Using Origami Activities to Teach Mathematics

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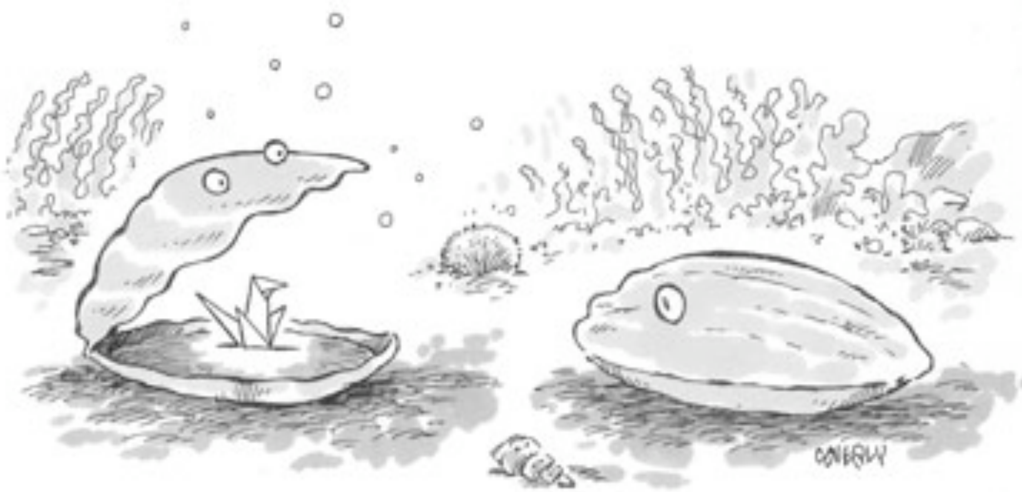
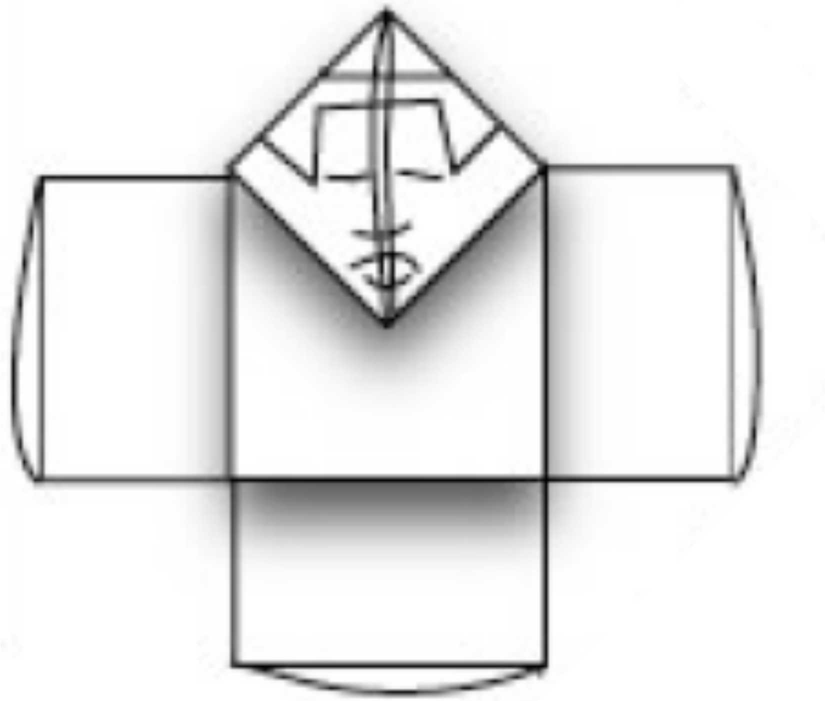
2012 NCTM Annual Meeting and Exposition
April 25-28, 2012



"I'M LOUSY AT PEARLS, BUT I'M GREAT AT ORIGAMI."

Origami Samurai Doll

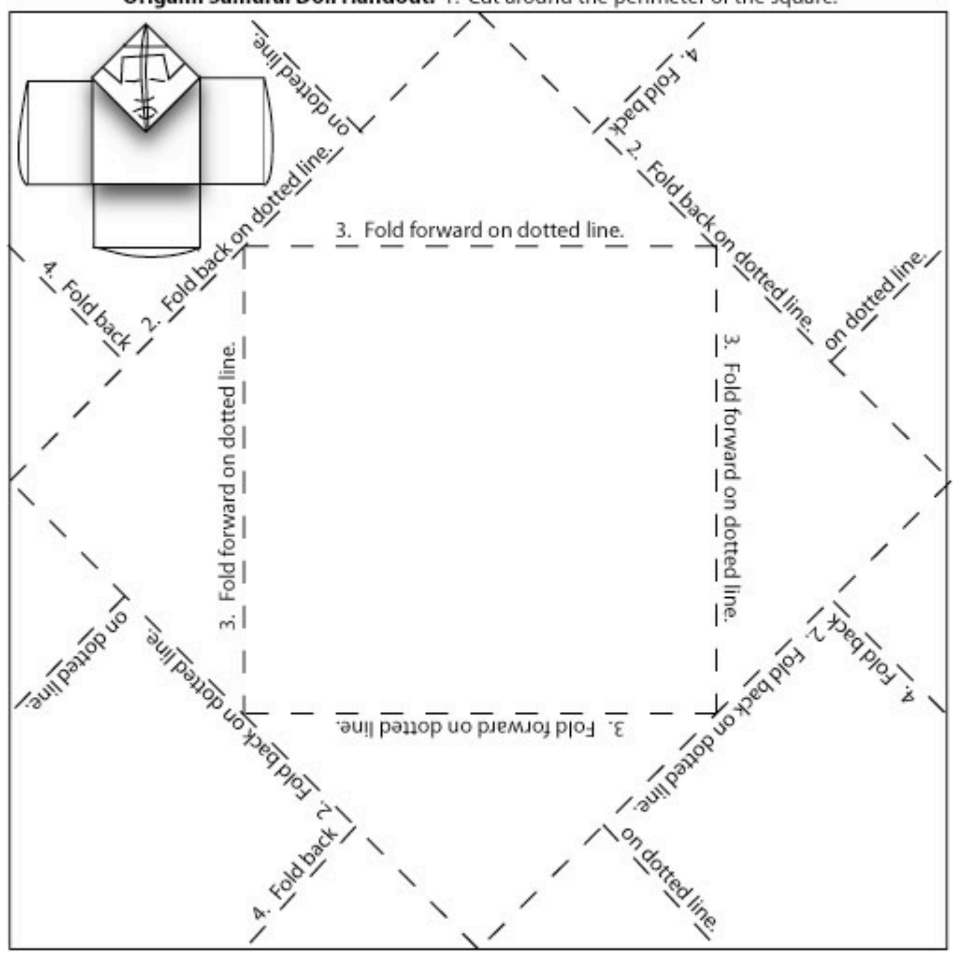
Objective: Students will construct a geometric shape, and identify geometric shapes and fractional parts of a region.



Origami Samurai Doll

- Before we start:
  - What geometric ideas are included in this figure?

Origami Samurai Doll Handout: 1. Cut around the perimeter of the square.



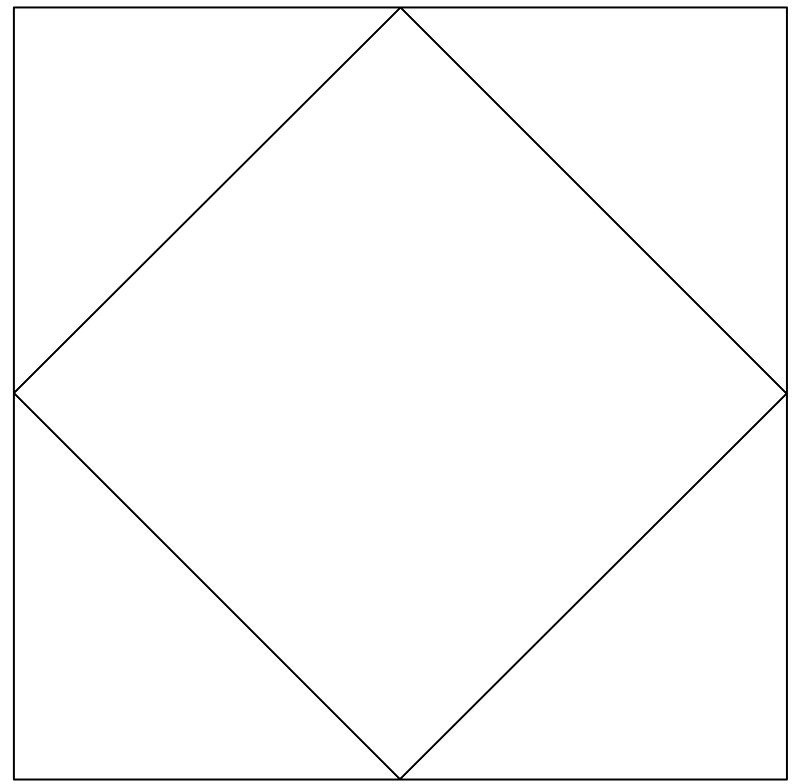
Origami Samurai Doll – Cont.

Procedures:

1. Cut around the perimeter of the square and make the folds as indicated. This is equal to one square unit.

Origami Samurai Doll – Cont.

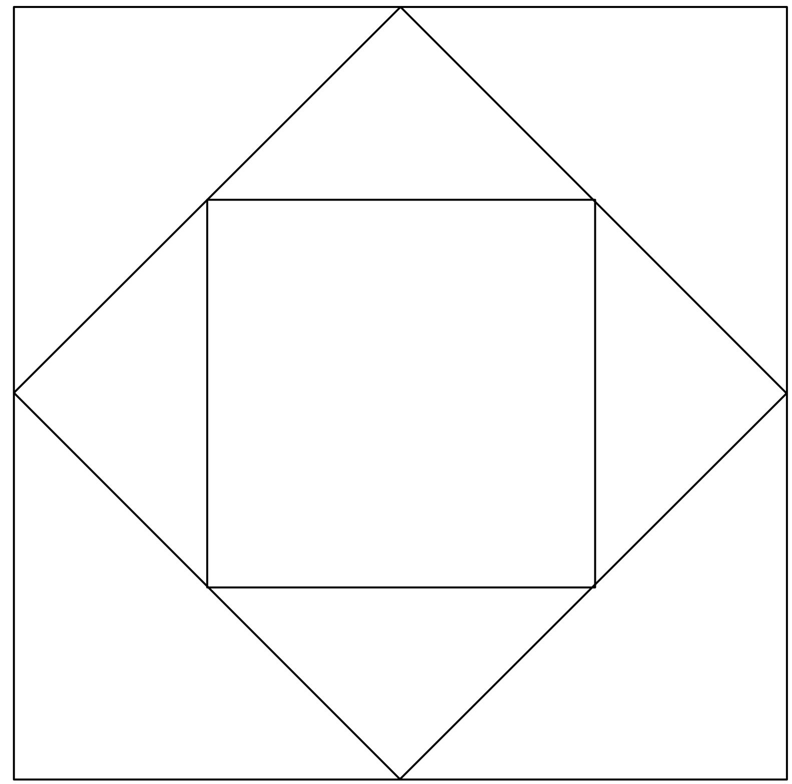
2. Fold back on the dotted line (corner to the center).
   - What shape have you made? Repeat this folding for each corner.
   - After folding the four corners, what shape have you made?
   - What fraction of the square unit is this new shape?
   - How can you express this fraction in exponential form?



$$\frac{1}{2} = \frac{1}{2^1} = 2^{-1}$$

Origami Samurai Doll – Cont.

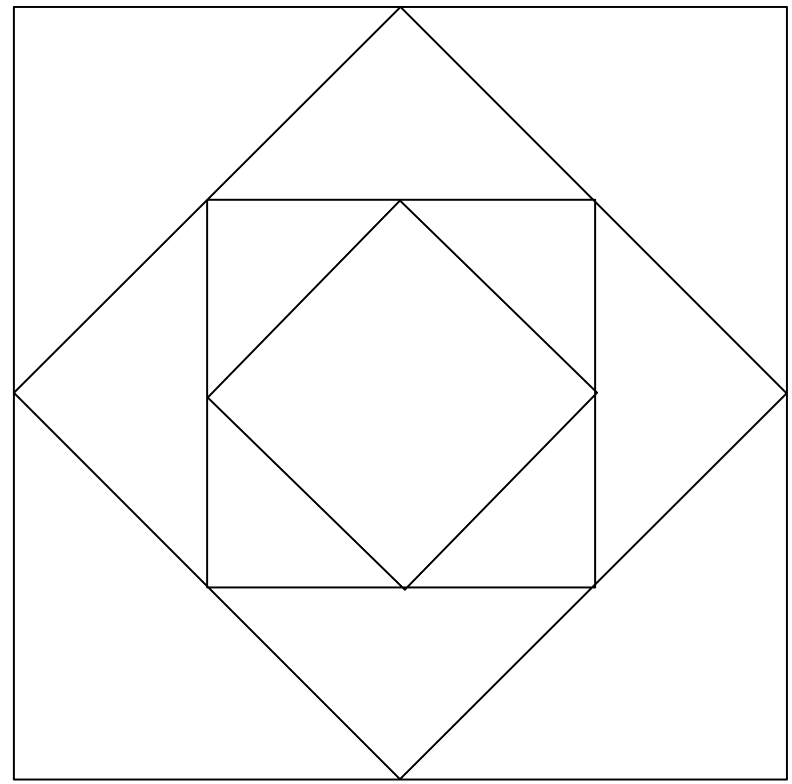
3. Fold forward on the dotted line (one corner to the center).
   - What shape have you made? Repeat this folding for each corner.
   - What fraction of the square unit is this new square?
   - How can you express this fraction in exponential form?



$$\frac{1}{4} = \frac{1}{2^2} = 2^{-2}$$

Origami Samurai Doll – Cont.

4. Fold back on the dotted line (corner to the center). Place a tape on the back of the square (were the words are written), and turn the square over again.
   - What fraction of the square unit is this new square, and express as a fraction?
   - What fraction of the square unit would be represented if you keep folding your paper in this manner, and express in exponential form?



$$\frac{1}{8} = \frac{1}{2^3} = 2^{-3}$$

Relationship between folds and squares being formed after folding:

$$\frac{1}{2} = \frac{1}{2^1} = 2^{-1}$$

$$\frac{1}{4} = \frac{1}{2^2} = 2^{-2}$$

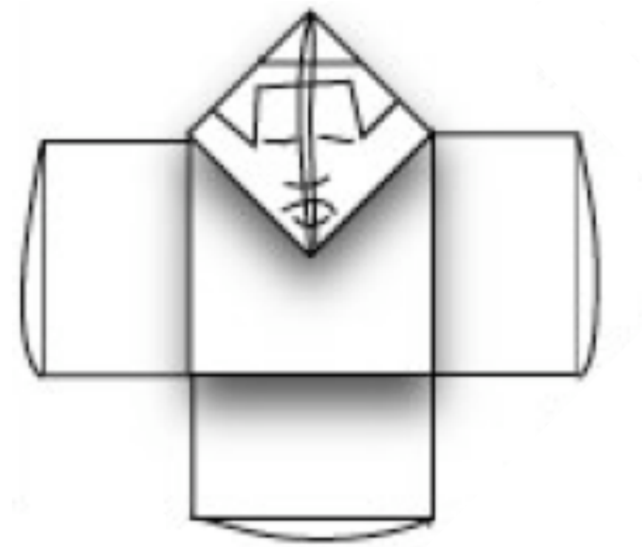
$$\frac{1}{8} = \frac{1}{2^3} = 2^{-3}$$

$$\vdots$$

$$\frac{1}{2^x} = 2^{-x}$$

Origami Samurai Doll – Cont.

5. Put your finger under one of the small squares and push outward.
   - What shape have you made by doing this?
6. Repeat step 5 with two more small squares.

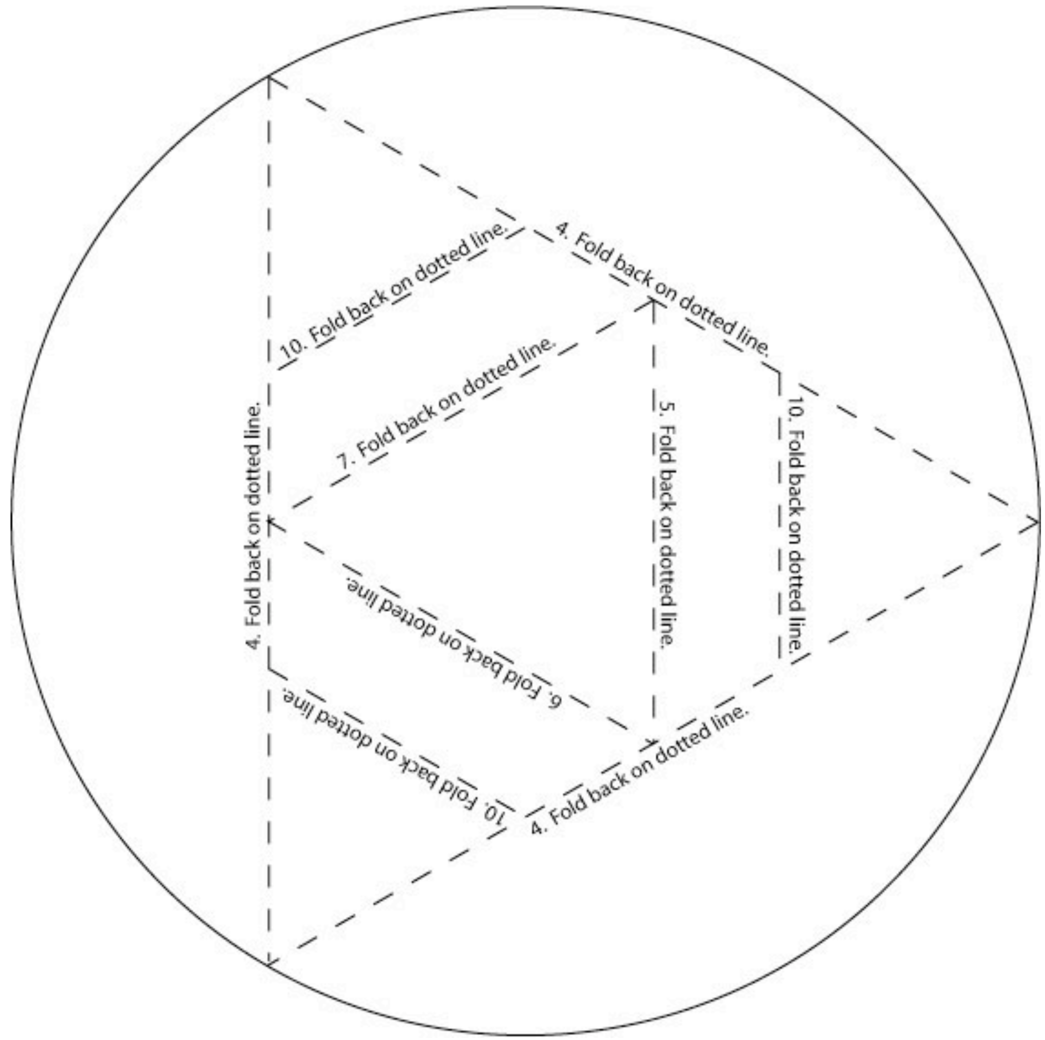


You have made a Momotaro!

The square is the head and rest the Samurai outfit.

Truncated Tetrahedron

- Step 3: Cut around the circumference of the circle.
  - What is the length of the chord?
  - What is the area of the folded region (flap)? Hint: Look at this one after working on the next question.

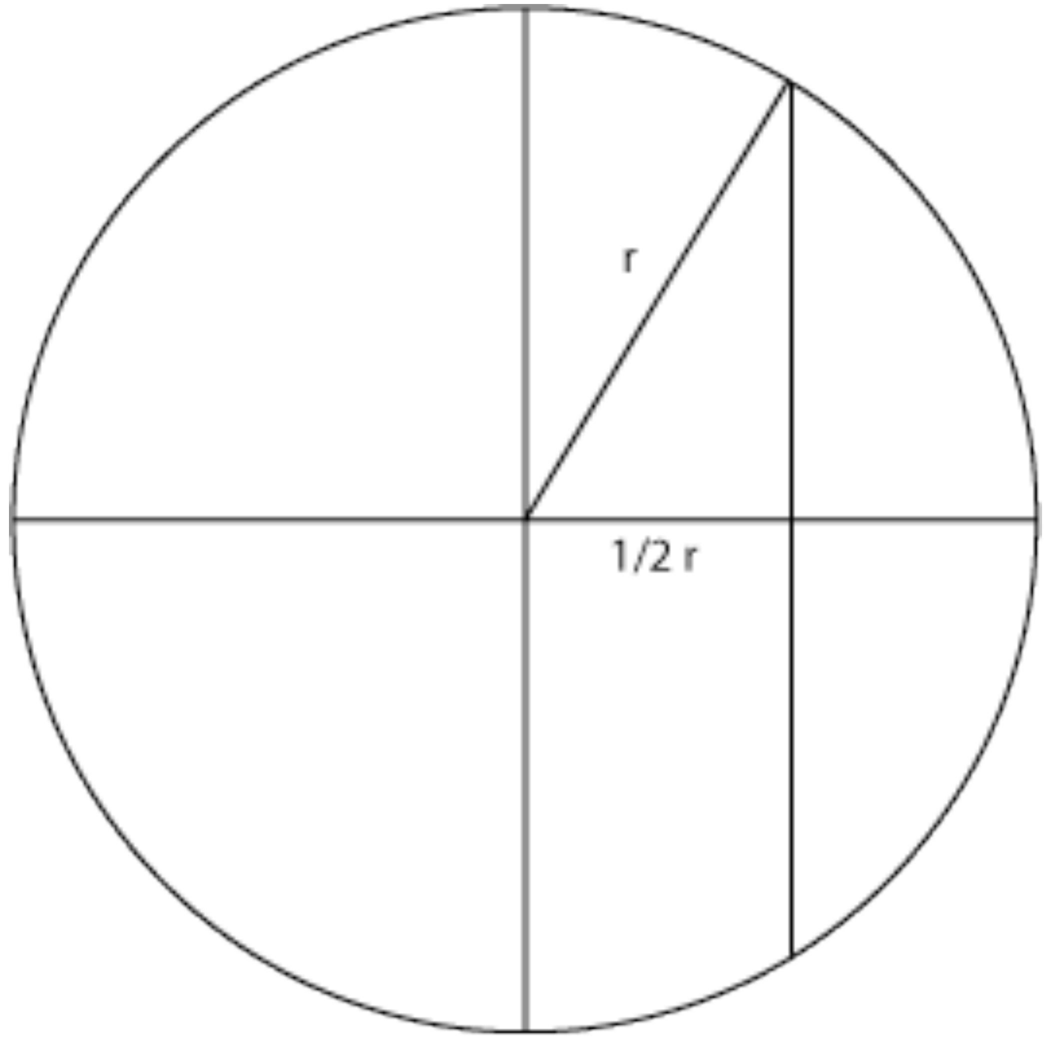


Possible Solution: Step 3:

a. Solve for x:

$$\left(\frac{1}{2}r\right)^2 + (xr)^2 = r^2$$

$$2\left(\frac{\sqrt{3}}{2}\right)r$$

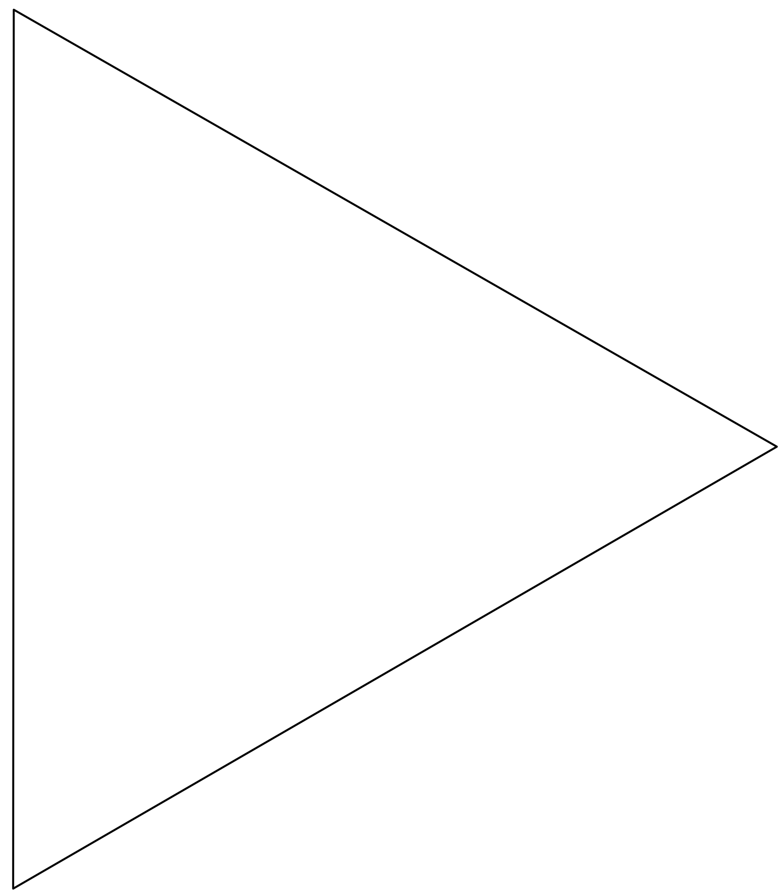


b. Area of flap:

$$\left(\frac{\text{Area} \bigcirc - \text{Area} \triangle}{3}\right) = \frac{\pi r^2 - 3\sqrt{3}r}{3}$$

Truncated Tetrahedron

- After step 4. What shape was formed?



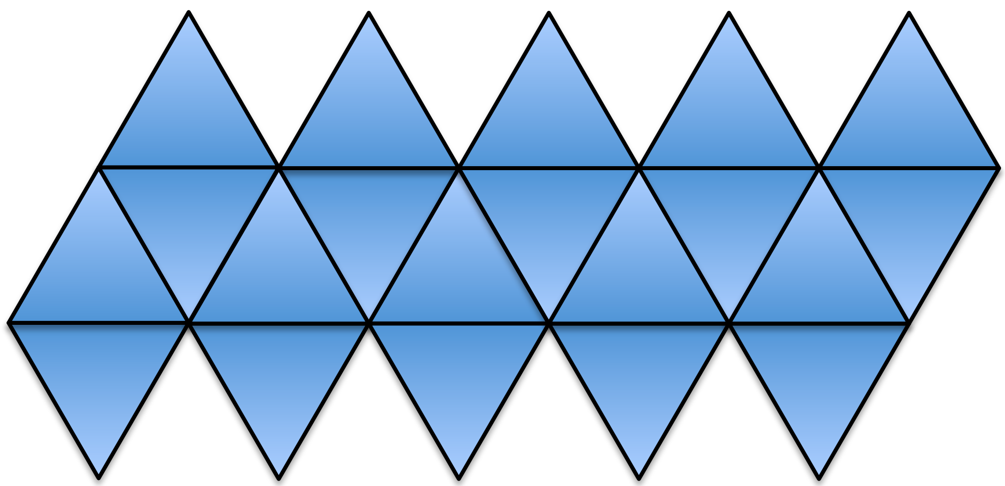
Platonic Solids

In Euclidean geometry, it is a regular, convex polyhedron, with the same number of faces meeting at each vertex. There are 5 platonic solids:

- Tetrahedron (4 faces)
- Cube or hexahedron (6 faces)
- Octahedron (8 faces)
- Dodecahedron (12 faces)
- Icosahedron (20 faces)

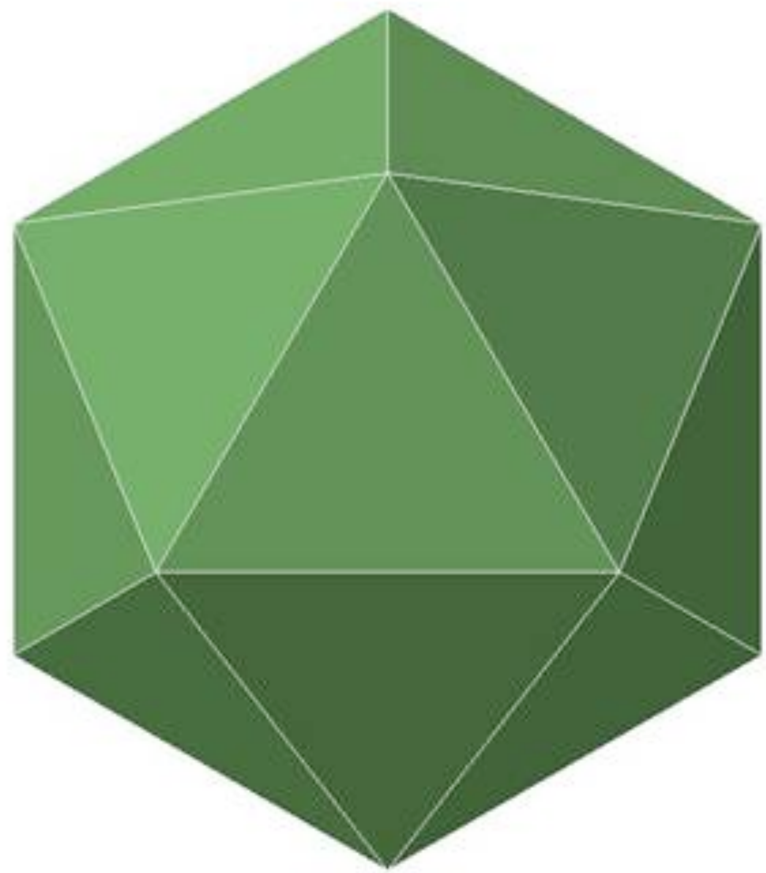
Icosahedron

- Make 20 truncated tetrahedron and organize them into a icosahedron. The following is a possible net for the icosahedron:



You may students find other possible nets.
Use the Euler's formula: $F + V = E + 2$ (F = faces, V = vertices, E = edges)

Icosahedron Image:



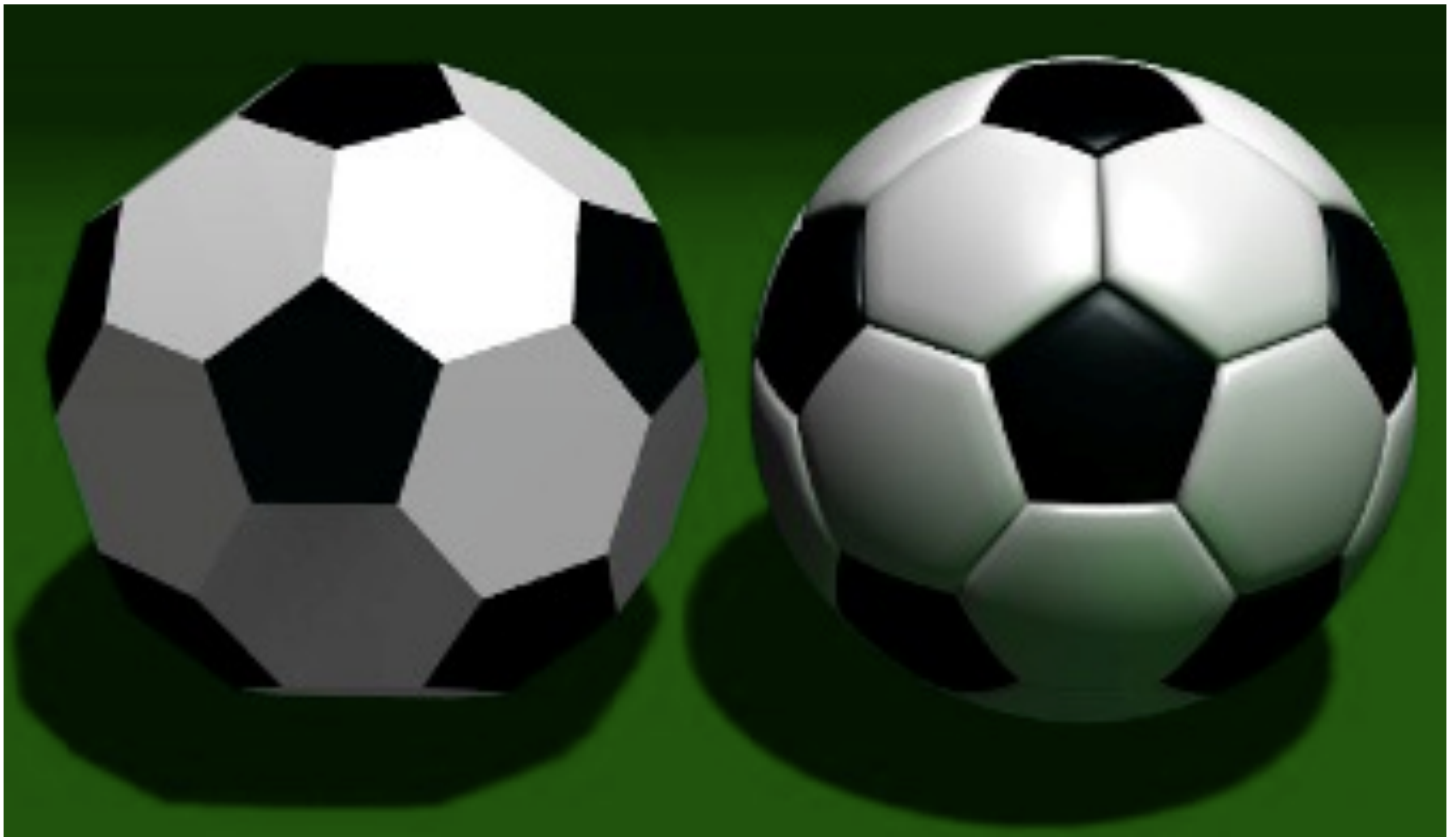
12 vertices, 20 faces, & 30 sides

Truncated Icosahedron

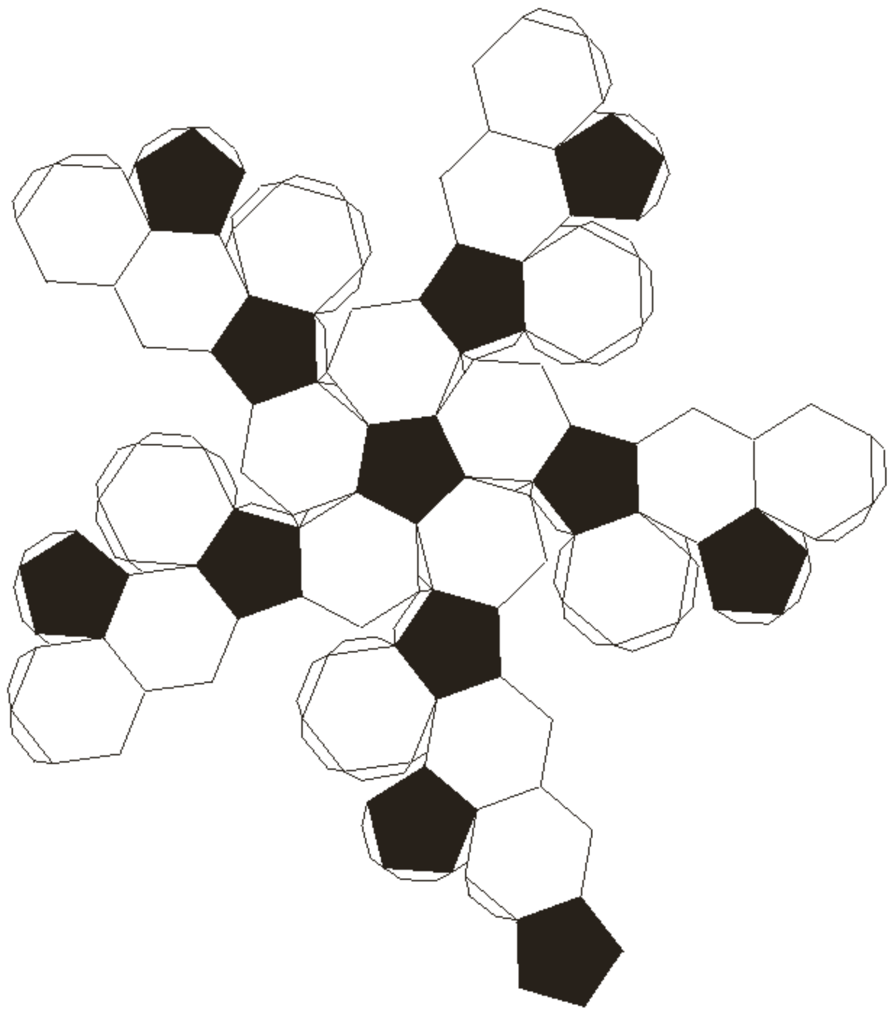
- It is an Archimedean solid, and one of 13 convex
- How many faces, edges, and vertices?
- What will be the resulting shape?

Truncated Icosahedron Image:
32 faces, 90 edges, 60 vertices

Truncated Icosahedron Football

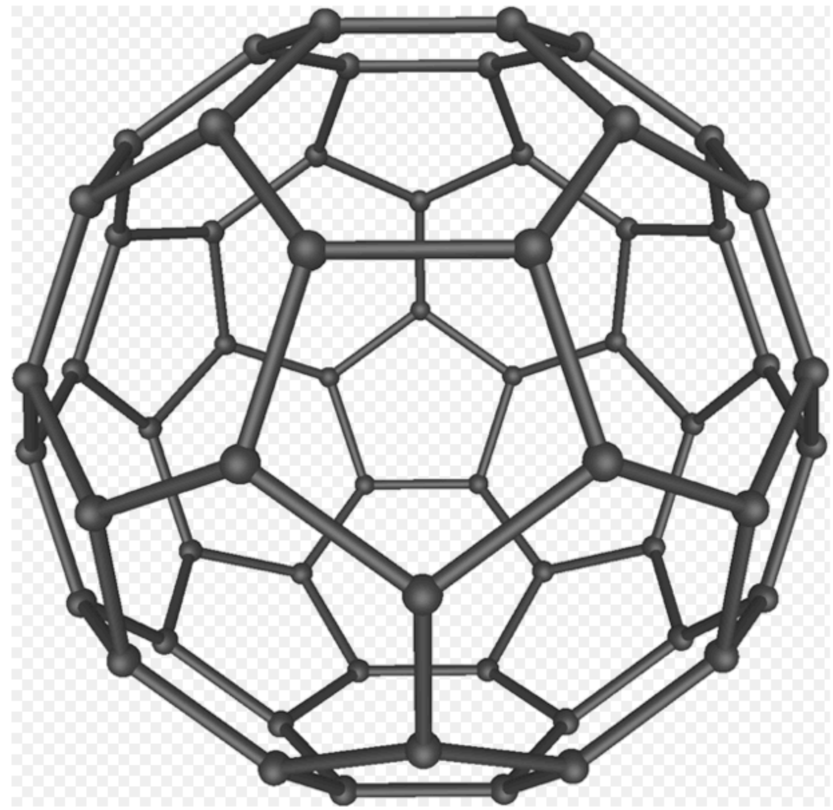


Possible Net for the Truncated Icosahedron:

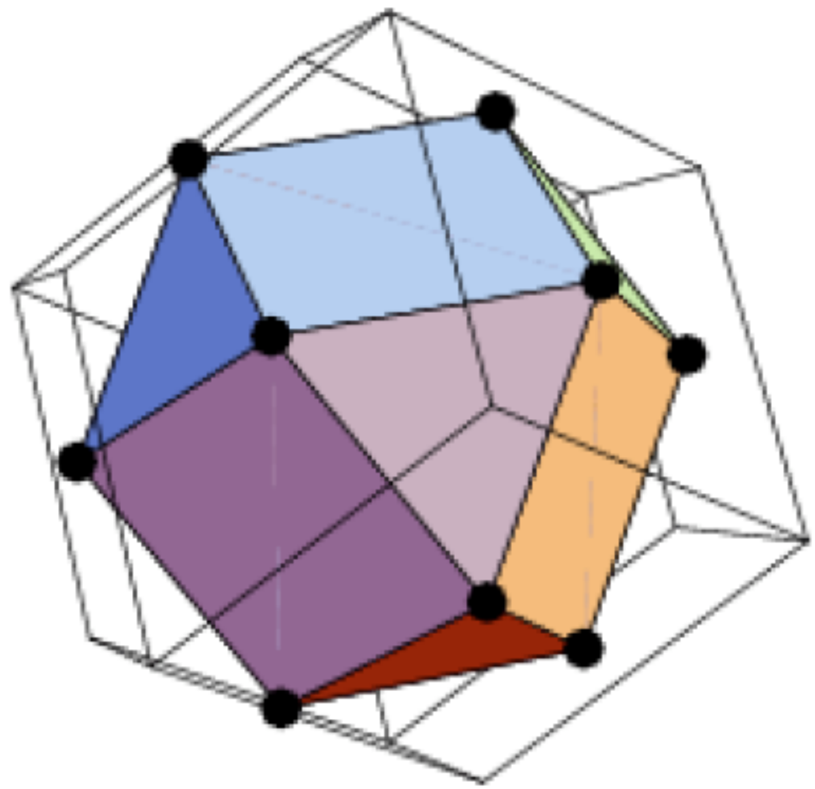


Buckminsterfullerene or $Carbon_{60}$ (C_{60})

The first fullerene (buckyball) – any molecule composed entirely of carbon, in the form of a hollow sphere, ellipsoid or tube – to be discovered, and the family's namesake, buckminsterfullerene (C_{60}), was prepared in 1985 by Richard Smalley, Robert Curl, James Heath, Sean O'Brien, and Harold Kroto at Rice University. The name was an homage to Buckminster Fuller, whose geodesic domes it resembles. The structure was also identified some five years earlier by Sumio Iijima, from an electron microscope image, where it formed the core of a "bucky onion." Fullerenes have since been found to occur in nature. More recently, fullerenes have been detected in outer space.

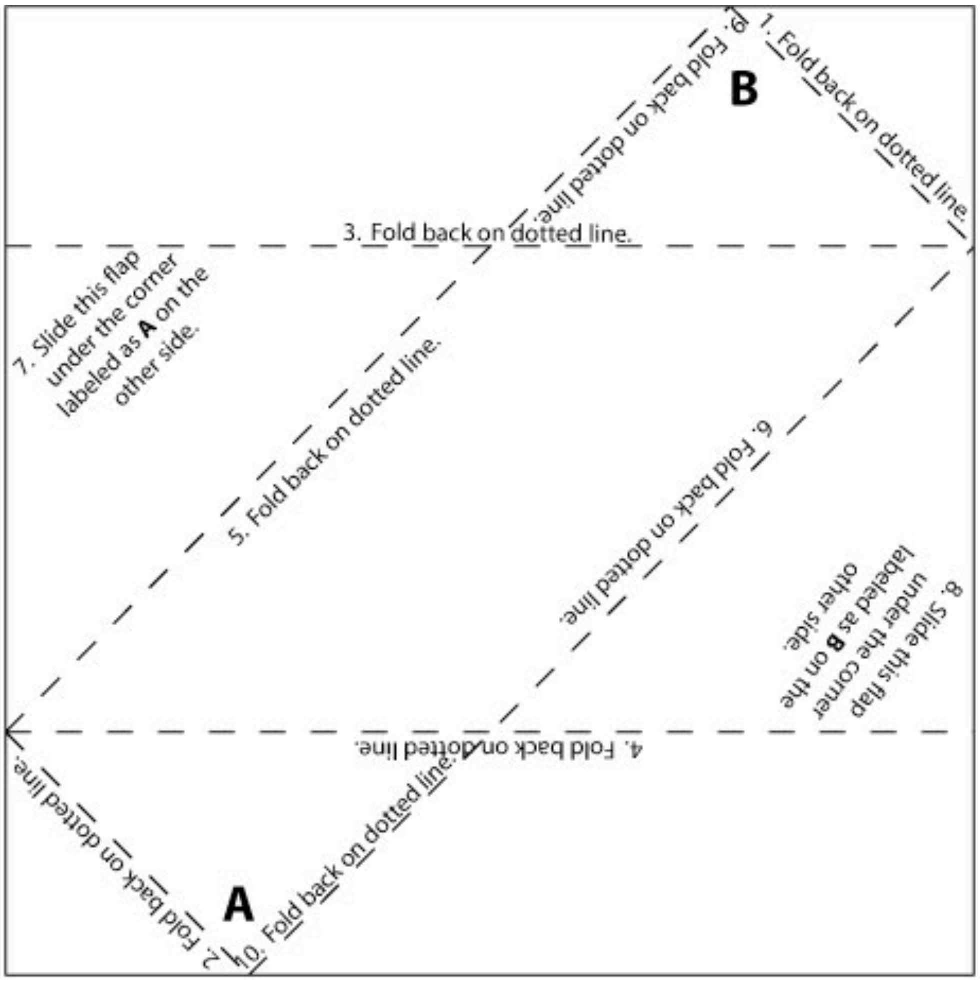


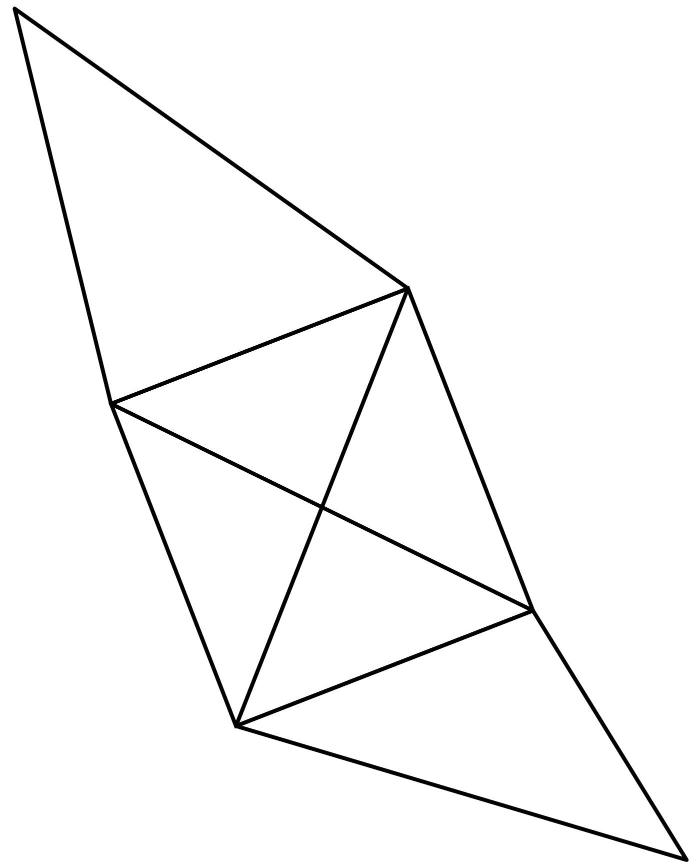
The cuboctahedron in the skeleton of its dual, the rhombic dodecahedron.



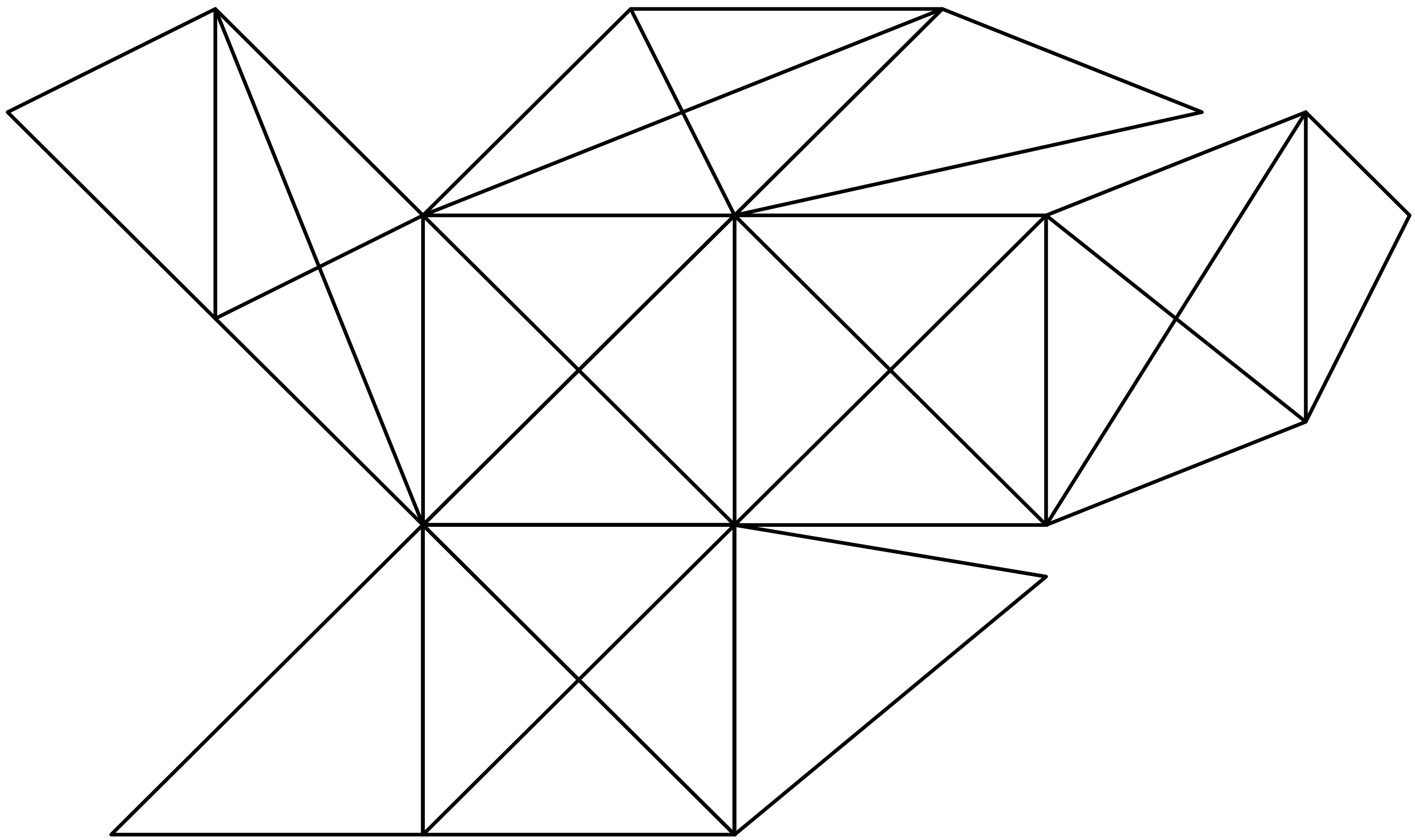
- *From: mathworld.wolfram.com/images/eps-gif/CuboctahedronInRhombicDodecahedron_700.gif*

Making an Origami Cube



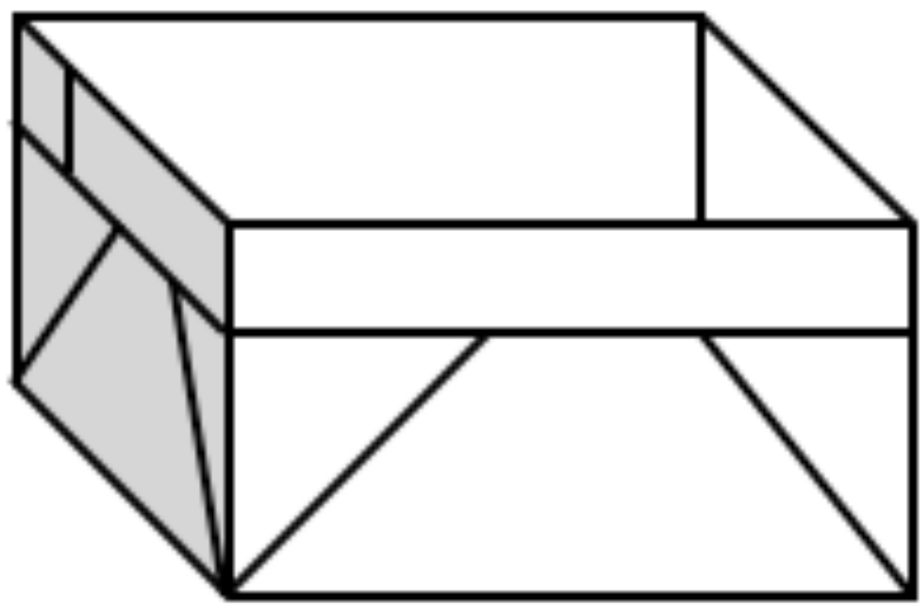


Making an Origami Cube



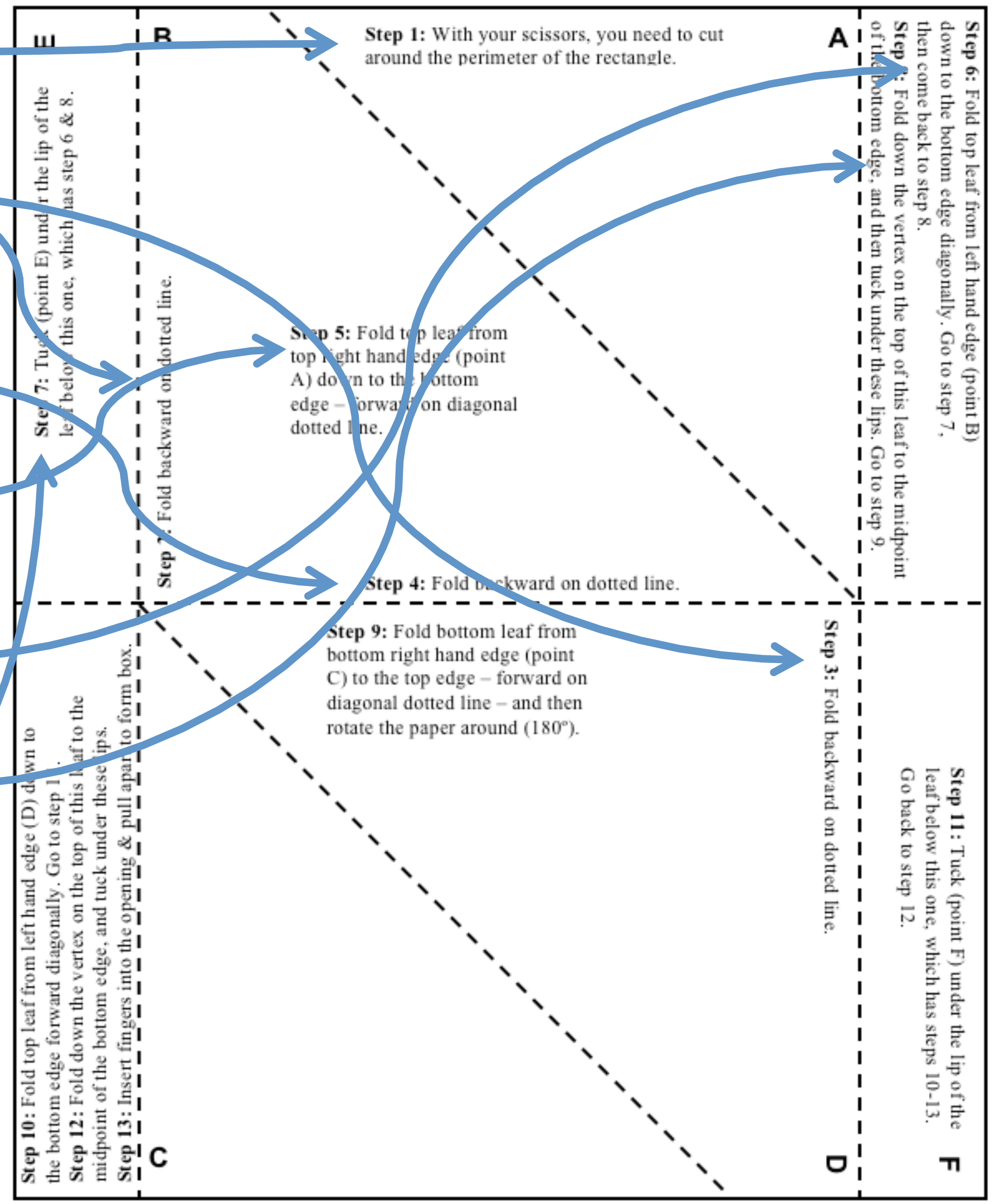
Making an Open Box

- Follow the steps and make your handy box.
- You can make another open box and place one inside the other to form a box.



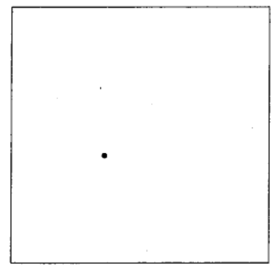
Steps for Open Box

- Step 1 was already done for you.
- Step 2 and 3: What type of lines (or line segments) these?
- Step 4: Is this a line of symmetry?
- Step 5: What type of triangle is the resulting leaf?
- Step 6: What is the resulting triangle?
- Steps 7 and 8: What is the resulting shape?
- Repeat steps: Steps 9-12. What is the resulting shape? Before opening.
- Step 13: What is the volume of the open box?

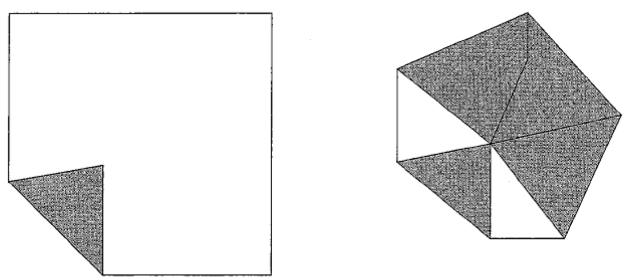


A Sample Manipulative Activity for Geometry
Adapted form Masunaga, D. (1998)

- Work individually and compile your results. Make a square out of the rectangular paper.
- Mark any point in the interior region of the square sheet of paper.



- Fold all vertices to a point marked, as shown in the diagram below.



- Look at the polygon you have folded. What do you notice? Describe and compare your polygon with another person.

A Sample Manipulative Activity for Geometry … Cont.
Masunaga, D. (1998)

- Repeat the process on 3 new sheets of paper by choosing another point inside the square’s edge, and at the square’s vertex.
- Complete the following chart.

Location of Point	**Type of Polygon**	**Scalene?**	**Symmetries?**
inside the square			
on the square’s edge			
at the square’s vertex			

A Sample Manipulative Activity for Geometry … Cont. Masunaga, D. (1998)

- One student in each group should make another square.
- Others in the groups plot their points on this square using the following symbols (9 to 12 participants):
  - If the point you chose yielded a folded quadrilateral, plot as a small black "circle" (●).
  - If the point you chose yielded a pentagon, plot that point as a small black "triangle" (▲).
  - If the point you chose yielded a hexagon, plot that point as a small black "square" (■).
  - If the point you chose yielded anything else, plot that point as small black "plus sign" (+).
- After everyone in the group plots their points on the square.
- Make conjectures about the relationship between the location of the points chosen and the types of polygons created (marked by the symbols above).

A Sample Manipulative Activity for Geometry … Cont. Masunaga, D. (1998)

“While most people create origami models of animals or other mirror symmetric objects, Professor Kazuo Haga of the University of Tsukaba in Japan has researched the geometrical nature of origami. … He call his scientific study of the mathematics of geometrical folds *origamics*, and this investigation is the first in as series of results he has discovered.” (p. 28).

- If a point is chosen at the center of a square,
  - then the four vertices folded to this point will create a square half the area of the original square.
- If a point is chosen anywhere in the interior of square except at the center,
  - then the result is a concave scalene pentagon or hexagon.
- If a point is chosen at one of the vertices of a square,
  - then the result is a square one fourth the area of the original square.
- If a point is chosen at the nonvertex edge of a square,
  - then the result is pentagon.

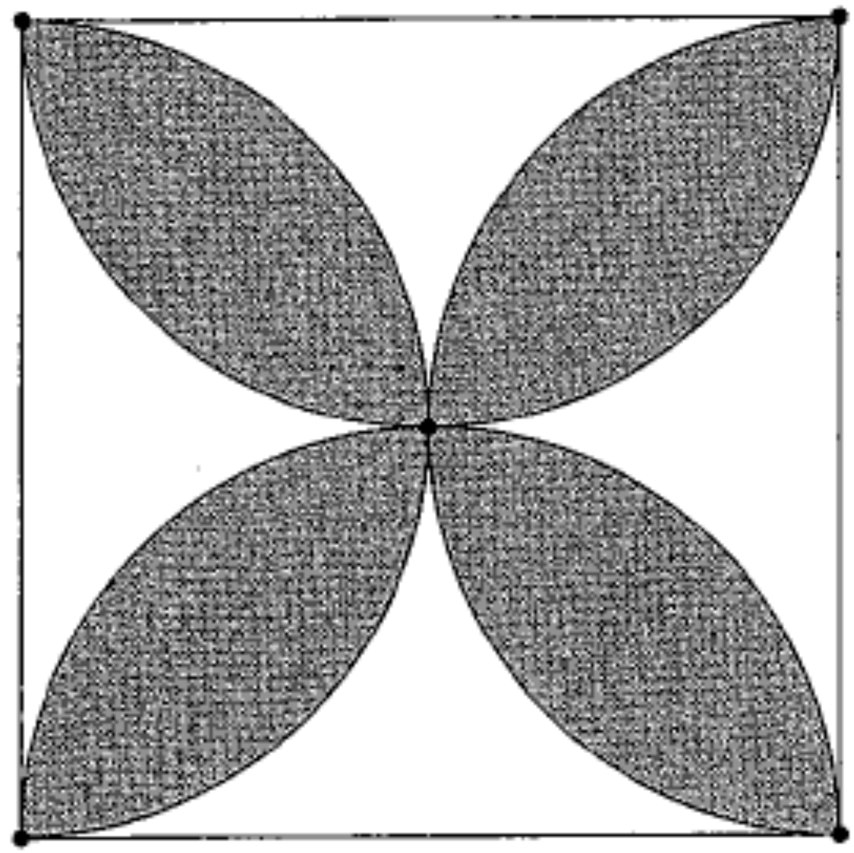
A Sample Manipulative Activity for Geometry … Cont. Masunaga, D. (1998)

What do you notice when the symbols from all of the groups are drawn?

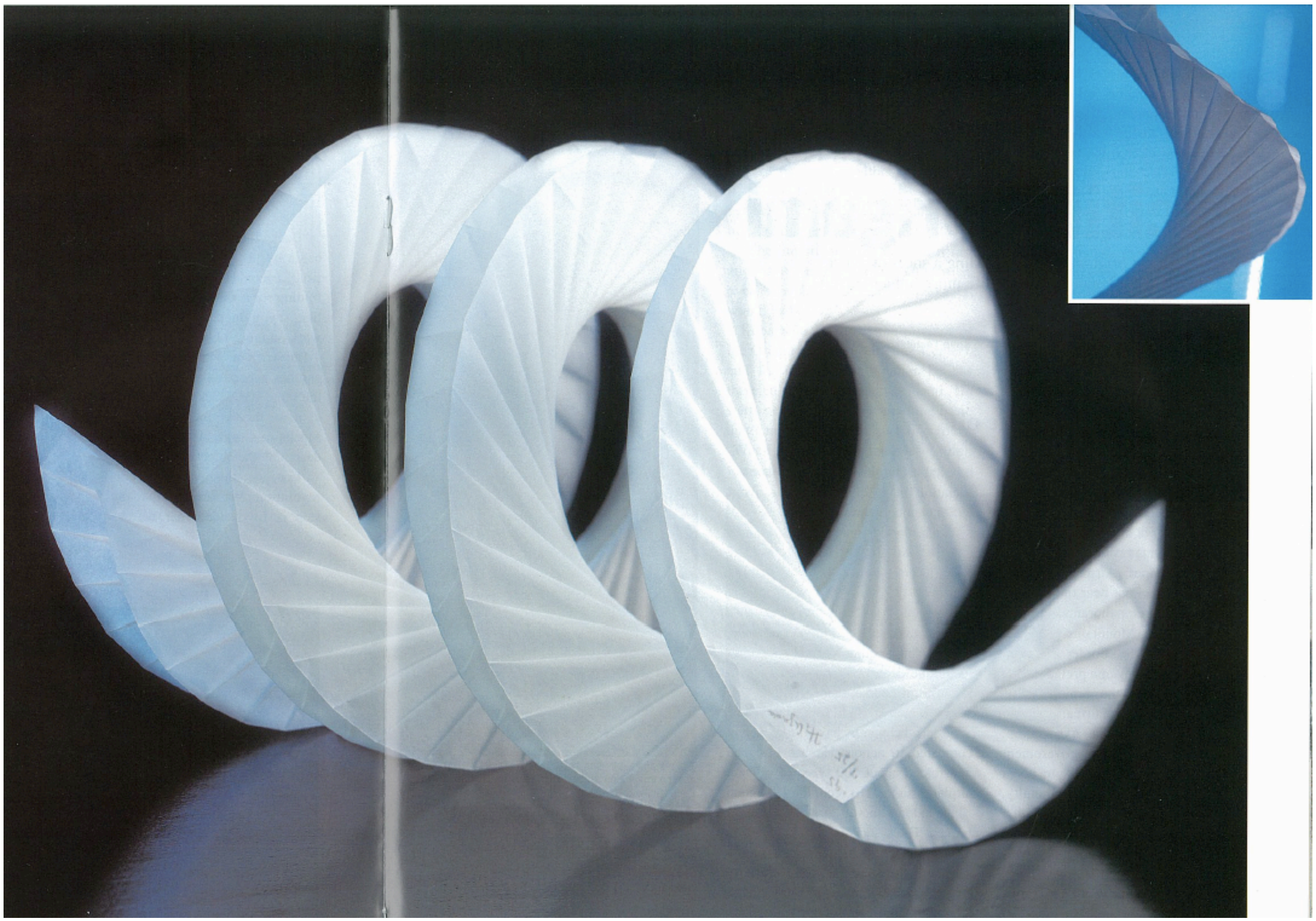
- Points chosen in the shaded area (■ symbols) yield
  - hexagons.
- Points chosen in the white region (▲ symbols) yield
  - pentagons.
- Points chosen at the vertices of the “petals” yield
  - squares.
- Why does this pattern occur?

Extensions:

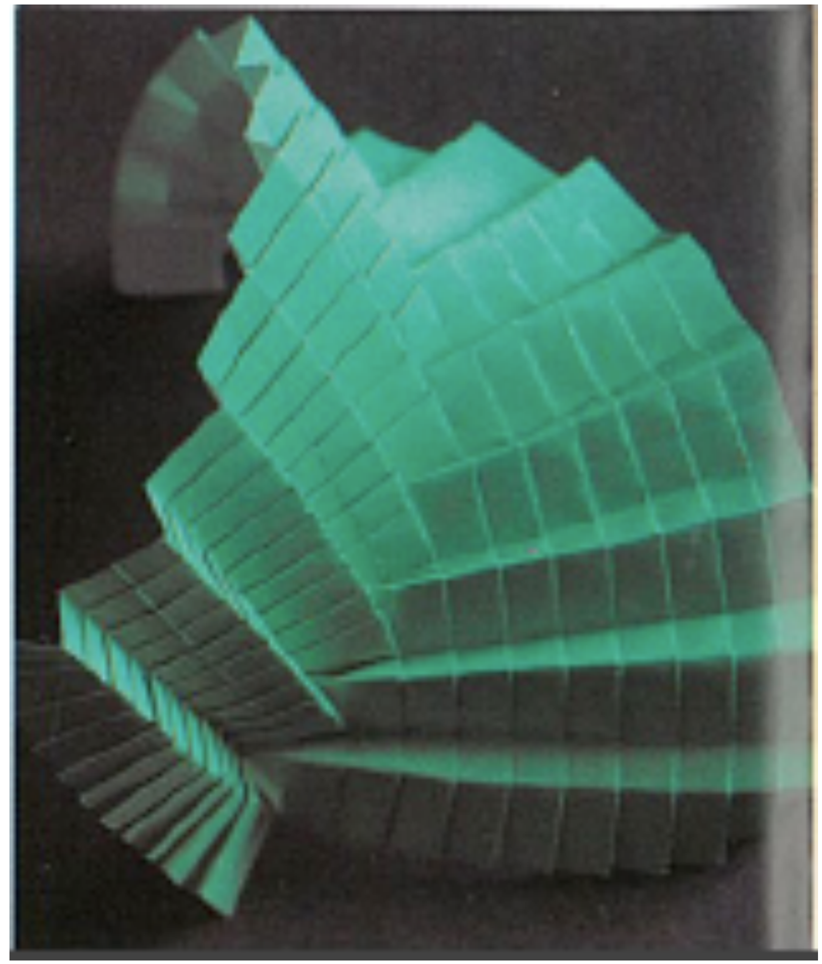
- What would the results look like if other regular polygons were chosen?
- How would the results change if more than one point were chosen?
- How would the results change if not all vertices were folded to the chosen point?
- Look up the Haga Theorem of paper folding. How does it apply to this investigation?



Convolution
by Azuma Hideaki



by Azuma Hideaki



Stunning "origami" dress

Japan was the theme of the Dior 2007 Spring/Summer Haute Couture Collection in Paris. What could be more imaginative than this gown in the collection? Inspired by origami, of course.

Resources

- Jeon, K. (March, 2009). Mathematics hiding in the nets for a cube. *Teaching Children Mathematics*.
- Wikipedia: Archimedean Solid: http://en.wikipedia.org/wiki/Archimedean_solid
- Wikipedia: Platonic Solid: http://en.wikipedia.org/wiki/Platonic_solid
- WolframMathWorld: Truncated Icosahedron: http://mathworld.wolfram.com/TruncatedIcosahedron.html

Resources … Continued …

Websites:

- http://www.origami.as/home.html

  Joseph Wu's Origami Page. This is the best first stop in origami web pages. Origami galleries, information, diagrams, and links. He is a popular speaker and well-known authority on all things folding.

- http://web.merrimack.edu/~thull/

  Tom Hull, Assistant Professor of Mathematics at Merrimack College is the originator of many polyhedral folds. His website gives links to origami math pages and also depicts his own beautiful models.

Resources … Continued …

- Masunaga, D. (1998). A sample manipulative activity for geometry. In Professional Handbook, Geometry: Explorations and applications. McDougal Littell (pp. 27-29).