition and Subtraction facts

Modifying the Activity: varying the content, the process, or the product

Example A

Applying a rule in the rule box

- Finding the rule
  - Students make up rule and complete the table using counters, then students make up the rule and fill in both sides of table, using counting numbers to see the patterns and then rolling number cube(s) for the input
  - Use a blank template. Both students create tables, write the rule on a different piece of paper, erase the rule on the other paper and trade with a partner, seeing if the partner can figure out the rule

Modifying the Activity: varying the content, the process, or the product

Example A

Directionality

- Graph the ordered pair. Then, draw a line and ask the students to create a table and generate the rule
- This seems to be a concern when the missing number is sometimes an input number and sometimes an output number. Have students indicate what the rule does, then have student tell if the number to go in the blank will be bigger or smaller than the number that is present.

Modifying the Activity: varying the content, the process, or the product

Inverse operations (working backwards)

- Practice with fact family triangles
- Write number sentences that correspond to the table
  - i.e. Rule +2, input unknown: ___+2 = 5

Modifying the Activity: varying the content, the process, or the product

Example A

Addition and Subtraction facts

- Change the numbers: Using larger/smaller numbers
  - Kids create a new problem to share with a partner
  - Set criteria (i.e. numbers between 100 – 200, certain criteria for the rule) for student to making own "What's My Rule" table using the blank template
  - Go to lessons from a previous year to find an appropriate strategy that teaches what the student is not able to do

Modifying the Activity: : varying the content, the process, or the product

Example A

Developing Conceptual Understanding and Strategic Knowledge

- Create word problems to depict the table
- Make a point graph of the row numbers
- Have students make a drawing, say a line where the input is a constant number of inches added (say 3) and the output is the length of the line. Then chart the data

How can we change the content, the process, and/or the product?

LESSON 4•6

Place-Value Puzzles

1. The digit in the thousands place is 6.

   The digit in the ones place is the sum of the digits in a dozen.

   The digit in the millions place is $\frac{1}{10}$ of 70.

   The digit in the hundred-thousands place is $\frac{1}{2}$ of the digit in the thousands place.

   The digit in the hundreds place is the sum of the digit in the thousands place and the digit in the ones place.

   The rest of the digits are all 5s. ___ ___ , ___ ___ ___ , ___ ___ ___

2. The digit in the tens place is 2.

   The digit in the ones place is double the digit in the tens place.

   The digit in the hundreds place is three times the digit in the tens place.

   The digit in the hundred-thousands place is an odd number less than 3.

   The digit in the millions place is $\frac{1}{3}$ of 15.

   The rest of the digits are all 9s. ___ ___ , ___ ___ ___ , ___ ___ ___

Everyday Mathematics, Grade 5 Math Journal 1, p 113

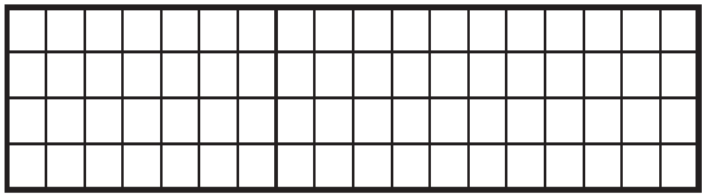
Example B: Math Trailblazers (MTB) Break-Apart Multiplication

Evaluating a student's response

Math Trailblazers
"Break-Apart Multiplication", Gr. 4 Field Test Edition

10. Find the number of squares in the rectangle below using the break-apart method.

- Break the rectangle into parts to make it easier to multiply. Write number sentences to show the number of squares in each part.
- Write a number sentence to show how you found the total number of squares in the large rectangle.



- What could this activity tell you about **student thinking and understanding**?
  - How might students **think** about this activity? **Respond**?
  - How could you **interpret** students' responses?

Example B

Math Trailblazers: Curricular "Key Ideas"

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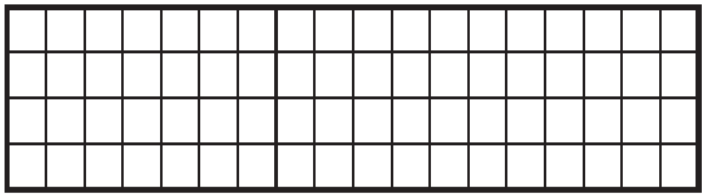

- Use arrays to solve multiplication and division problems
- Use break-apart products to solve multiplication problems with larger numbers

“Break-Apart Multiplication”

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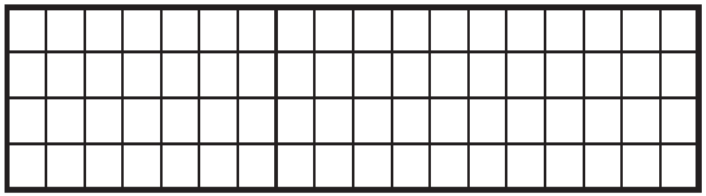
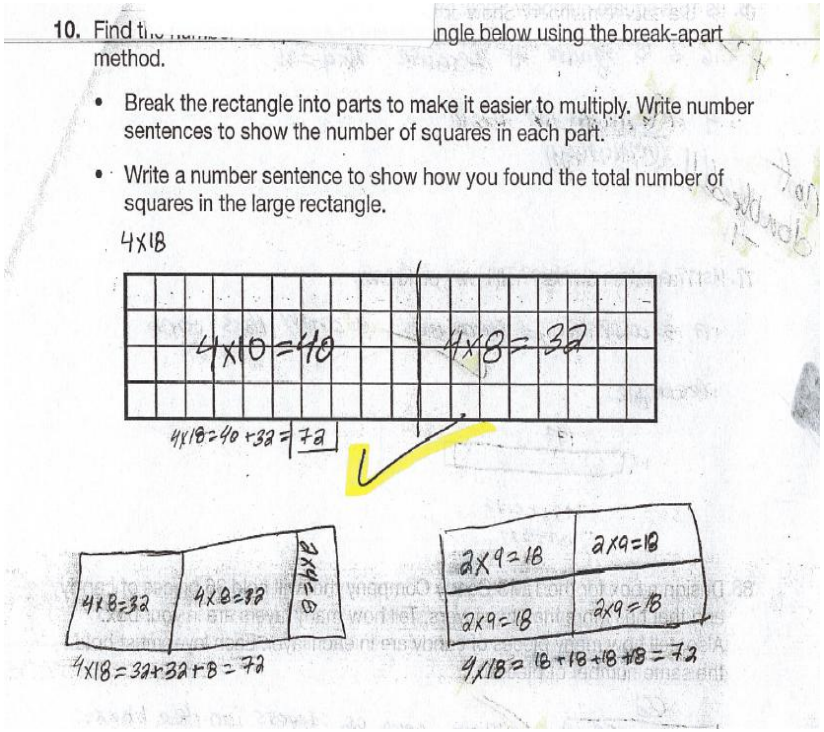
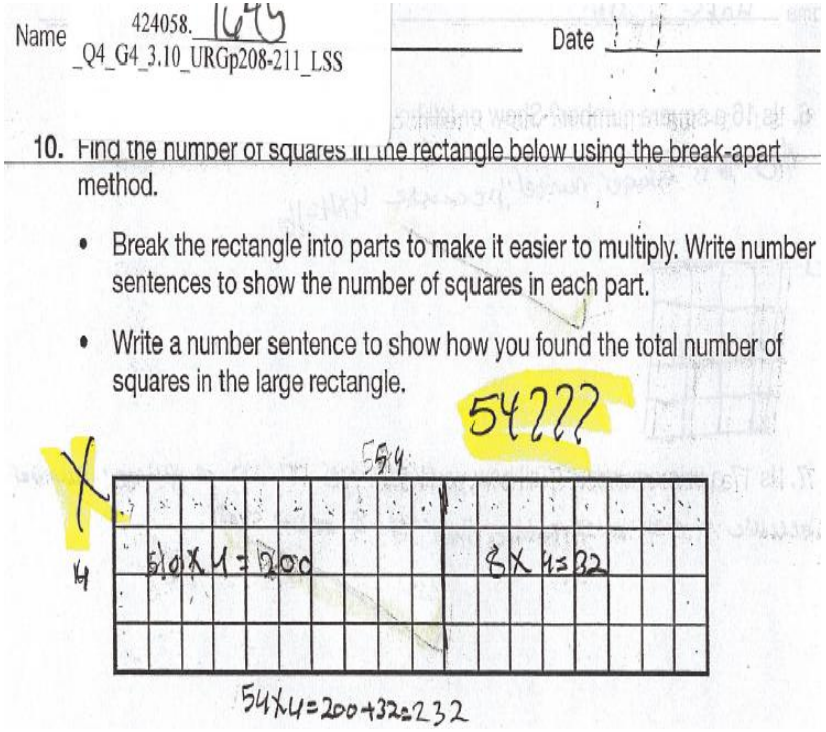


- What types of errors would you expect?
- What will we learn when the answer is correct? Incorrect? Partially correct?

Example B

Examples of Student Work





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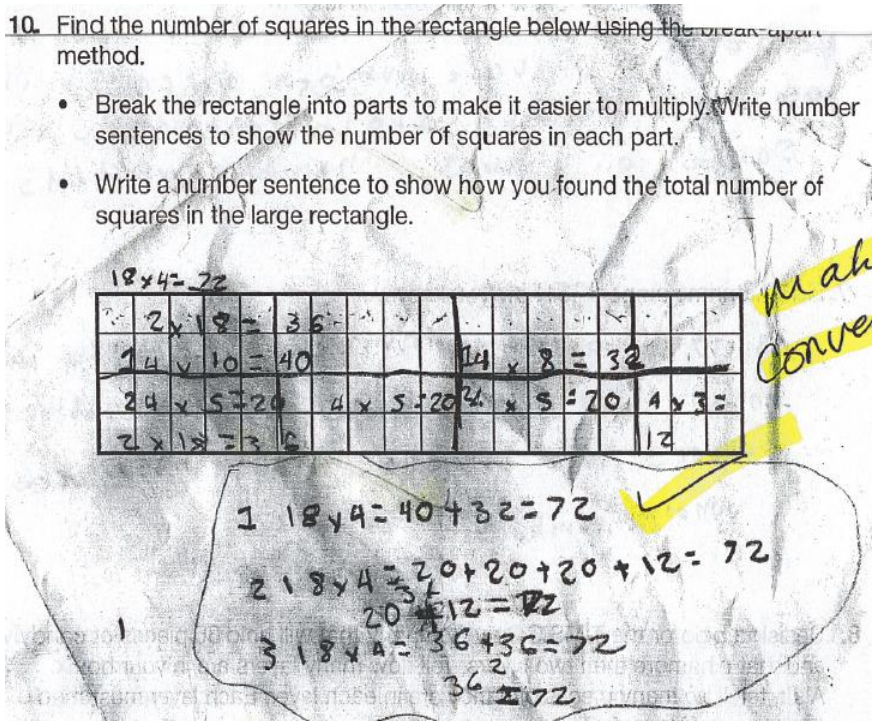

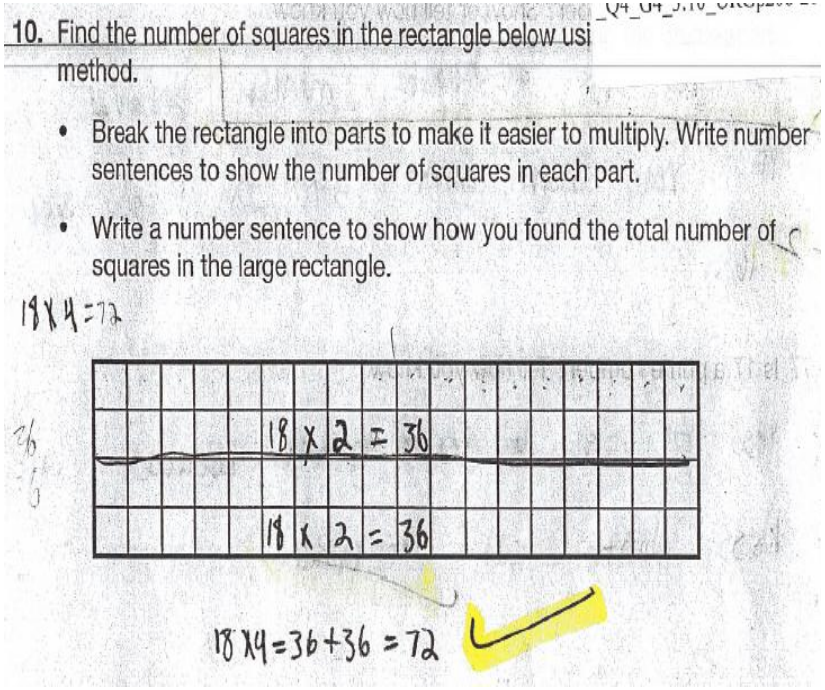

Samples of Student work

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Differentiating Instruction

- Feedback
- Modifying the Activity:
  - Using Curricular Materials
  - Using Supplemental Materials

Example B

Using Learning Progressions to understand where your students are at

Where do the students' responses fit in the progression?

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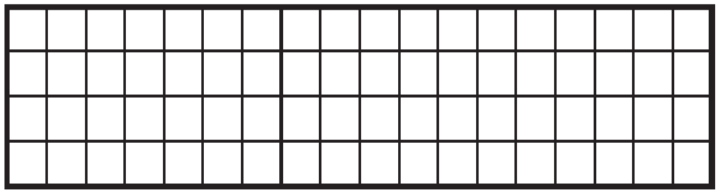

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Expressions with letters can be read as uncalculated numbers	3(x + 1) + 5(x+1) = 8(x+1)

(Daro, 2011)

Example B

Modifying the Activity: varying the content, the process, or the product

- **Identify student understanding of key concepts and skills**

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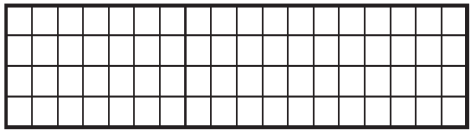


- Counting
- Multiplication
- Multiplication facts
- Using Arrays
- Using break-apart products to solve multiplication problems

Example B

Modifying the Activity: varying the content, the process, or the product

Counting

- Student draws a dot when the square is counted, or shades in the column while skip-counting (for students who have trouble keeping track of the count, or who are not sure if the line or the area of the square is what is being counted)
- Discuss need for staying focused on the count- importance of being accurate – have two students count squares and see if come up with same answer

Example B

Modifying the Activity: varying the content, the process, or the product

Multiplication, Multiplication facts, and Using Arrays

- Make arrays with tiles
- Create smaller/larger arrays

- Use known "factor" number for column, or use the factor that is being practiced

Example B

Modifying the Activity: varying the content, the process, or the product

Using break-apart products to solve multiplication problems

- Use grid paper instead of picture in the book and cut the grid paper into parts, then multiply and add amounts

Example B

Modifying the Activity: varying the content, the process, or the product

Developing Conceptual Understanding and Strategic Knowledge

- Student writes story to match the array, or uses a story to make the array.

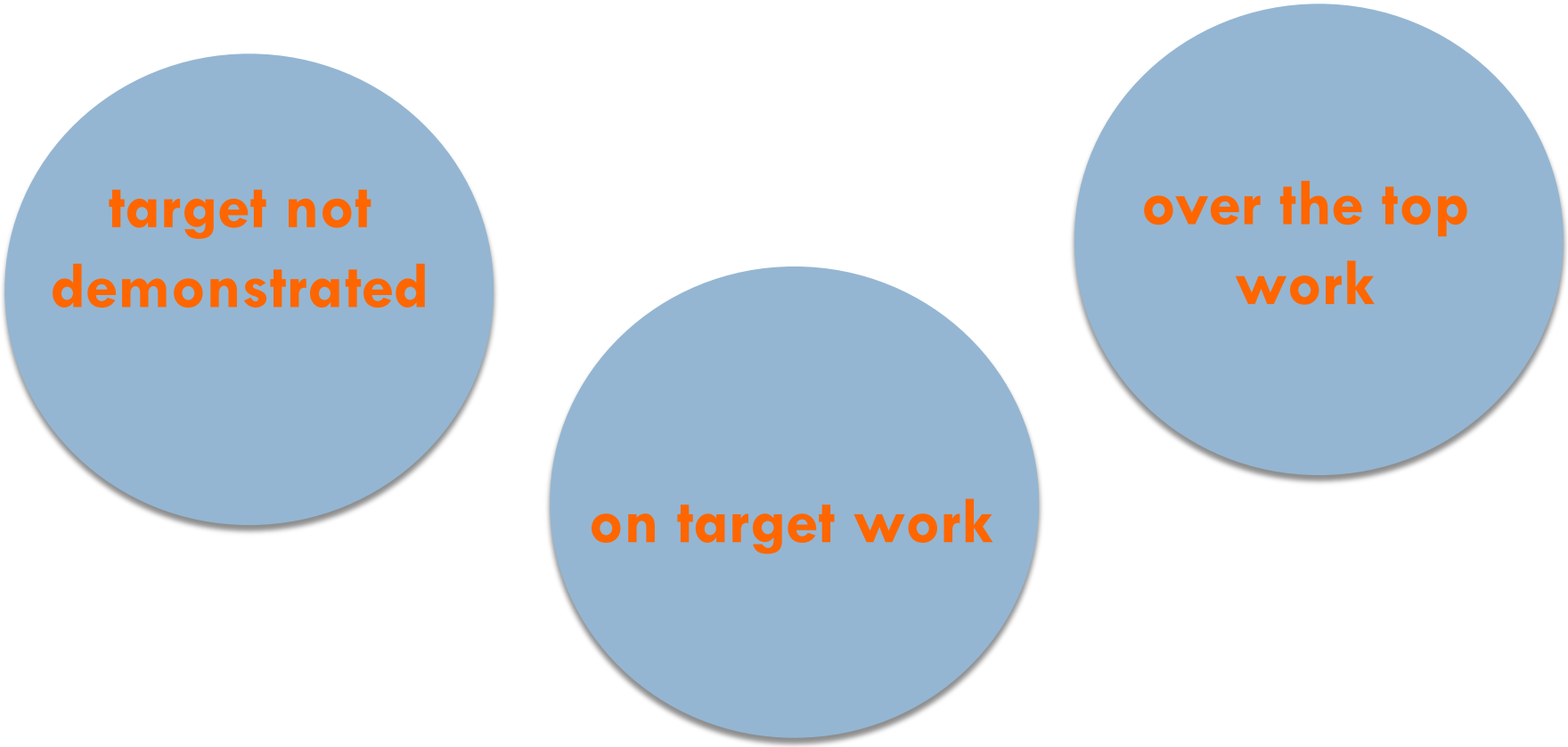
More resources for differentiation

Reflecting on student work

- Did the student's work reflect understanding of the goals?
- Did the student exhibit typical responses? Did the student show more or less understanding?
- Did the student interpret the activity differently than intended (i.e. varying knowledge and cultural backgrounds)?
- What was the range of student responses within the classroom? How does this inform subsequent instruction?

Differentiating Instruction: Grouping

- The “3 piles” method – make three piles



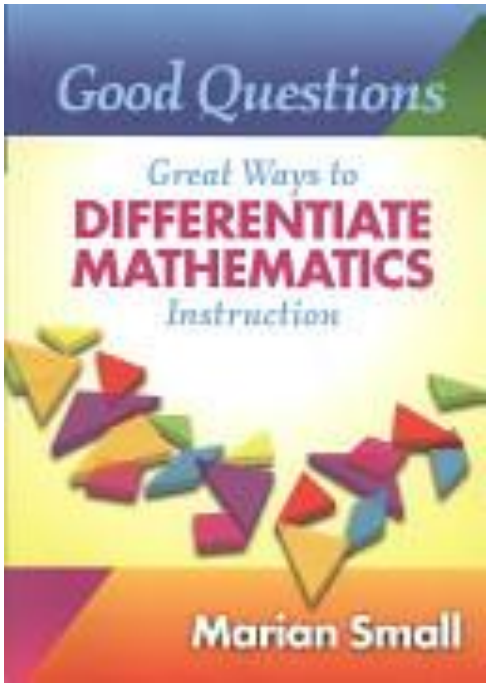
Differentiating Instruction: SCAMPER

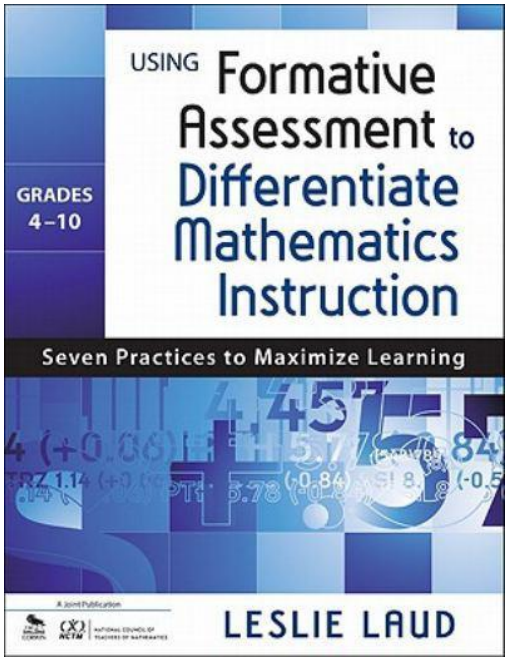
- **SCAMPER is based on the notion that everything new is a modification of something that already exists.** Each letter in the acronym represents a different way you can play with the characteristics of what is challenging you to trigger new ideas:
- **S** = Substitute
- **C** = Combine
- **A** = Adapt
- **M** = Magnify/Minify/Modify
- **P** = Put to Other Uses
- **E** = Eliminate (or Minify)
- **R** = Rearrange (or Reverse)

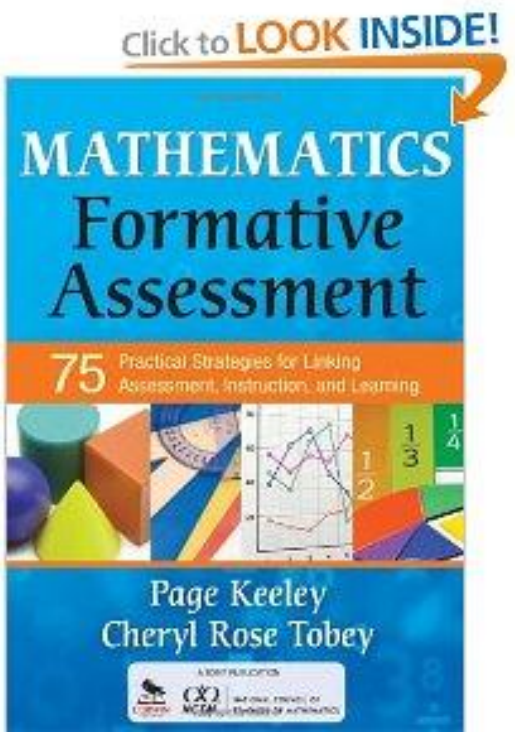
Retrieved at: http://litemind.com/scamper/

Also helpful: http://www.brainstorming.co.uk/tutorials/scampertutorial.html

NCTM Resources



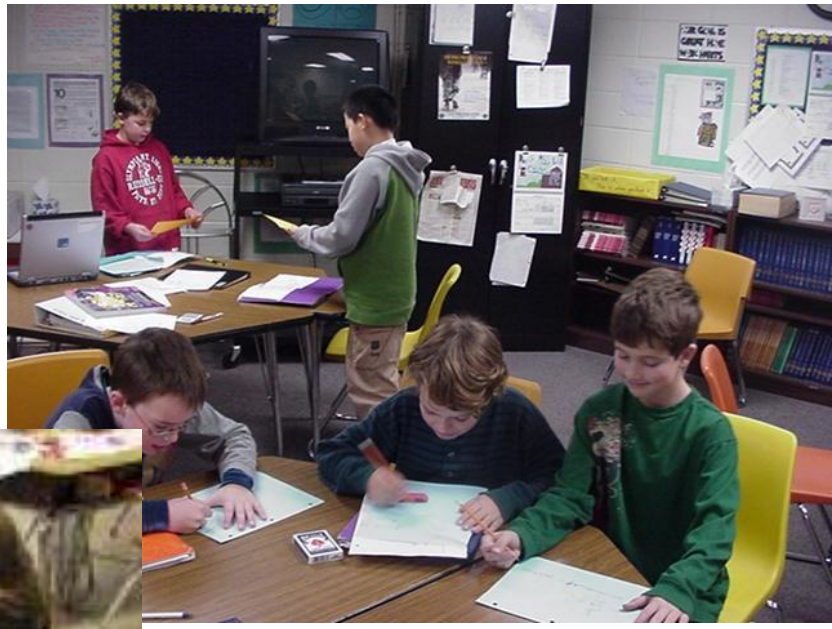




Summary

- Learning progressions/trajectories enable us to evaluate learning along a continuum
- Student work and patterns of errors provide valuable information about particular facets of understanding
- Meaningful differentiation not only focuses on content and process, but also works towards the development of strategic thinking, problem solving, and students disposition towards mathematics

Building an Environment for Self-Directed Learning



- How learning progressions/trajectories can help improve instruction
- How to use class work more effectively
- How to differentiate instruction and find resources
- Comments? Feedback?

References

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- Hess, K (2012). *Learning Progressions in K-8 Classrooms: How Progress Maps Can Influence Classroom Practice and Perceptions and Help Teachers Make More Informed Instructional Decisions in Support of Struggling Learners*. Minneapolis: National Center on Educational Outcomes.
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CCSS

- Common Core State Standards http://www.corestandards.org/
- Model Content Frameworks http://www.parcconline.org/parcc-content-frameworks
- Compare NAEP to Common Core http://www.achieve.org/comparing-common-core-state-standards-mathematics-and-naep-framework
- Achieve http://www.achieve.org/PARCC
- Smarter Balanced Assessment Consortium http://www.smarterbalanced.org/resources-events/faqs/

CCSS related Math Progressions

http://ime.math.arizona.edu/progressions/#products

Draft 3–5 progression on Number and Operations—Fractions

Data part of the K–5 progression on Measurement and Data

Draft K–5 Progression on Number and Operations in Base Ten

Draft K–5 Progression on Counting and Cardinality and Operations and Algebraic Thinking

Draft 6–8 Progression on Expressions and Equations

Draft 6–7 Progression on Ratios and Proportional Relationships

Draft 6–8 Progression on Statistics and Probability

More Learning Trajectory/Progressions

- Counting:
  - Clements, D. & Sarama, J. (2009). Learning and teaching early math: The learning trajectories approach. New York: Routledge.
- Geometric Reasoning:
  - Battista, 2007, 2009; Clements & Battista, 1992
- Place Value:
  - Battista, 2007, 2009; Clements & Battista, 1992
- Modeling Data & Other Concepts:
  - Lehrer, Kim, & Schauble, 2007; Lehrer, Konold, & Kim, 2006
- Equi-partitioning:
  - Confrey, 2008; Confrey, Maloney, Nguyen, Mojica, & Myers, 2009
- Learning Progressions connected to the CCSS in poster format ($20 each) http://www.wirelessgeneration.com/posters

Resources for Teachers

- Student-created problems – grade the exercise one year, then edit and use to differentiate instruction the following year
- Ken Ken from Math Forum
- Illuminations or other web-based resources (i.e. Sheppard Math)
- Figure This! Problems from http://www.nctm.org/
- Dynamath Magazine; Scholastic math
- TOPS math cards
- Logic problems
- Marcy Cook tile problems
- Critical Thinking Press: Building Thinking Skills –
  - http://www.criticalthinking.com
- Great Source - Daily Math
  - http://www.greatsource.com