

Taking the Guesswork out of Computational Estimation



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The Problem



“When asked to estimate $12/13 + 7/8$, only 24 percent of thirteen-year-old students in a national assessment said the answer was close to 2.”

National Council of Teachers of Mathematics (2000, p. 35)

The Purpose of this Study



- What is an estimate?
- Why is estimation important?
- How do students of different estimation levels think about mathematics?
- Look for innovative ways to teach and utilize estimation.

What is an Estimate?



- Is it simply Guessing?



LeFevre, Jo-Anne, Stephanie Greenham, and Hausheen Waheed (1993),
Rubenstein (1985), Sowder and Wheeler (1989)

What do these definitions for estimation and computational estimation tell us?



- To estimate students have to look at context clues and use problem solving skills.
- Students have to recognize approximate numbers.
- Students have to recognize that estimation can be done in multiple ways and receive multiple answers.

Sowder and Wheeler (1989) & Lefevre (1993)

What conceptual math knowledge do students need to estimate computationally?



- Knowledge of arithmetic facts
- Mental computation
- Understanding base 10
- Understanding place value
- Ability to make size comparisons

Sowder (1989)

What is the importance of computational estimation?



- How can students work with and manipulate numbers if they do not have a conceptual understanding of their relative size and relation to one another?

Van de Walle, Karp, and Bay-Williams (2010)

The Study



- We created an estimation skills test that is designed to look at students' abilities to utilize different estimation strategies. It is split into two sections:
 - 1) Written Section
 - 2) Verbal Section
- The test was conducted with the 35 Berry College Middle School students for the written section and 14 of those students were then selected to partake in the verbal section.

How the Test Questions were Derived



- We wanted different questions to be more conducive to specific methods of problem solving and estimation.
- The test did not specifically say “estimate.” We instead wanted the test to imply estimation through words like “approximate” and “about.” This way one could look at the student’s ability to problem solve or look at the context of the question.
- The questions look for student reasoning rather than simple numerical answers.

The Analysis



Results are based on a student's ability to:

a) utilize and recognize multiple methods of problem solving and estimating—

- Rounding
- Benchmarks
- Compatible Numbers
- Front-End Method
- Clustering/Averaging
- Invented
- Compensation

b) the student's conceptual understanding of what an estimate is.

How were the estimation skills tests assessed?



A rubric was formed for the verbal section. It gives points to students for their ability to:

- explain how and why they solved in a certain way.
- solve through exact methods, use an estimation strategy, or adjust (no points were given if a student was incapable of finding a solution).
- recognize that multiple answers and strategies were acceptable.

Example Student Work from the Test



For the purposes of this presentation we will focus on two students in particular: One who shows exemplary estimation skills and one who struggled on the estimation skills test. We will be using two sample questions from the verbal section of the test. So that the students remain anonymous, we will refer to both of them using the feminine pronoun.

Question 1



Your younger sibling is having trouble with some math homework one night and your mother asks you to help because you are really great at math. Your younger sibling's first math problem says the following:

- Suzanne is at the computer store and sees a computer that is normally \$325.72. It is now half price because of a weekend sale. About how much is the computer, now that it is on sale?

Your younger sibling is confused because he/she says that the class has never worked with such large and complicated numbers before. He/she also says that the teacher told them not to use a calculator! What would you advise your sibling to do?

Student One's Response

Handwritten student work showing a division problem and a final answer. The work is written on a white background with a blue border. At the top left, there is a small circle with a red outline. The work includes the following:

- A division problem: $8 \overline{) 325.72}$ with a dollar sign on the left.
- A subtraction step: $\begin{array}{r} 32 \\ -32 \\ \hline 00 \end{array}$ with a downward arrow.
- A subtraction step: $\begin{array}{r} 8 \\ -8 \\ \hline 0 \end{array}$ with a downward arrow.
- A subtraction step: $\begin{array}{r} 7.7 \\ -7.7 \\ \hline 0 \end{array}$ with a downward arrow.
- A subtraction step: $\begin{array}{r} 452 \\ -48 \\ \hline 4 \end{array}$ with a downward arrow.
- A final answer: 40.715 with a dollar sign on the left.
- A note: "R4" written to the right of the final answer.

- “You would multiply \$325.72 by...no...you would reduce the number by 2...I’m sorry, you would divide by 5 because 5 goes into 35..wait divide by 8 because there is 32.”
- When asked to solve for an answer the student solved as shown on the left. The student struggled to explain how she came to her answer.

Student Two's Response



- “I would probably say round to \$300 if it doesn't have to be exact. So that would be \$150 and then take a little higher than \$150. If they were really young then, round to \$400. If they are closer to my age then they can round to \$350.”

Question 2



At the beginning of math class one day, your teacher places a math problem on the board. You are placed into groups of four to discuss the problem and come up with a group answer. The question is:

- Brian owns a catering business and was just hired for a very large birthday party. The people throwing the party order 37 party platters. The people ordering the platters want Brian to quickly tell them about how much the total is for their order. What should Brian tell them if he knows that 1 platter costs \$11.56?

What would your answer be? Why? How did you get that answer?

Student One's Response



“I think you would times it. I think dividing would take too long. He could estimate but it wouldn't be exact.” When asked how she would divide for the problem, the student responded, “you would say what times 37 goes into \$11.56.” When asked how the student would estimate, responded, “I would round,” but could not give an answer or explain how she would round.

Student Two's Response



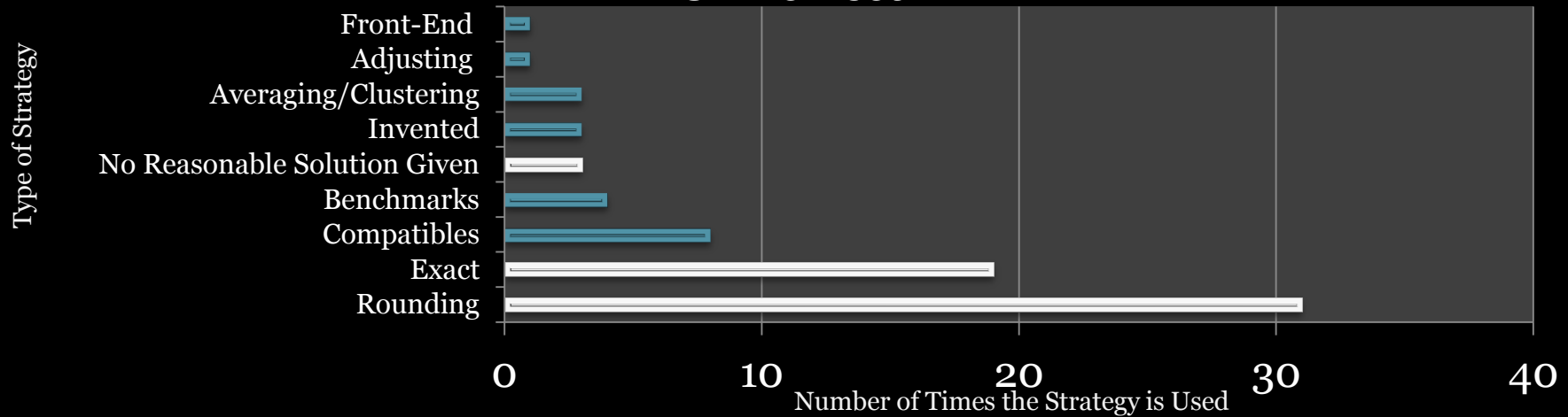
- “If it needs to be quick round up to \$40 and down to \$11...it’s \$440.”
- When asked why the student rounded this way she responded, “If I rounded them both up or down it would change the price more. In the real world I would round down to make it seem cheaper.”

What do these two particular students show us from their responses?

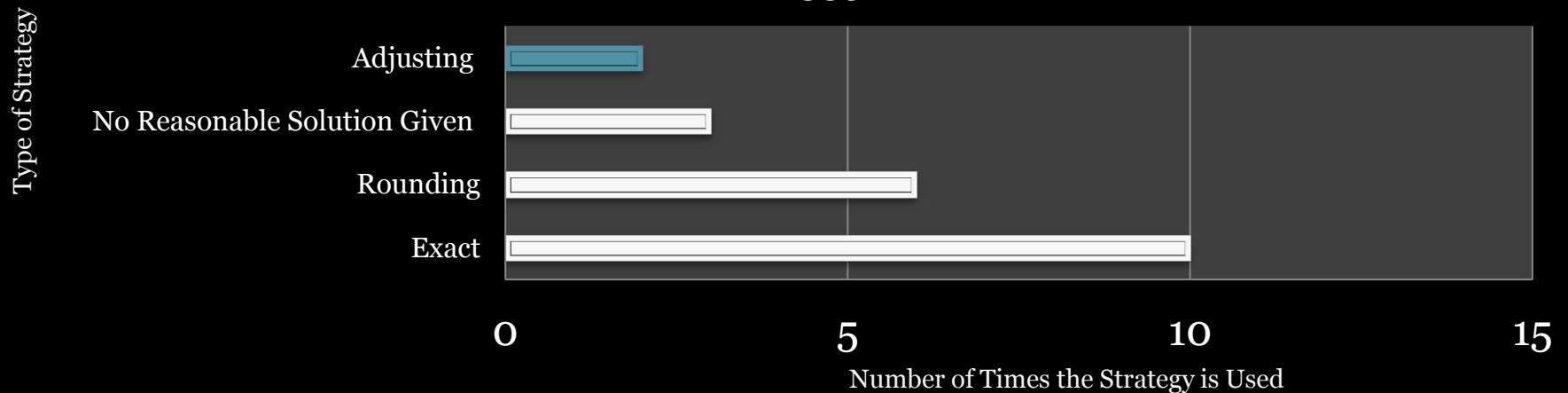


What were the overall results of the study?

Student Thought Processes on the Written Estimation Skills Test



Student Thought Processes on the Verbal Estimation Skills Test



What did the rubrics show?



- Out of the 14 students who participated in the verbal section, 5 passed/excelled on the estimation skills test.
- Of the 5 that passed, only one excelled meaning that they utilized adjusting/compensation.

Where should the study go from here?



- Based on the results of the study, we decided to implement a series of lessons which would foster mathematical thinking. The lessons are aimed to focus on the use of using a variety of strategies related to estimation. They are additionally connected to the Common Core Standards.
- At a local public elementary school, we are currently going into one fifth grade classroom of 10 boys and 8 girls. We are giving a pre-test similar to the test given in the first half of the research project. We will then begin to implement a series of 7 lesson plans linking estimation strategies to the fifth grade Common Core Standards. In the end, we will give a post-test slightly different from the pre-test to show the results of math workshop instruction in estimation. From the results, we can see the effects of estimation instruction on problem solving skills.

What did the seven lesson plans cover?



- The lessons each start with a KWL chart which is expanded upon over the course of the semester.
 - **Lesson 1: *Rounding/Front-End Method*** (adding decimals- 5.NBT.4, 5.NBT.7)
 - **Lesson 2: *Compatibles Method*** (adding decimals-5.NBT.7)
 - **Lesson 3: *Rounding Method*** (multiplying decimals- 5.NBT.4, 5.NBT.5)
 - **Lesson 4: *Compatibles Method*** (division- 5.NF.3)
 - **Lesson 5: *Averaging*** (statistics/graphing- 5.MD.2)
 - **Lesson 6: *Benchmarking*** (adding/subtracting fractions- 5.NF.2)
 - **Lesson 7: *Adjusting*** (adding/subtracting fractions- 5.NF.2)
- The unit is completed with an estimation jeopardy review game.

Let's do an example lesson from the unit!



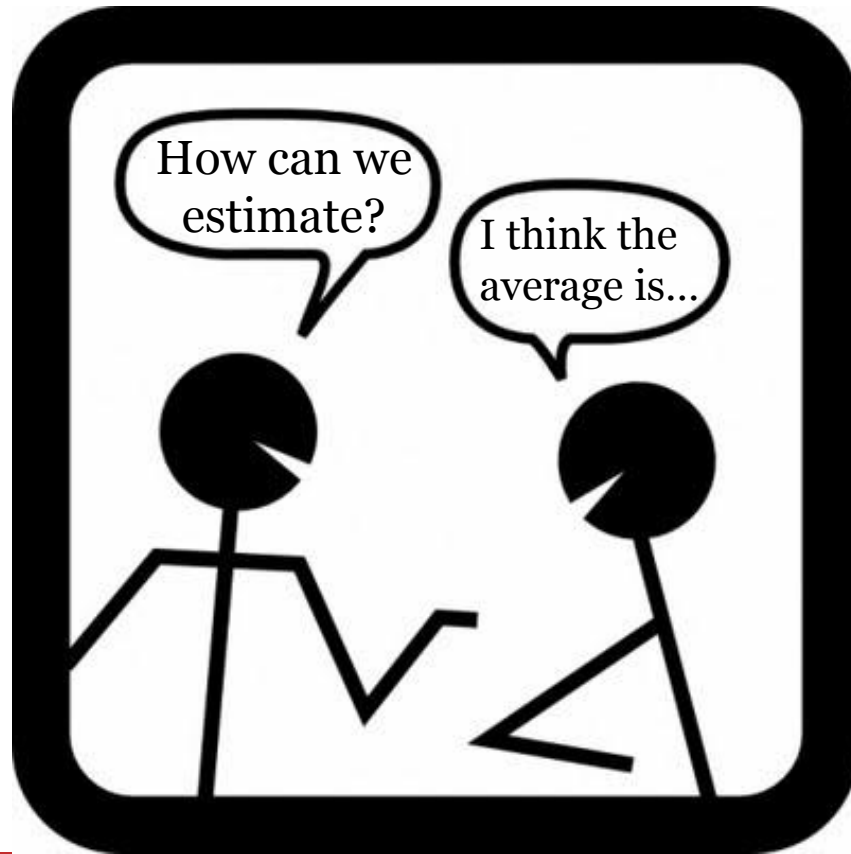
AVERAGING (*STATISTICS AND GRAPHING*)

- 1) THE STUDENTS WILL MEASURE EACH OTHER'S HEIGHTS IN FEET AND PLACE THE INFORMATION ON A CLASS LINE PLOT (**6.SP.4**).
- 2) THE STUDENTS WILL ESTIMATE THE AVERAGE HEIGHT OF THE CLASS BASED ON THE INFORMATION SHOWN ON THE LINE PLOT (**6.SP.2**).

With the person sitting next to you, measure and record each other's height in inches using the measuring tape found at your table. When you have finished, come up to the front to record your measurement...



With your partner, look at all of the measurements on the board and our class line plot. Discuss how you could estimate the average height of the class. What is your estimate? Write down and explain your reasoning.



What are your thoughts about this lesson?



- Multiple strategies were acceptable, though averaging was highlighted.
- Hands-on
- Discussion based
- Connected to real world
- Linked to standards

Let's Review!



JEOPARDY!

Conclusion



- We hope that today's presentation inspired you to incorporate estimation lesson plans in your classroom.
- Estimation is an important skill related to number sense and problem solving.
- We hope to share the results of the remaining section of our study in the future.

Works Cited



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Estimation Jeopardy!

CREATED
BY
MS. HARTMANN

Rounding or Benchmarks

Compatibles

Front-End

Clustering/ Averaging

10

100

100

100

0

200

200

200

200

300

300

300

300

400

400

400

400

500

500

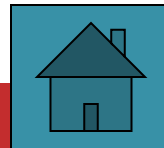
500

500

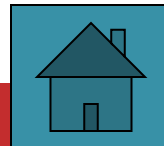
Describe rounding
and using
benchmarks. When
would you use these
strategies?



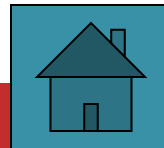
What is the
compatibles
strategy? When
might you use this
strategy?



How would you solve an estimation problem using the front-end method?



What is the clustering/averaging strategy? When would you use this strategy?



Round to the nearest tens place...

396.764



Which two numbers add up to make a benchmark?

a) $16 + 28$

b) $58 + 42$

c) $65 + 43$

d) $73 + 62$



Add up the numbers listed below using the front-end method. What is the estimated total? Explain your work.

4,892
16,033
2,596
8,246



What is your estimate for the average of the numbers listed below?
How do you know?

25, 20, 19, 12, 20, 15, 20 ,
20



Use benchmarks or rounding to solve the estimation problem below:

Before going to the grocery store, Mrs. Bowman wants to know about how much she will spend. About how much will she spend based on the sale prices shown below in the weekly sale advertisement?

Sale Items of the Week!

Milk	\$1.98
Broccoli	\$2.43
Cheese	\$1.14
Bread	\$1.50



Use the compatibles method to solve the estimation problem below:

Gary works for the post-office shipping packages in his truck. He can only take a total 2000kg in weight. Gary needs to quickly tell if the large boxes in his shipment weigh too much in total. Can Gary carry all of these boxes in his truck? About how much do they weigh in total?

Box 1: 115

Box 2: 203

Box 3: 475

Box 4: 283

Box 5: 450

Box 6: 197

Box 7: 247



Use the front-end method to solve the estimation problem below:

Sadie is at an electronics store and is purchasing a new computer she found on sale. The computer was originally \$678 and is now \$398. About how much did Sadie save on the computer?



Use the averaging/clustering method to solve the estimation problem below:

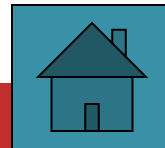
Kylie has a lemonade stand. On her last day selling lemonade her father asks her about how much money she made on average each day. About how much did she make each day? How do you know?

Monday	\$3.42
Tuesday	\$2.12
Wednesday	\$3.78
Thursday	\$2.50
Friday	\$2.98



Use the rounding method to solve the estimation problem below:

In the Braves opening baseball game 3,622 people attended. Tickets for the game cost \$12.30. About how much did everyone spend on the game in all?



Use the compatibles method to solve the estimation problem below:

Tessa is at a local farm stand. She needs to buy a few oranges but wants to know what the price is per orange. About how much does one orange cost?

18 oranges \$3.48



Use the front-end method to solve the estimation problem below:

Mr. Williams and his family just got a new puppy! His daughter agreed to buy some new things for the dog. She only has ten dollars to spend. Does she have enough money to purchase all of the items shown below? How do you know?

Dog Bowl	\$1.86
Dog Bed	\$7.43
Dog Toy	\$1.05



Use the averaging method to solve the estimation problem below:

John is the head camp counselor at a local nature preserve. He needs to know approximately how many children attended throughout the summer.

First session	79
Second session	64
Third session	69
Fourth session	74
Fifth session	72
Sixth session	63

***Pause and Think:** Is this question asking for an average? How can you use the averaging method to solve?



Use the rounding method to solve the estimation problem below:

Mrs. Simmons is holding a bakes sale. Her first customer purchases 3 loaves of bread and one piece of apple pie. About how much did the first customer spend?

Cookies	\$1.47
Apple Pie	\$1.75
French Bread	\$1.19

*Do you think you estimate is lower or higher than the exact answer? Why/why not? Can you adjust your answer?



Use the compatibles method to solve the estimation problem below:

Michael Johnson is the fastest human runner recorded. He can run 400 meters in 43.18 seconds! About how many meters can he run in just one second?

*Do you think your estimate is lower or higher than the exact answer? Why/why not? Can you adjust your answer?

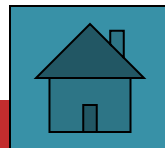


Use the front-end method to solve the estimation problem below:

The mayor wants to know approximately how many people voted in the election in total. About how many voted?

Candidate X	42,958,376
Candidate Y	34,732,149
Candidate Z	5,552,349

*Do you think your estimate is lower or higher than the exact answer? Why/why not? Can you adjust your answer?



Use the averaging method to solve the estimation problem below:

Ryan played his favorite video game four times this weekend.
About how much is his average score for the game?

Game 1 147

Game 2 156

Game 3 139

Game 4 162

*Do you think your estimate is lower or higher than the exact answer? Why/why not? Can you adjust your answer?

