"Green" Geometric Modeling: Capture Interest and Address Common Core Standards

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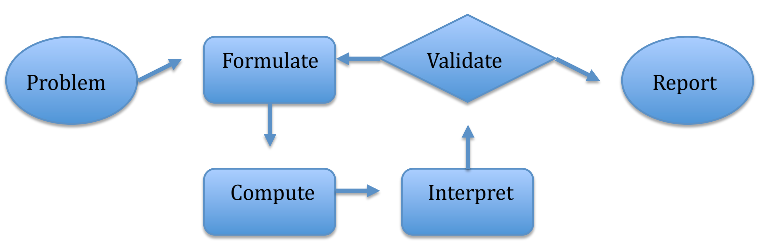
Our Goals for Today's Workshop

- Introduce you to Mathematical Modeling
- What does the CCSS say about **Modeling** and **Geometry**?
- NCTM's Focus on Reasoning and Sense Making
- Activity 1: Maximizing use of shelf space for sport-drink bottles
- Activity 2: "Green" packaging of sport-drink bottles
- Activity 3: "Green" packaging of bike helmets
- Extensions: What else can we package more efficiently?
- Questions and Recap

Mathematical Modeling



Common Core State Standards Initiative. (2010). *Common Core State Standards for Mathematics*. Washington, DC: National Governors Association Center for Best Practices, Council of Chief State School Officers, Council of Chief State School Officers. Retrieved from www.corestandards.org

Mathematical Modeling in the Common Core State Standards (CCSS)

"Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards"

Common Core State Standards Initiative. (2010). *Common Core State Standards for Mathematics*. Washington, DC: National Governors Association Center for Best Practices, Council of Chief State School Officers, Council of Chief State School Officers. Retrieved from www.corestandards.org.

Common Core State Standards for Modeling using Geometry

- Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).
- Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost).
- Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems
- Use volume formulas for prisms, cylinders, pyramids, cones, and spheres to solve problems.

Common Core State Standards Initiative. (2010). *Common Core State Standards for Mathematics*. Washington, DC: National Governors Association Center for Best Practices, Council of Chief State School Officers, Council of Chief State School Officers. Retrieved from www.corestandards.org.

Common Core State Standards (CCSS)

CCSS also emphasizes in its Geometry standards that students:

"Visualize relationships between two-dimensional and three-dimensional objects"

Common Core State Standards Initiative. (2010). *Common Core State Standards for Mathematics*. Washington, DC: National Governors Association Center for Best Practices, Council of Chief State School Officers, Council of Chief State School Officers. Retrieved from www.corestandards.org.

NCTM Geometry Standard on Modeling (Grades 9-12)

Use visualization, spatial reasoning, and geometric modeling to solve problems.

- Draw and construct representations of two- and three-dimensional geometric objects using a variety of tools.
- Visualize three-dimensional objects from different perspectives and analyze their cross sections.
- Use geometric ideas to solve problems in, and gain insights into, other disciplines and other areas of interest such as art and architecture.

National Council of Teachers of Mathematics (2000). *Principles and Standards for School Mathematics*, Reston, VA: NCTM.

NCTM: Reasoning and Sense Making

"A high school mathematics program based on reasoning and sense making will prepare students for citizenship, for the workplace, and for further study." (p.3)

National Council of Teachers of Mathematics (NCTM). *Focus in High School Mathematics: Reasoning and Sense Making*. Reston, Va. : NCTM, 2009.

Key Elements of Reasoning and Sense Making in Geometry

1) Conjecturing about geometric objects.
2) Construction and evaluation of geometric arguments
3) Multiple geometric approaches.
4) **Geometric connections and modeling. Using geometric ideas, including spatial visualization, in other areas of mathematics, other disciplines, and in real-world situations.**

National Council of Teachers of Mathematics (NCTM). *Focus in High School Mathematics: Reasoning and Sense Making*. Reston, Va. : NCTM, 2009.

Efficient Use of Space

Question

Find an optimal method for packaging sport drinks when your primary concern is to…

1. use shelf space well;
2. use the least amount of packaging material;
3. use the least space possible (think 3-D);
4. make the packaging most attractive to consumers.

Activity 1: Maximizing the use of shelf space for sport drink bottles

Questions

1. What ways can you arrange bottles on a shelf to use space efficiently?
2. How would you decide if one arrangement is more efficient than another?
3. How would you define efficiency in this context?

Activity 1: Maximizing the use of shelf space for sport drink bottles

Efficiency Rating (ER) for two-dimensional area can be calculated as:

Area covered by object(s) $\div$ Area available

** ER values close to 1 are desirable

Activity 2: "Green" packaging of sport drink bottles

Questions

1. Design a rectangular prism to package 12 sport drink bottles (7cm diameter, 20cm height).
2. Which arrangement of 12 bottles would produce a rectangular box that uses the least amount of material?
3. What other arrangements might use less material?

Activity 2: "Green" packaging of sport drink bottles

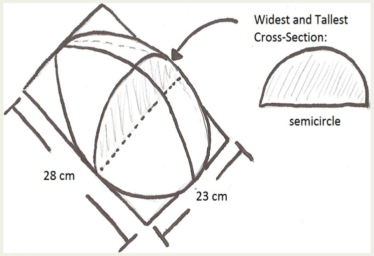
Question 4

One brand of sport drink bottle begins to taper at about 13 cm from the bottom and has a cap that is 4cm in diameter. Can you design a more efficient package in terms of surface area and volume if the package also tapers at the top? Show your design and calculations.

Activity 3: "Green" packaging of bike helmets

The **On-The-Go** bike helmet company is looking to use environmentally friendly packaging. They have already determined a relatively "green" material to package their helmets, but now they are looking to determine a packaging design that reduces the amount of material used. Not only will this be more "green" but also more cost effective.

Activity 3: "Green" packaging of bike helmets



The package needs to fit the helmet and a 28x23 sheet of bike decals that are promised free with any helmet purchase. If this sheet is bent or curled in any way, it could damage the decals, so it must lay flat underneath the helmet. The thickness of the sheet itself is insignificant.

Activity 3: "Green" packaging of bike helmets

Two Packaging Designs to Consider:

1. a regular semi-hexagonal prism with smallest dimensions possible such that it is able to hold both the helmet and the bike decals with a 0.25cm buffer around the helmet across the top and sides (it will sit flat on the sheet of decals);

2. a half-cylinder with smallest dimensions possible such that it is able to hold both the helmet and the bike decals with a 0.25cm buffer around the helmet (it will sit flat on the sheet of decals).

Activity 3: "Green" packaging of bike helmets

<u>Questions</u>

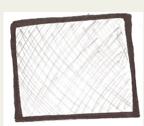
1. Draw a 2-D net for each 3-D package design.
2. Identify the necessary dimensions of each package design.
3. Calculate the total amount of material necessary for each package design. Which design is more "green"?

Activity 3: "Green" packaging of bike helmets

<u>Extension</u>

The "green" material for the helmet packaging only comes in 150cm by 150cm square sheets.

Suppose you have exactly one sheet. How many of each package can you make using only this one sheet?

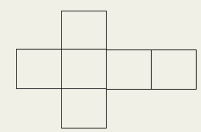


150 cm x 150 cm

Another Investigation: NETS

A 2-D net for a 3-D object can be folded to become the object.

- *2-D net for a cube design would be:*



- *Consider 2-D nets for various packages of headphones (or other objects of interest to your students)*

Summary

- Key mathematical concepts
  - Area of circles, rectangles, triangles, hexagons and various other shapes
  - Volume of cylinders, rectangular prisms, trapezoidal prisms, etc.
  - Nets and their relation to 3-D objects, surface area, etc.
  - Spatial reasoning/visualization
  - Calculating efficiency ratings
- CCSSM, the modeling process, and mathematical reasoning

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