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Where am I Ever Going to Use This Stuff?

When a student asks this question, a teacher can honestly answer, "I don't know, I'm not a psychic!" However, teachers should be able to address this question by indicating where "somebody" uses a given topic from the NYS performance indicators in a real world application. The purpose of this column is to provide actual applications of mathematical topics from the mathematics curriculum. Anyone who would like to share how a specific real world application can be used in a classroom to enhance a given topic from any grade level should contact the editor at robert.rogers@fredonia.edu .

Financial Algebra: Real-World, Real Math, Real Numbers

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Most Americans aren't fluent in the language of money…It's clear that most of us need some help, preferably starting when we're still in school…All of this raises the question: How many schools even broach the topic? As it turns out, for a country that prizes personal responsibility, we're doing very little. - NY Times, April 9, 2010

What do we know? What should we know? What does the average person remember? We have given surveys to over a thousand adults and teenagers over the past few years, and received enlightening, but not surprising, answers to questions such as:

- What team won the last World Series?
- What famous Hollywood actress recently got married?
- What rock band played at the last Super Bowl?
- What collision deductible do you have on your automobile insurance policy?
- Did you know that educators can deduct $250 for educator expenses on their income taxes?
- Can you deduct the cost of a program to stop smoking on your income tax?
- Do you read your electric meter, and check it against your bill each month?
- Do you check the finance charge on your monthly credit card statement?
- What is an insurance floater policy?
- What is the cost of a funeral?

When we go over the questions with our audiences, everyone smiles as they realize that they might want to adjust some of their priorities! One person told us that he just took out a nine-month CD at an interest rate of 0.5%. We went right on the Internet and found online banks that pay double that rate for their regular savings accounts that don't come with the CD restrictions! He didn't know information that directly affected his income, but admitted to knowing several facts about Paris Hilton. Perhaps this after-the-fact learning is not the best way to make smart decisions! Can mathematics teachers help?

Throughout the 1970s, general and consumer math classes existed in virtually all high schools. Certain students were relegated to tracks offering courses that were basically arithmetic-based. Many of these students graduated never having taken an Algebra 1 course. The presumption that these students could never handle an algebra course was unwarranted. All students deserved to complete an algebra course, even those requiring prior remediation. Yet many did not.

In the 1980s, the "algebra for all" movement corrected the disservice and inequity of depriving certain students access to Algebra 1. With the advent of this movement, consumer math class enrollment began a decline which, in many schools, ended in their demise in the 1990s. Raising the bar and including everyone in an algebra course was a solid idea whose time had come. Unfortunately, with it came an unfair, implied assumption that consumer mathematics and algebra were mutually exclusive. As national, state, and district standards rose, consumer mathematics, traditionally devoid of any algebra, virtually disappeared.

As high school teachers for over three decades, we have always embraced financial applications. Finance and algebra are a very natural fit. So, in the new millennium, we created Financial Algebra, a course that includes topics such as income taxes, insurance, credit, banking, investments, etc., in an advanced-algebraic setting. Because there was no existing curriculum, we compiled hundreds of our own activities over the course of many years and published them in 2011 as a textbook entitled *Financial Algebra*. [The book, by Gerver and Sgroi, is available from Cengage/South-Western, www.cengage.com.] The prerequisite is completion of Algebra 1. The course takes selected topics from Algebra 2, precalculus, statistics, and probability as they are needed to investigate the financial topics. The financial applications are incredibly rich in algebra, and financial topics are inherently relevant to young adults. We will sample a dozen examples here, with the hope that these mathematical riches will inspire mathematics teachers to find out more about studying advanced algebra via financial applications. These examples and many more are covered in detail in the book.

Financial situations that affect our daily lives can be modeled, and better understood, using algebra. Did you ever realize that…

1. Cell phone charges form a piecewise function that incorporates the ceiling (round up) function. In the following example, m represents the number of minutes. The charges $c(m)$ are \$0.16 for the first three minutes and \$0.11 for each minute, or part of, above three minutes. This gives us the formula

$$c(m) = \begin{Bmatrix} 0.16m & \text{if } m \leq 3 \\ 0.16m + 0.11\lceil m-3 \rceil & \text{if } m > 3 \end{Bmatrix},$$

where $\lceil m-3 \rceil$ represents the smallest integer greater than or equal to $m-3$.

2. The FICA tax function is a piecewise function whose graph has a cusp. The FICA tax for 2010 is 6.2% of the first \$106,800 of income. After that is reached, no more FICA tax is taken out for the year. Let x represent taxable income and let $t(x)$ represent the Federal income tax on a taxable income of x dollars. The FICA tax function is

$$t(x) = \begin{Bmatrix} 0.062x & \text{if } x \leq 106{,}800 \\ 6{,}621.60 & \text{if } x > 106.800 \end{Bmatrix}$$

The graph of $t(x)$ has a cusp at the point (106800, 6621.60) [Figure 1].

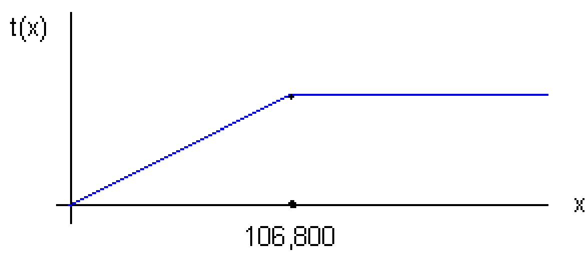

Figure 1: A cusp in a "real-world" application.

3. Planning for a future expense by making savings account investments can be examined from four different vantage points: the future values of a single or periodic deposit investment, as well as the present values of a single or periodic deposit investment. A periodic deposit is a deposit that is made in equal increments of time (every month, every year, every two weeks, etc.). Each time, the payment is made at the end of the payment period. In the formula for the present value of a periodic deposit investment, P represents the periodic deposit to be made n times per year, for t years to realize a balance of B dollars. The annual (nominal) interest rate, compounded n times per year, expressed as an equivalent decimal, is r. Students can make the following table to see the pattern.

End of period	Balance
1	P
2	$P + P\left(1+\frac{r}{n}\right)$
3	$P + \left(P + P\left(1+\frac{r}{n}\right)\right)\left(1+\frac{r}{n}\right) = P + P\left(1+\frac{r}{n}\right) + P\left(1+\frac{r}{n}\right)^2$
⋮	⋮
nt	$B = P + P\left(1+\frac{r}{n}\right) + P\left(1+\frac{r}{n}\right)^2 + \cdots + P\left(1+\frac{r}{n}\right)^{nt-1}$

Utilizing the formula for a geometric series, we get $B = P\left(\frac{1-\left(1+\frac{r}{n}\right)^{nt}}{1-\left(1+\frac{r}{n}\right)}\right)$. Solving for *P*, we can find a formula to determine the deposit *P* which will realize a balance of *B*.

$$P = \frac{B \cdot \frac{r}{n}}{\left(1+\frac{r}{n}\right)^{nt} - 1}$$

4. As with the table above, students can compound interest without using formulas to gain an understanding of "making interest on your interest," before any formulas are derived. Students are exposed to annual, semiannual, monthly, weekly, daily, and hourly compounded interest. Then they make the leap to the formula for continuous compounding, in which *P* represents the initial principal, *t* is the number of years, and *r* is the annual interest rate (force of interest) expressed as a decimal.

$$B = Pe^{rt}$$

5. The formula for continuous compounding requires that students become familiar with the natural base *e*. This is developed using limits, which students have been introduced to earlier in the Financial Algebra course.

$$e = \lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x$$

Students can utilize tables on graphing calculators to see that as the value of *x* increases, the value of $\left(1+\frac{1}{x}\right)^x$ approaches *e*. The "battle" going on between the increasing exponent and the simultaneously decreasing base can be discussed.

6. There is a formula that can be used to compute the numbers of *BTUs* (British Thermal Units) required when purchasing an air conditioner for a room . In the formula, *l* represents length, *w* represents width, and *h* represents height. The variable *i* reflects the quality of insulation, as is defined as an index by the power company. The variable *e* represents exposure (which direction the room faces). This is also defined by an index supplied by the power company. The formula is

$$BTUs = \frac{while}{60}$$

7. If you are purchasing or selling a car, you would likely compile a list of the prices of several comparable cars. You could create a modified box and whisker plot of the prices, and compute the range, interquartile range (*IQR*), quartiles, and use the outlier formulas to determine if there are any outliers. Recall that Q_1 is the first quartile (the 25th percentile) and Q_3 is the third quartile (the 75th percentile). We have

$$\text{Boundary for lower outliers} = Q_1 - 1.5(IQR)$$
$$\text{Boundary for upper outliers} = Q_3 + 1.5(IQR)$$

Students can then create a modified box plot. They should also know how their calculator makes a modified box plot [Figure 2].

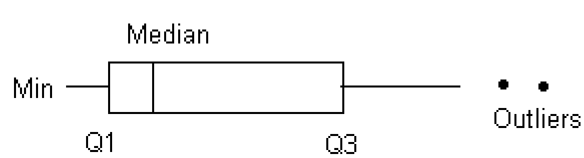

Figure 2: Modified boxplots give information on the spread of the distribution.

When they negotiate a mutually agreeable price, they will have hard data to help their case, and the other party will also know that they are aware of the availability of other, similar cars!

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8. When negotiating the buying or selling of a used car, you can search for cars of the same make, model, and year, and compile bivariate data on the mileage (x), and the price (y). A regression model can be created, and each potential deal can be compared to the regression model. A predicted "competitive" price could then be computed for a given mileage. The buyer or the seller will be able to use this data to their negotiating advantage when the actual price can be compared to the predicted price based on the mileage.

9. The cost C of an automobile purchase can be modeled by a linear equation of the form $C(x) = 12mx + b$ where b is the down payment, $12m$ the total yearly amount paid for the loan (12 times the monthly payment m), and x is the number of years the car is owned. Depreciation can be modeled by an exponential function $E(x) = p(1 - b)^x$ where p represents the original price of the car, b is the exponential depreciation rate, and x is the number of years the car is owned. Graphing this system of equations yields the point where what you have paid to that date is equal to what the car is worth [Figure 3].

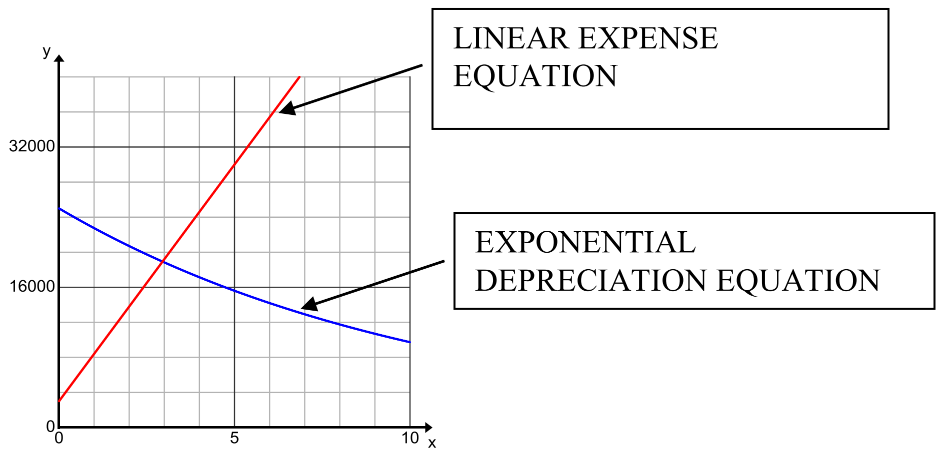

Figure 3: Cost and value of a car after x years.

10. Expense, revenue and profit projection scenarios for a new product can best be modeled by creating, examining, and interpreting linear and quadratic graphs based on the following relationships:

- The quantity q demanded will depend on the price p that the manufacturer sets.
- The manufacturing expenses E depend on the number of units, q, being produced.
- Since q depends on the price p and E depends on q, E ultimately depends on p.
- Revenue is the product of price and quantity: $R = pq$.
- Since q is a function of p, the revenue R is a function of the price p.
- Since profit P is the difference between revenue R and expenses E, $P = R - E$.

Substitution is used to get all variables expressed in terms of the price p, and they can be graphed on the same axes. There are linear and quadratic equations inherent in this development, and the company should set a price p that maximizes the profit [Figure 4].

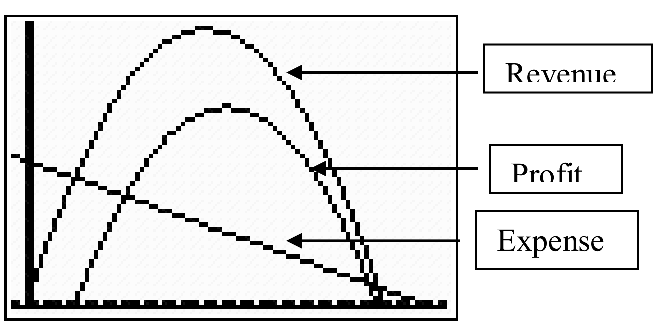

Figure 4. Maximum profit occurs to the right of maximum revenue.

Students can investigate domains, intersection points, maximum values and zeros from the graphs in the context of a new product's profitability analysis.

11. Our progressive tax rate system can be modeled by piecewise functions. The tax schedule shown here can be interpreted by 6 distinct linear equations that form a continuous piecewise function defined on specified domains representing a tax payer's adjusted gross income.

Schedule Y-1—If your filing status is Married filing jointly or Qualifying widow(er)

If your taxable income is: Over—	But not over—	The tax is:	of the amount over—
$0	$15,100	.... 10%	$0
15,100	61,300	$1,510.00 + 15%	15,100
61,300	123,700	8,440.00 + 25%	61,300
123,700	188,450	24,040.00 + 28%	123,700
188,450	336,550	42,170.00 + 33%	188,450
336,550	....	91,043.00 + 35%	336,550

The following piecewise function models the tax schedule shown above.

$$f(x) = \begin{cases} 0.10x & 0 < x \leq 15100 \\ 1510 + 0.15(x - 15100) & 15100 < x \leq 61300 \\ 8440 + 0.25(x - 61300) & 61300 < x \leq 123700 \\ 24040 + 0.28(x - 123700) & 123700 < x \leq 188450 \\ 42170 + 0.33(x - 188450) & 188450 < x \leq 336550 \\ 91043 + 0.35(x - 336550) & 336550 < x \end{cases}$$

When graphed, students identify the cusps and see that the function is indeed a continuous one [Figure 5].

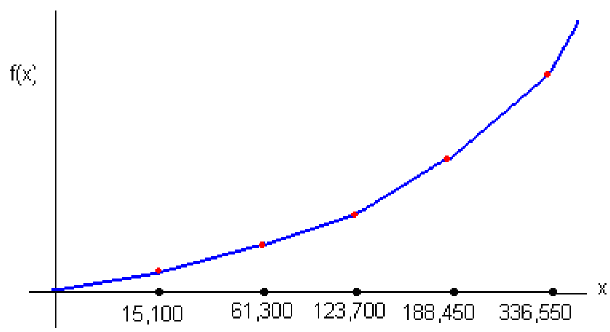

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Figure 5: The slopes in each interval represent tax rates.

12. The Front-End Ratio is one figure used by banks as a barometer of credit worthiness when people apply for loans. It is a rational function; a ratio of selected monthly housing expenses to gross monthly income.

$$\text{Front-End Ratio} = \frac{\text{monthly housing expenses}}{\text{monthly gross income}}$$

If the front-end ratio exceeds a prescribed percent (often said to be around 28%), the loan will be rejected.

These twelve examples are just a sample of the mathematically-rich applications available. The financial context is the prime focus and the mathematics is called upon to model and make sense of that context. The applications *drive* the mathematics.

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Teachers and schools that consider implementing a Financial Algebra program should examine some facets of the course that help facilitate its success.

Pedagogy Essential to the Financial Algebra Teaching Experience

Financial Algebra is an algebra-based, applications oriented, technology-dependent course that requires Algebra 1 as the prerequisite. It is a hybrid of algebra 2, precalculus, statistics and calculus. The course is appropriate for students of all mathematical abilities. We suggest that Financial Algebra be taken by upper classmen as a 3rd or 4th year course, or concurrently as an elective with another math course.

What strategies can the classroom teacher incorporate to maximize success? We have identified a number of contributing factors that we recommend being incorporated into the course in order to enhance the learning experience for the students. Let's take a look at some strategies we have used for over three decades.

1. Discussion

Humanities teachers regularly get to draw their students into discussions that touch both the topics they teach and the lives that students live. Financial Algebra does just that. The mathematics is motivated by reality and it is that reality which sparks discussions leading to the recognition of the necessity of the mathematics. Students often exhibit emotional responses when confronted with policies of the IRS, banks, credit card companies, insurance companies, the Social Security Administration, etc. This emotion fuels *passionate* discussions that make the course exciting to teach.

2. Reading

To be a lifelong learner, especially on financial matters, you need to rely on reading. In Financial Algebra, reading is not only encouraged; it is required. Students read passages from the text, from the Internet, from brochures, from the newspaper, etc., both aloud and silently. Then, the discussion starts. Reading and discussion are integral partners in this course.

3. Interpreting and Analyzing Quotes

We begin each chapter and each section of the chapter with a quote. There are many excellent websites offering quotes on a variety of topics. Initially, the quote is discussed from the vantage point of the novice. Students have yet to study the topic so they speak only from their beliefs and experience. The quote is revisited after the lesson is taught and students are asked to interpret it in light of the knowledge they have gained from the lessons.

Examine this sampling of quotes and the Financial Algebra topics to which they relate. How might students interpret the quote before and after the topic is studied?

The Stock Market: *Never try to walk across a river just because it has an average depth of four feet.* Milton Friedman, American economist

Automobile Insurance: *Never lend your car to anyone to whom you have given birth.* Erma Bombeck, Humor Writer

4. Posing Essential Questions

Essential questions are focal questions that lay the groundwork for inquiry about the topic to follow. Examine this sampling of essential questions and the Financial Algebra topics to which they relate. How might students answer the essential question pre-instruction? How might students answer the essential question post-instruction?

Banking: *How can you effectively plan for the future balance in an account?*
Automobile Depreciation: *At what rate does your car's value decrease?*
Taxes: *What is the difference between tax evasion and tax avoidance?*

5. Using Outside Resources

Since this is truly a real world/real math course, it is only natural to tap into resources outside the classroom to motivate the mathematics. We urge you to use the internet, newspapers, and financial magazines. Look around your community for places to take your students on a Financial Algebra field trip (to a stock exchange, to a bank, etc.) Local businesses love to be involved with the schools.

6. Field Work

Too often, students resort exclusively to online sources for their information. We have had hundreds of students over the years price a car, insurance and a loan for that car, price a funeral or a wedding, interview a lawyer about wills, or interview an accountant about tax loopholes by actually visiting local businesses. Students create a Powerpoint or poster presentation on their field work project. These projects offer students an alternative assessment opportunity through which they can show what they have learned in a non-traditional way. It also provides them with a chance to speak to an expert in the field.

The relationship between fiscal responsibility and financial education is undeniable. This "fiscal call to arms" is making headlines routinely in print, on television, and throughout cyberspace, as shown by the following:

- *Promoting Financial Success in the United States: National Strategy for Financial Literacy 2011* published by the Financial Literacy and Education Commission.
- President Obama's 2010 *Proclamation for National Financial Literacy Month,*
- The Council for Economic Education's bi-annual report *Survey of the States 2009: The State of Economic, Financial and Entrepreneurship Education in our Nation's Schools,*
- The vision and mission statements of the JumpStart Coalition for Personal Financial Literacy, headquartered in Washington, DC.

Offering financial education through the mathematical money management models that anchor a Financial Algebra curriculum can build fiscal confidence and responsibility as well as help students see, enjoy and implement mathematics in their everyday lives. As for their perpetual cries with the problem of "when are we ever going to use this?" Problem solved!

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