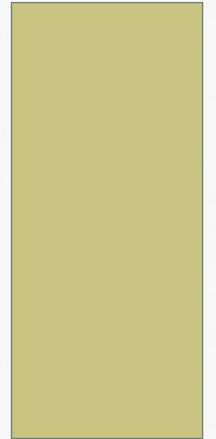


# REASONING AND SENSE MAKING: TRANSLATING VISION INTO PRACTICE

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# WHAT IS REASONING AND SENSE MAKING?

- *Focus in High School Mathematics: Reasoning and Sense Making* (NCTM, 2009)
- Calls high school math programs to shift away from a focus on memorization of procedures.
- Suggests that fostering students' reasoning and sense making abilities develops mathematically literate citizens

## FRAMING THINKING ABOUT CURRICULAR RECOMMENDATIONS:

- “The crucial point is that the proposal is not to be regarded as an unqualified recommendation but rather as a provisional specification claiming no more than to be worth putting to the test of practice.” (Stenhouse, 1975, p. 142).

What are you willing  
to put to the test in  
your teaching?

# TEACHERS MAKING CHANGES

- 7 math teachers from 6 different high schools
- All interested in testing these recommendations in their practice
- Teaching high school math content
- 0-11 years experience (mean: 3.5)
- Met every 3 weeks during the school year
- We discuss R&SM and then other related articles from NCTM journals
- Plan actions, share progress and struggles

# WHAT IS REASONING AND SENSE MAKING?

- “Reasoning can be thought of as the process of drawing conclusions on the basis of evidence or stated assumptions” (p. 4)
- “Sense making is developing understanding of a situation, context, or concept by connecting it with existing knowledge” (p.4).

## MEANINGFUL EXCERPT

“A focus on reasoning and sense making...will ensure that students can accurately carry out mathematical procedures, understand why those procedures work, and know how they might be used and their results interpreted” (p. 3).

# MEANINGFUL EXCERPT

“Reasoning and sense making should occur in every mathematics classroom every day. In such an environment, teachers and students ask and answer such questions as “What’s going on here?” and “Why do you think that?” Addressing reasoning and sense making does not need to be an extra burden for teachers struggling with students who are having a difficult time just learning the procedures” (NCTM, 2009, p. 5-6)



# THE GAP ACTIVITY

My vision: Lecture discussion consists of equally dispersed curious questions about why math works the way it does. All questions are accepted regardless of how mundane and collectively the students are able to answer each other's questions or thoughts as a group. Basic prerequisite knowledge is known but not necessarily mastered, and homework is less practice based but more skill and application based. Reality: Silence, mostly uncomfortable, reigns supreme. Usually broken by either a bored student complete with eyes rolling or myself having lost patience. Time is often spent on basic arithmetic skills. Homework is often drill based to make sure that they can simply do it. (10/25/10, meeting 1).

## CONTEXT OF PETER'S WORK:

- Frustrated with the way the teacher teaches
- Really wants to see his students start critically thinking. I swear that I could say, "Your lesson today is to learn that  $5 + 8 = 22$ ." And they will just write  $5 + 8 = 22$ , and not even think a thing about what they're actually writing, whether it even makes sense at all.
- "I want to see my students start critically thinking."
- Introduced tasks/problems that require discussion

# FOCUS OF PETER'S WORK

- “keep students evaluating ideas”
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I want to make it so that they cannot just memorize what I say, because what I say may not be correct. And they have to think, “Is this right? Is this not? Why does this work? Why does it not?” And hopefully they’ll start spitting out ideas, and I’ve been starting to fire their questions back to them, instead of just answering the questions. I just say, “Well, what do you guys think?” And I know sometimes I accidentally answer the questions, because I’m just - because I’m a teacher and I always thought that’s what I’m supposed to do.

# PETER'S CHALLENGES

One of the biggest challenges of pursuing my dream class environment is that, despite the fact that it is a better research-based practice and that it will be better for the everyone, it will (and has already) be fought by the students partly because it will require more mental in-class work than they are used to and partly because it will go against the norm of what my students have likely experienced for the better part of a decade, especially in math classes, and changes from such an expected routine are not often met with initial positive reactions.

# PETER'S FINDINGS

- Student... to great discussions

- Student... I'm shutting up more and more. And  
co... I'm catching myself shutting up,  
which is good. Because I can clearly  
tell that there is situations where I want  
to and have to specifically keep my  
mouth shut. And, for the most part, it's  
gone pretty well. I allowed a wrong  
concept to sit on my board during the  
brief notes time for like 15 minutes.
- Student...
- Av... stu...
- Te...

# CONTEXT OF LOGAN'S WORK

- Lectures and formal notes
- Struggled with engagement in “lower-level” math courses
- Appreciated the collegiality with math teachers
- Interested in “baby steps”

# FOCUS OF LOGAN'S WORK

- Reasoning: “Using things you know to solve problems you don't know how to do”
- Transform lectures into discussions
- Connect new concepts to prior knowledge
- Use wait time
- Focus on “lower level” classes

# LOGAN'S CHALLENGES

- student
- funny
- Pro

This semester is more difficult because of the topics... teaching Logs right now. Where do all these logarithm things come from? That's been a struggle. I feel like I've given too much information... just kind of said "Here it is." Which is something I worked so hard to get away from last nine weeks... Kind of backtracked a bit.

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to answer... question or wait it out.



# LOGAN'S FINDINGS

- Techniques for accountability
- Developed a new awareness

I think I changed more than the students did through my action research. More than anything, I became more aware about myself and how I was teaching. I found myself not giving enough wait time, not making everyone get involved, and not making all of the students reason in my math class. Now when I am teaching, I am so much more aware of the things that I am doing. It is as if things are slower and more clear as I am teaching.

# CONTEXT OF SARAH'S WORK

- Curriculum based on textbook
- Lessons involved short-answer questioning focused on right answers
- Concerned about students' inability to think critically
- Students struggled with story problems

# FOCUS OF SARAH'S WORK

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In the past, I would ask students, "What is the next term of the resulting polynomial?" I decided to change the way I discuss this topic. This year, I asked, "How do you get the next term?" This focused more on the justification instead of the answer.

- In

*I am constantly trying to change my focus from procedures and answers to communication and justification.*

their own

SOLUTIONS

# SARAH'S CHALLENGES

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- Engo
- Givi
- mista

I have noticed that I tend to give the answer when a student states something that is incorrect. About 50% of the time, I will correct the statement myself and move on. I need to work on continuing the problem even if someone makes a mistake. Then, I can see if the students can catch the mistake themselves. I need to always be conscious of this.

# A REASONING “CITING”

Students were given an equilateral triangle with a side length of 10 units. They were asked to find the area of the triangle. Many groups tried to visualize the triangle in different ways. They tried three methods: using the formula for the area of a triangle, using the formula for the area of a square, and using the formula for the area of a circle. Three groups gave the answer 50. They went with the answer that had shown up twice. After their selection, they discussed the reasons why the third method did not work. (journal entry, 3/8/11)

Exploring wrong answers provides good opportunities for fostering reasoning and sense making.

# SARAH'S FINDINGS

- Students gradually change

Before, I wouldn't let them do it the way they wanted if they were wrong. A lot of times, I would just say, "Let me see this?" instead of just letting them do it their way. You know, I think it's okay to let them do it a different way. Even if it's the hard route. Just let them be, because that's the way they understand. Giving them that freedom.

(Sarah, final interview)

*I just feel that what I have been doing has been working. So, if it's working, why stop?*

## ALEXIS'S FINDINGS

“I've learned that I can forego the example problems I used to rely on. Instead, a well designed discussion can replace a lecture and be more effective. When I give examples, each example only applies to one kind of problem. However, if we spend the entire class discussing a problem, brainstorming a solution, and interpreting what we have found, the students can later apply that knowledge to the entire homework assignment.”

# COMMONALITIES

## CHALLENGES

- Student resistance
- Every Mathem Classroom Ev Day?

## OPPORTUNITIES

- 5 member

*The changes in my classroom have just begun. ... now the real work actually begins.*  
-Logan

ort



# COMMON GAINS

- Teachers progressively became more aware of their ways of influencing student's reasoning.
- Students participation gradually changed.
- Once opportunities for reasoning were developed, students surprised them with their creativity and ideas

# CONCLUDING THOUGHTS

- Teachers concluded reasoning is a thought process developed through experience
- Variations existed in how inherent reasoning and sense making was to doing mathematics
- Elements that teachers tested in their practice were a reflection of those elements that held meaning for them.

# REFLECTIONS ON R&SM (2009)

- First three chapters help teachers

I think the NCTM document is lacking in terms of being relevant to my classroom. Everything in the vignettes seems to be staged and works out perfectly. Well, my classroom is nothing like that. A lesson plan that I create may be completely thrown out the window in five minutes, or the whole class gets stuck on a certain aspect of the discussion or task. Plus, what if my students do not have the same level of work as the elaborate examples in the NCTM articles?

# BRAINSTORM!

- How can you foster more reasoning and sense making in your current classroom?

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# QUESTIONS FOR DISCUSSION

- Is student resistance to reasoning and sense making to be expected? Is it avoidable?
- How can curricular recommendations such as *Reasoning and Sense Making* better support teachers in the process of getting “there?”
- How can mathematics teacher educators support teachers interested in making changes?
- What do you think are the essential features in order for change to occur?

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