Interviewing students to learn about their algebraic reasoning

NCTM Regional Conference
October 2012

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Interview Assignment from *Connecting Arithemtic to Algebra* on-line course

One way to learn about students' thinking is to listen carefully to individuals as they articulate their thoughts while doing mathematical tasks. The math interview is one strategy for obtaining evidence about students' ideas. Think about the range of learners in your classroom and choose a student you would like to learn more about. In your interview, explore the student's thinking about the meaning of one of the operations by posing interview tasks and questions. Include these two components:

1. Ask the student to create at least two representations to illustrate a particular arithmetic expression, such as 4×5 or $18 + 16$. Representations should include a story situation, a drawing or visual representation, and/or actions with cubes. Include questions that elicit the connections between different representations.

2. Expand the work to looking at arithmetic sentences by posing one of the following: Is this number sentence true? $18 + 16 = 20 + 14$ or How do you know they are equal? 4×5 and 2×10. Encourage the student to use the representations developed in the first part of the interview to explain his or her thinking.

Although you will need to plan math tasks and questions in advance, you will also need to listen carefully to what the student does and says during the interview, so that you can follow up with questions or problems that seem appropriate. Keep in mind that your job in the interview is to find out as much as you can about the student's ideas; at this time, you are not necessarily trying to teach the student anything. Tape-record the interview so you will be able to listen to it later. Choose a portion of the interview to transcribe, analyze and write about.

Your posting should include the task(s) you chose, a transcription of a portion of the interview with a narrative to describe what happened, and a discussion of what you learned about the student's ideas, including what surprised you about the student's thinking. Conclude the writing with a paragraph or two of reflective comments describing what you learned from the experience and what questions this work raises for you.

Example of interview summary and reflection

I interviewed a student in my 4th grade classroom whom I have had questions about since the beginning of the year. Theresa has always been comfortable participating in class discussions and willing to share her thinking, but she is not proficient at articulating her ideas. I chose to give her the expression 4x5 and then the equation 4x5=2x10. We have been working with multiplication and representation of multiplication and I wanted to hear Theresa's ideas about this in an individual setting.

Ms. Franklin: Theresa, I would like you to make two different representations of 4x5 (point to expression on paper). You can write a story situation, draw a picture, or use the cubes.

Theresa: Wait, do you mean solve the problem and draw a picture.

Ms. Franklin: I don't mean for you to solve the problem, I would like you to show me what 4x5 means (and then repeat directions above.)

Theresa: ok, I am going to write a story and then draw a picture.

1. *Jack has 4 packages of bottles, each pack has 5 bottles in it. How many bottles does Jack have?*

2. 

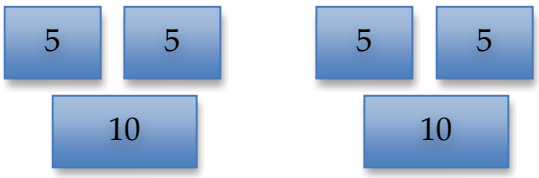
Ms. Franklin: Now look at this equation: 4x5=2x10. Is it a true equation?

Theresa: Is this like one of those "no answer" problems?

Ms. Franklin: Yes! I would like you to explain your thinking using a representation.

Theresa: Well, I know this is true so I am going to draw another picture.

Ms. Franklin: ok, and you may use the drawing from the previous thinking.

Theresa: 

Ms. Franklin: Tell me about your representation

Theresa: I know they are equal because 10 is (2x5). So I put two 5's together and that makes one ten, and the other two fives makes the other 10.
Ms. Franklin: How do you think your picture proves they are equal?

Theresa: I know because my picture shows they are the same size. See how the four 5's is equal to the two 10's?

Ms. Franklin: Can you think of another equation that would work like this?

Theresa: ummm….with different numbers? I think you could do 4x6=2x12. The picture would look the same except 6's in the small boxes and 12 in the two big boxes. Its like a pattern.

I learned a lot from having this one on one conversation with Theresa about her thinking. I found that she was able to articulate her ideas and ask clarifying questions in this setting, which was not happening with her in the whole class discussions. She demonstrated a strong understanding of multiplication in the first set by writing an accurate story situation to illustrate 4 groups of 5. She also drew a picture of 4 packages with the number 5 in each to show the meaning of that expression. With the second part of the question, Theresa applied her knowledge to think about whether the equation was true. She was able to recall the idea of the activity from our whole class practice with a similar equation. Theresa drew the same 4 boxes of 5 in each and then drew arrows from 2x5 to 1x10. I was surprised that Theresa was able to draw an accurate representation that did prove that the equation was true. She used a form of an array, but didn't fully articulate them as arrays, but rather stuck to her original problem about packages. Theresa's mathematical thinking is more sophisticated than I thought from in-class and formal assessments.

I am now curious as to whether Theresa would be able to articulate the generalization of halving and doubling with a couple more exposures to the same idea. I was impressed that she was able to use different numbers by doubling 6 to 12 and halving 4 to 2. She saw this idea as a pattern instead of as a general idea about math that will happen every time. "If you double one factor and halve the other, you will get an equivalent equation." I had to end our session together because of a time constraint, but I feel like if I were able to probe further, and use more examples, she may have seen the rule. I also was surprised about her representation, mostly because I have normally shown halving and doubling to my students with an array. This interview raised some questions for me about whether this type of one-on-one interview would be beneficial to get a true assessment of all students' thinking. It made me realize how the whole class discussions are not always an accurate portrayal of a student's knowledge. I have been thinking about setting up an interview or small group discussion about these ideas during a math workshop time, which is when students are working on different activities or games during the unit individually or with a partner. I feel like I could push their thinking and get a true sense of each students' ideas in a small group or one on one setting.