

Multisensory Geometry: a Hands-on Approach to Reasoning, Proof and Logic

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Nadia Carrell, PhD
The Multisensory Training Institute of ASDEC
www.asdec.org
Rockville MD

Reasoning and Proof Standard for Grades 9-12

- recognize reasoning and proof as fundamental aspects of mathematics;
- make and investigate mathematical conjectures;
- develop and evaluate mathematical arguments and proofs;
- select and use various types of reasoning and methods of proof.

Mathematical Practices

Construct viable arguments and critique the reasoning of others.

“compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is “

Common Core State Standards

Critique the reasoning of others:



It's all about the language

conditional antecedent
And disjunction MODUS TOLLENS
 conjunction contrapositive consequence
statement Modus Ponens **Or** Conjecture
consequent postulate

Our Challenge

Take abstract concepts and make them concrete in order to make them understandable and memorable

CRA Instructional Sequence

- Concrete teaches the concept
- Representational provides practice and makes memorable
- Abstract calculates with numbers and symbols

Venn Diagrams and the Inclusive "OR"

- Do you like ice cream or lima beans?
- Do you like cabbage or candy?
- Do you like dog biscuits or liver?

Venn diagrams

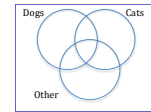
Concrete: construct diagrams using attributes of class (sports, where are our grandparents, clothing, etc)

Representational: Shade regions “and”, “or”, “not” (handout)

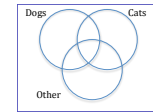
Abstract: Analyze diagrams with numerical values

VENN DIAGRAMS

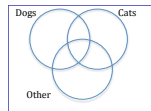
Students who have at least one dog and at least one cat



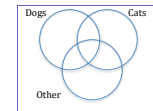
Students who do not have any pets or students who have a pet that is not a dog or a cat



Students who have a dog or cat or other pet



Students who have a pet that is not a dog or a cat



Truth Tables

Concrete/Kinesthetic: double-sided markers replace T and F (handout)

Determining Truth Values

Code the statements:

21 is divisible by 3 and 21 is not prime

Blue is a color and 7+3=10

				p
				b
				$\sim p$
				$b \vee \sim p$

		p
		$\sim p$

TRUTH TABLES

Conditional Statements

- Simple statements
 - statements vs. compound sentences
- Language: make it relevant
 - conditional
 - Converse
 - Inverse
 - contrapositive

Conditional statements

- One statement per index card
- Construct conditional
- Write out the compound sentence
- Code the sentences
- Write in symbols
- Repeat with converse, etc.

Modus Ponens and Modus Tollens

- Coding the statements aids in pattern recognition
- Code each statement (p,q,~q, etc) and conclusion (handout)
- Look for pattern of MP or MT

MODUS TOLLENS

If p you get enough money, then q you can buy a new car.

~q

You cannot buy a new car.

~p

Therefore, you don't have enough money.

$$p \rightarrow q$$

$$\sim q$$

$$\therefore \sim p$$

Proofs a real crowd pleaser

C is the midpoint of \overline{BF}

1. C is the midpt
2. $\overline{AC} \cong \overline{CE}$
3. $\angle ACB = \angle ECF$
4. $\overline{BC} \cong \overline{CF}$

1. Given
2. Def of Midpoint
3. Vertical Angle Theorem

Visual Theorem Bank

Reflexive Property = and \overline{AA}	$x = x$ $x + 5 = 17$ $2x = 2x$ $BA = BA$	$ST = AB$ $AR + TU = SU$ $BA = AB$
Symmetric Property = and \overline{BA}	$a = b$ $b = a$ $10 = 20$ $20 = 10$	$AB = BA$ $BA = AB$
Transitive Property = and \overline{AC}	$a = b$ $b = c$ $a = c$	$10 = 20$ $20 = 5$ $5 = 10$ $AB = DD$ $DD = FZ$ $AB = FZ$
Definition of Congruent Segments	$\overline{PQ} \cong \overline{RS}$ $PQ = RS$	equal Same measure

Visual Theorem bank

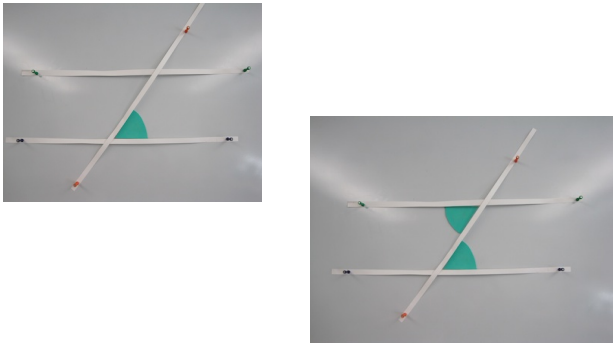
Complement Theorem	$m\angle DBA + m\angle ABC = m\angle DBC$ complementary angles $m\angle 1 + m\angle 2 = 90^\circ$
Def of Linear Pair	A pair of adjacent angles whose non-common sides are opposite rays
Supplement Theorem	supplementary angles $m\angle 1 + m\angle 2 = 180^\circ$
Vertical Angles Theorem	two nonadjacent angles formed by two intersecting lines. Vert \angle s are \cong $\angle 1 \cong \angle 3$ - vertical angle $\angle 2 \cong \angle 4$
Definition of Angle Bisector	A ray that divides an angle into two congruent angles.

Parallel Lines and Transversals

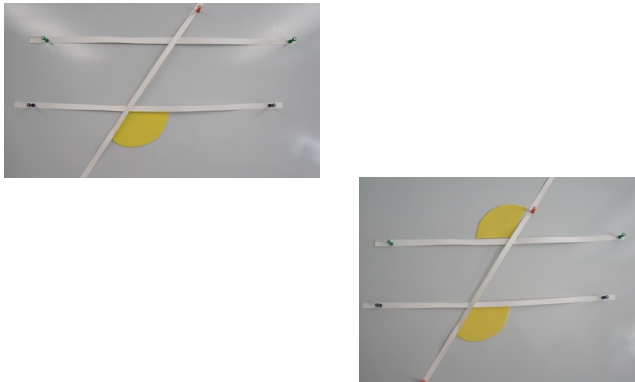
Interior	Exterior
Alternate	Corresponding
Vertical	Transversal

These terms must have meaning for the students

Concrete: Discovering Angle Relationships



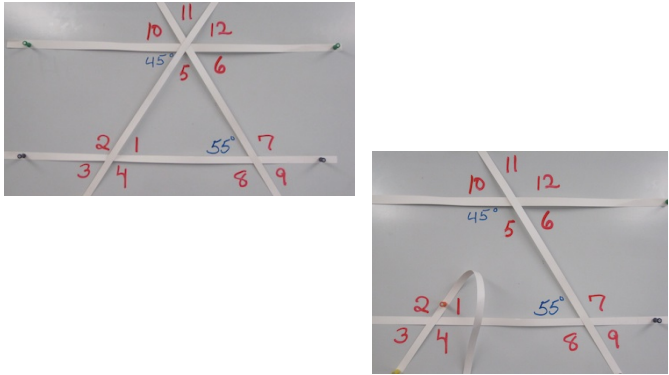
Discovering angle relationships



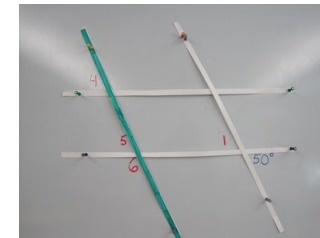
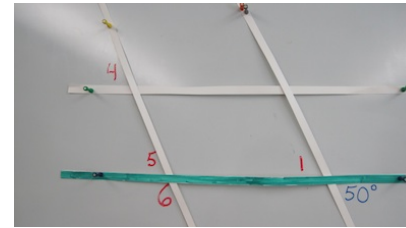
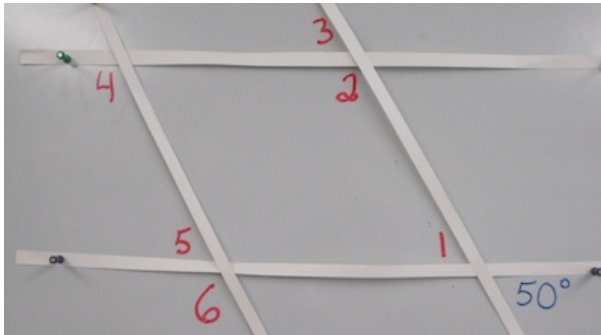
Discovering angle relationships



Clarifying angle relationships



Clarifying angle relationships



It really is all about the language..

*Include
Vocabulary Development Activities*

**“Hit my Rhombus
One more Time”**

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ncarrell@asdec.org

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