

How to Win Games Using Math

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This handout and additional handouts for students with teacher resources are available at <http://campus.houghton.edu/webs/employees/kcamenga/teachers.htm>

Central Questions in Game Strategy:

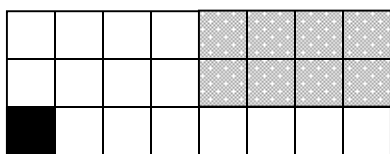
- Does the game have a winner? Does the game end? *These questions have some similarity to the mathematical question of whether an object is 'well-defined'.*
- Does one player have a winning strategy? If so, what is it? *If there is always a winner, one player or the other will have a winning strategy – but it can be VERY complicated to find. Combinatorial games are simple enough that students can analyze at least portions of the strategy. In these games, there are opportunities for students to think through different possibilities and map out game trees, to recognize similar or symmetric situations, and to reason using a variety of representations.*

Game Descriptions:

1. Chomp

Chomp is played on a rectangular grid, such as squares of a candy bar. (It can be helpful to use a piece of graph paper to play.) The lower left square is considered “poison”. Players take turns picking a square. With each choice, all squares above and to the right of the picked square are no longer available – they are eaten. The person forced to take the “poison” square loses.

Example: Playing on a 3x8 grid, the lower left square (in black) is the poison square. The first player chooses the square in the 2nd row and 5th column of the grid and all the shaded squares are eaten.



2. Nim

Nim is played starting with piles or rows of objects. Players take turns taking any number of objects from one of the piles. Whoever takes the last of the objects wins.

For example, suppose we start with three piles of M&Ms, the first with 10, the second with 8 and the third with 13. The first player can choose to take 8 M&Ms from the first pile. However he/she could not take 8 from the first pile and 2 from the second pile, since he/she can choose only from one pile.

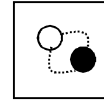
3. Sprouts

Start with a few dots (vertices). For the first few games, start with 2-4 vertices. Players take turns connecting two vertices with a curve (edge) and placing a new vertex along this edge. This is done following two rules:

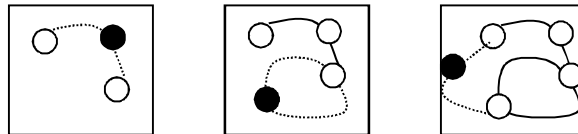
- 1) Each vertex can have at most three edges coming from it.
- 2) Edges must be drawn so that they do not cross or touch any other vertices than the two they are connecting.

If a player is not able to draw an edge according to the rules, the other player wins.

Note: You can draw an edge connecting a vertex to itself, such as:



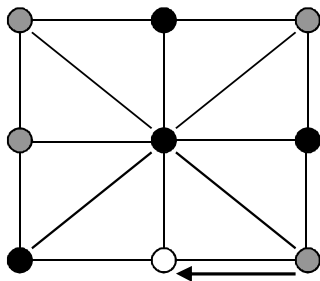
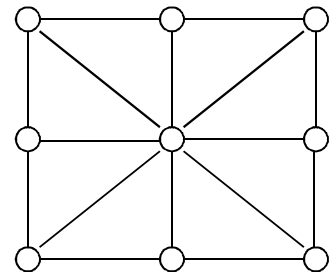
A game might start like:



(The new vertex is shown in black and the new edge is dotted in each step.)

4. Achi

This is a game played by the Asante people of Ghana, West Africa. It is played on a board like the one to the right. Each player starts with four counters (like X's and O's) and takes turns placing them on the board as in Tic-Tac-Toe, with the goal of getting a 3-in-a-row. However, if the game is a draw after each has played their four counters, they take turns sliding a counter along the lines into the space left empty. The winner is the first player to get 3-in-a-row.



For example, if the players have played their counters as at the left – first player in gray and second in black – and the game is a draw, the first player then must slide the counter in the bottom right corner into the open (white) space, and then the second player slides one of his/her pieces, etc.