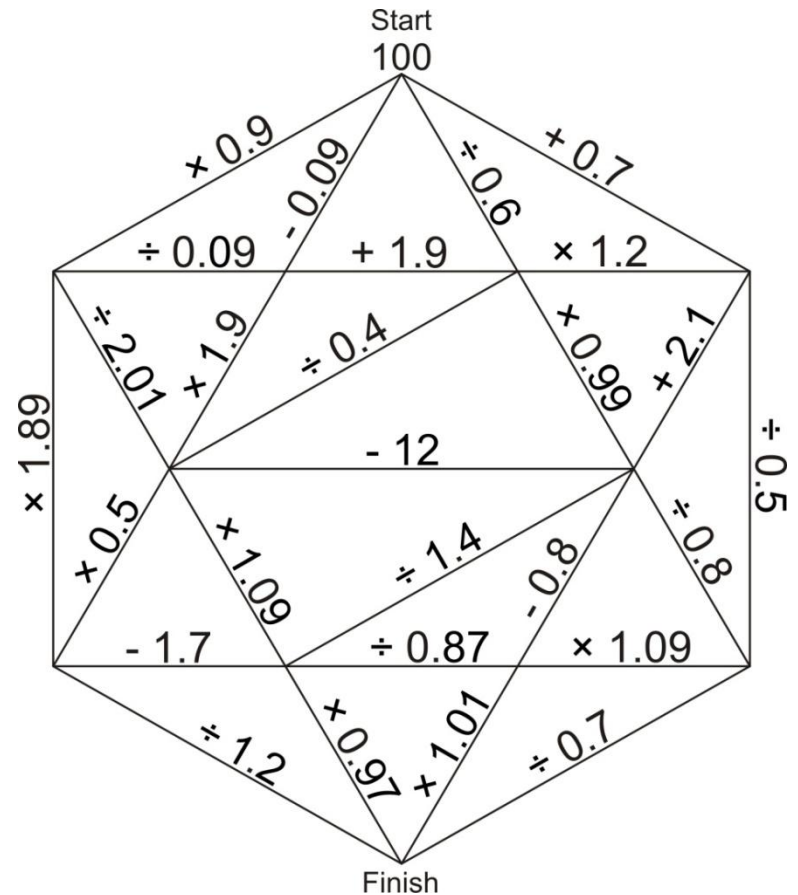# Warm-up: Decimal Maze

- Begin with a value of 100.

- Move down or sideways from **Start** to **Finish**.

- As you cross a segment, perform the indicated operation.

- You may not go up. You may not cross a segment more than once.

- *What is the largest possible value when you reach **Finish**?*

Start
100

× 0.9   – 0.09   ÷ 0.6   + 0.7

÷ 0.09   + 1.9   × 1.2

÷ 2.01   × 1.9   ÷ 0.4   × 0.99   + 2.1

× 1.89   – 12   ÷ 0.5

× 0.5   × 1.09   ÷ 1.4   – 0.8   ÷ 0.8

– 1.7   ÷ 0.87   × 1.09

÷ 1.2   × 0.97   × 1.01   ÷ 0.7

Finish

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Welcome

~~23~~	(94)	~~18~~	~~13~~
~~31~~	~~102~~	~~26~~	(21)
(31)	~~102~~	~~26~~	~~21~~
~~40~~	~~111~~	(35)	~~30~~

What'd You Get?

181

Whoa!

How does that work?

Engaging and Free Online Resources for Teaching Operations and Fractions

October 26, 2012

Sarah DeLeeuw

sdeleeuw@nctm.org

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.

2. Reason abstractly and quantitatively.

3. Construct viable arguments and critique the reasoning of others.

4. Model with mathematics.

5. Use appropriate tools strategically.

6. Attend to precision.

7. Look for and make use of structure.

8. Look for and express regularity in repeated reasoning.

Reasoning and Sense Making

- It is very important for teachers to lead scholars into the habit of **attending to the process going on in their own minds** while solving questions, and of **explaining how they solve them**. [...] It is next to impossible for a person to direct another's thoughts unless he knows the channel in which they are already flowing.

 – Warren Colburn, *Teaching Arithmetic in the Method of Pestalozzi*, **1830**

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Decimal Maze

- Begin with a value of 100.

- Move down or sideways from **Start** to **Finish**.

- As you cross a segment, perform the indicated operation.

- You may not go up. You may not cross a segment more than once.

- *What is the largest possible value when you reach **Finish**?*
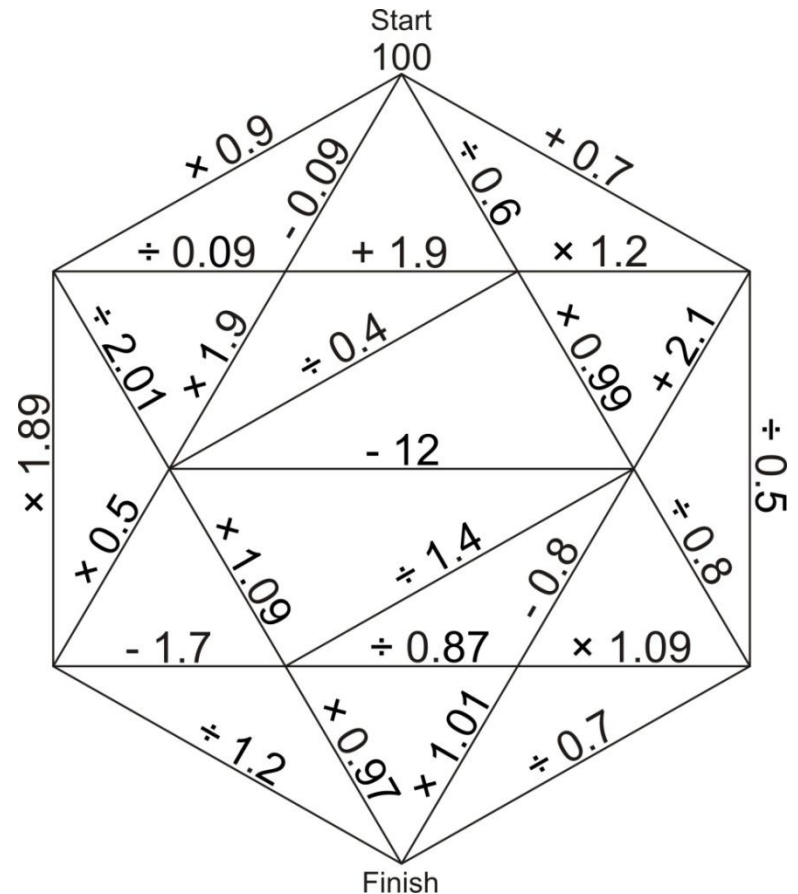
Decimal Maze

Here's some help:

- The **red lines** are beneficial.

- The **gray lines** are detrimental.

- Addition and subtraction are inconsequential.
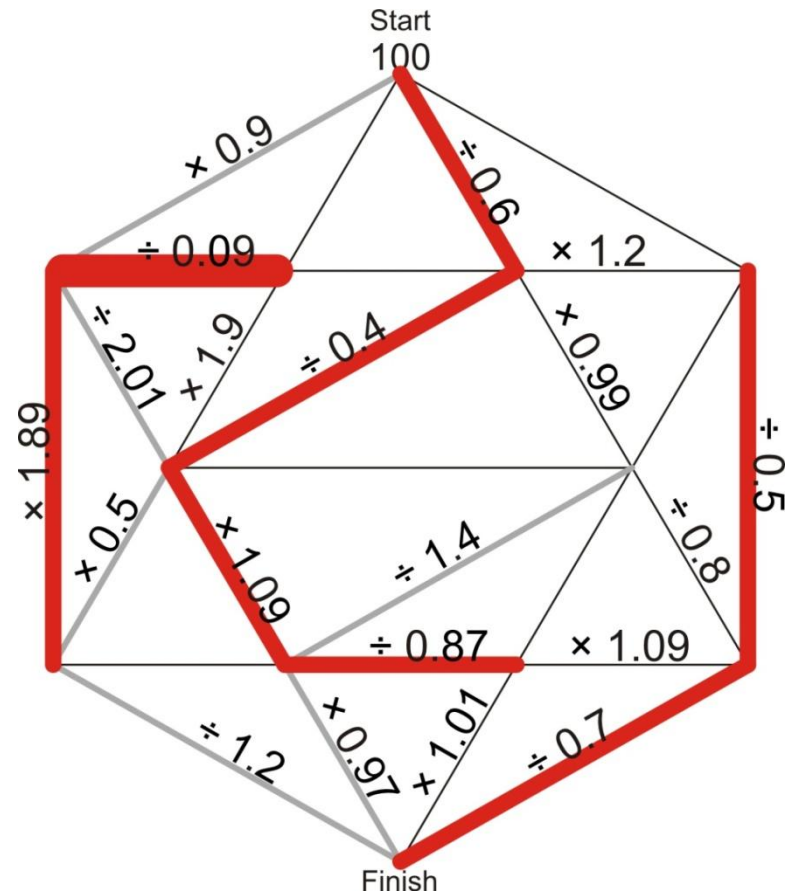
Decimal Maze

- Maximum value: **6332**

- Minimum value?

- Finish value closest to 100?

- How many paths from Start to Finish?

- How else might you modify this activity?

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Decimal Maze

Pick-a-Path

Pick-a-Path

- http://illuminations.nctm.org/pickapath

Play Anywhere. Learn Everywhere.
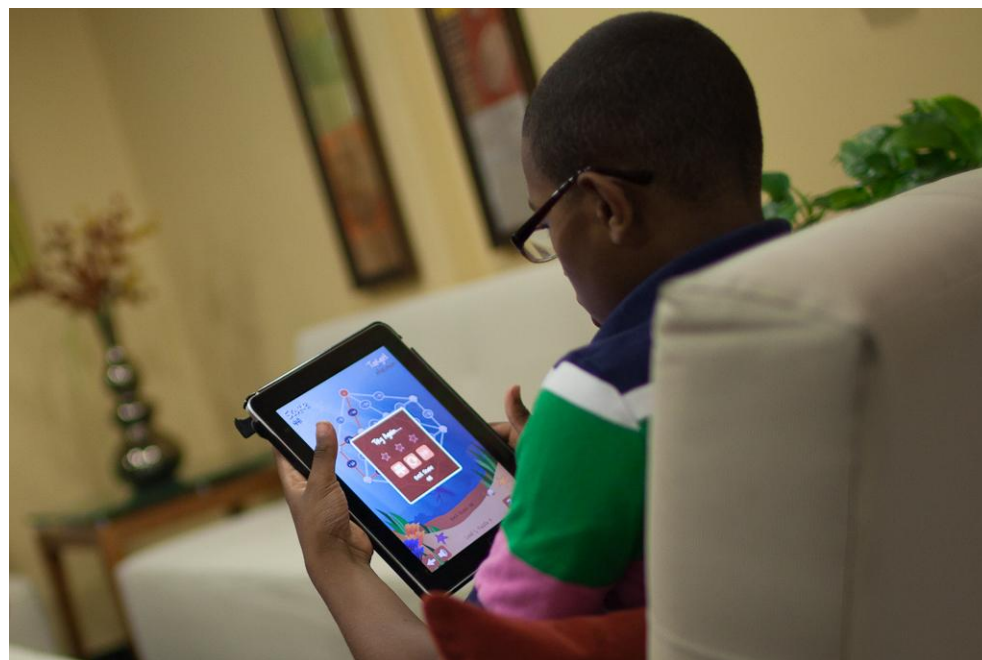
Dollar Nim

- Start with a dollar
- Remove any coin:
  - Penny
  - Nickel
  - Dime
  - Quarter
- Player to take the last coin wins

Dollar Nim

- What is the winning strategy for this game?

- How could you modify this game for use with your students?

Extension from NY Times Blog

Since Dollar Nim is played with real money, it makes sense for the participants to keep the change they remove. This confers a reward for removing larger denominations. To offset this, the winner must be given an extra monetary reward. What should be the **minimum** prize money for the two-player game so that no matter what happens, the winner comes out ahead?

Enrichment: Eleven Nim

- Start with a dollar
- Remove any coin:
  - Penny
  - Dime
- Player to take the last coin wins

John Mason, Math 2.0 Listserve

"Just because I play a game, it does not follow that I become aware of what I am doing [or the] underlying mathematical thinking. ...the value of playing a 'mathematical game' may lie not in the playing so much as in the reflective consideration of effective and ineffective actions."
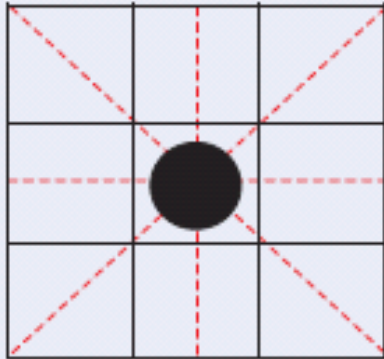
Three C's of Game Play

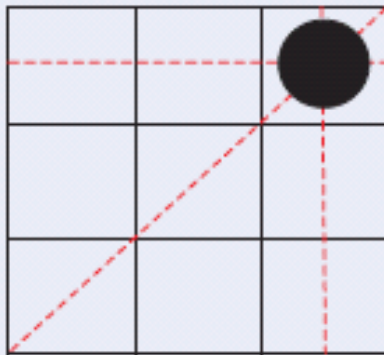- Competition

- Collaboration

- Communication

Even one-player games can spark rich discussion of strategy.
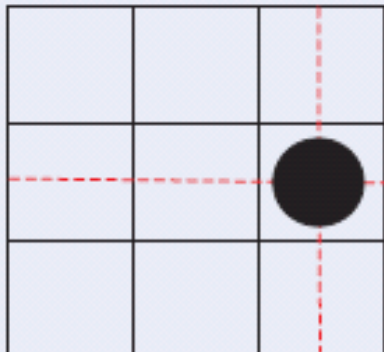
Tic Tac Toe
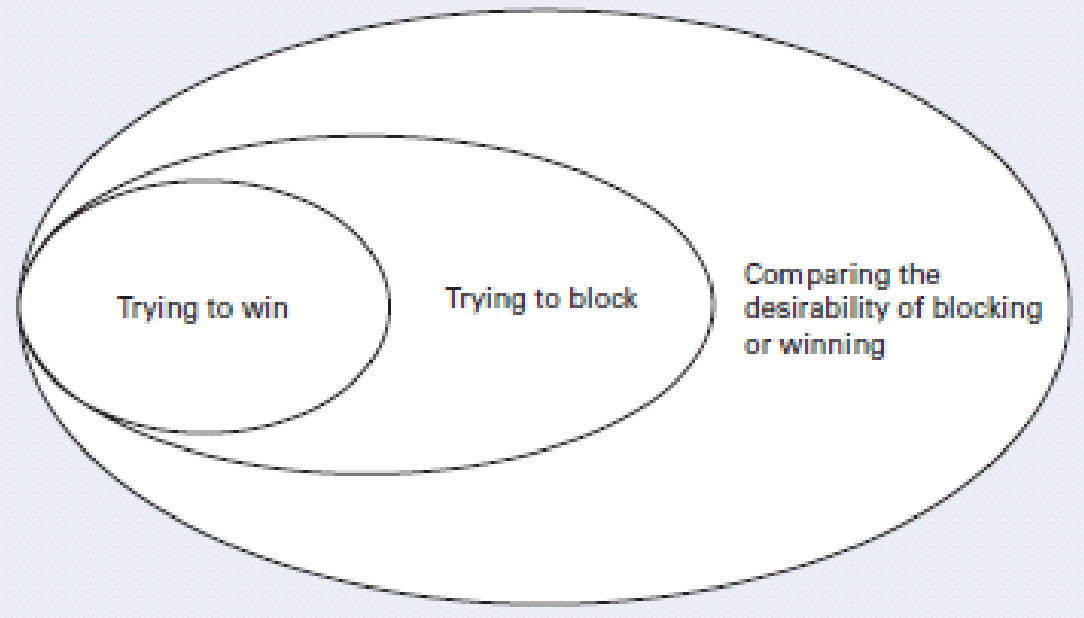

(a) The most advantageous place

(b) The next most advantageous place

(c) The least advantageous place

The hierarchical development of the three levels

Trying to win

Trying to block

Comparing the desirability of blocking or winning

Kamii, C. The Educational Value of Tic-Tac-Toe for Four-to Six-Year-Olds. *Teaching Children Mathematics,* May 2008.

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Dig It

Dig It

- What are the best numbers to try to get?

- What number(s) are easiest to get?

- Which points on the number line can be created in the least number of ways?

- How many fractions can be created with a value less than 1?

- Which digit is the best to get?

Calculation Nation®

- **An online world of math strategy games**

- One- and two-player games:
  Challenge others. Challenge yourself.

- Basic registration process:
  - Username
  - Email
  - Password

- Can play games as a guest without registration

Calculation Nation™
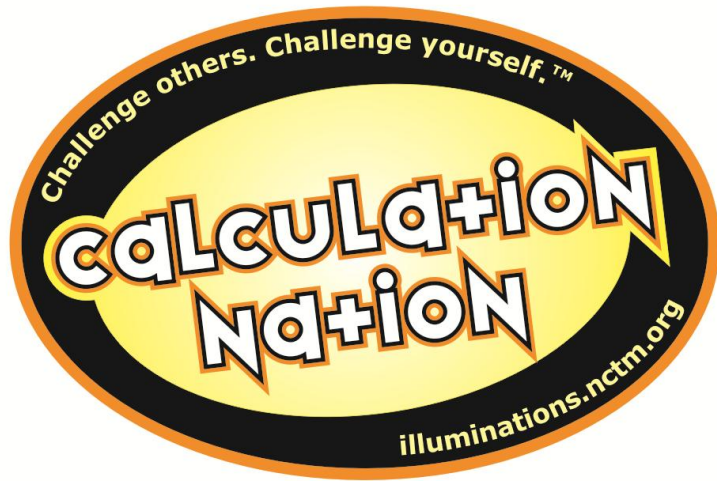
Official Launch

April 22, 2009

To Date: 1,209,527 Visitors

September 2012: 1,500 Visitors/Day

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Calculation Nation™

- Idea Inspired by Teachers
- Played the "Product Game" Online Using Instant Messenger
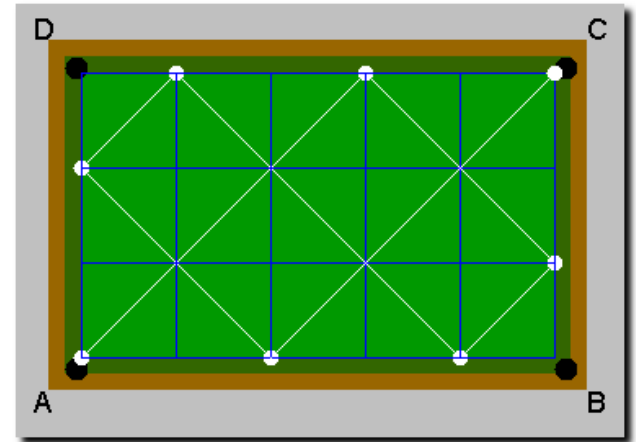
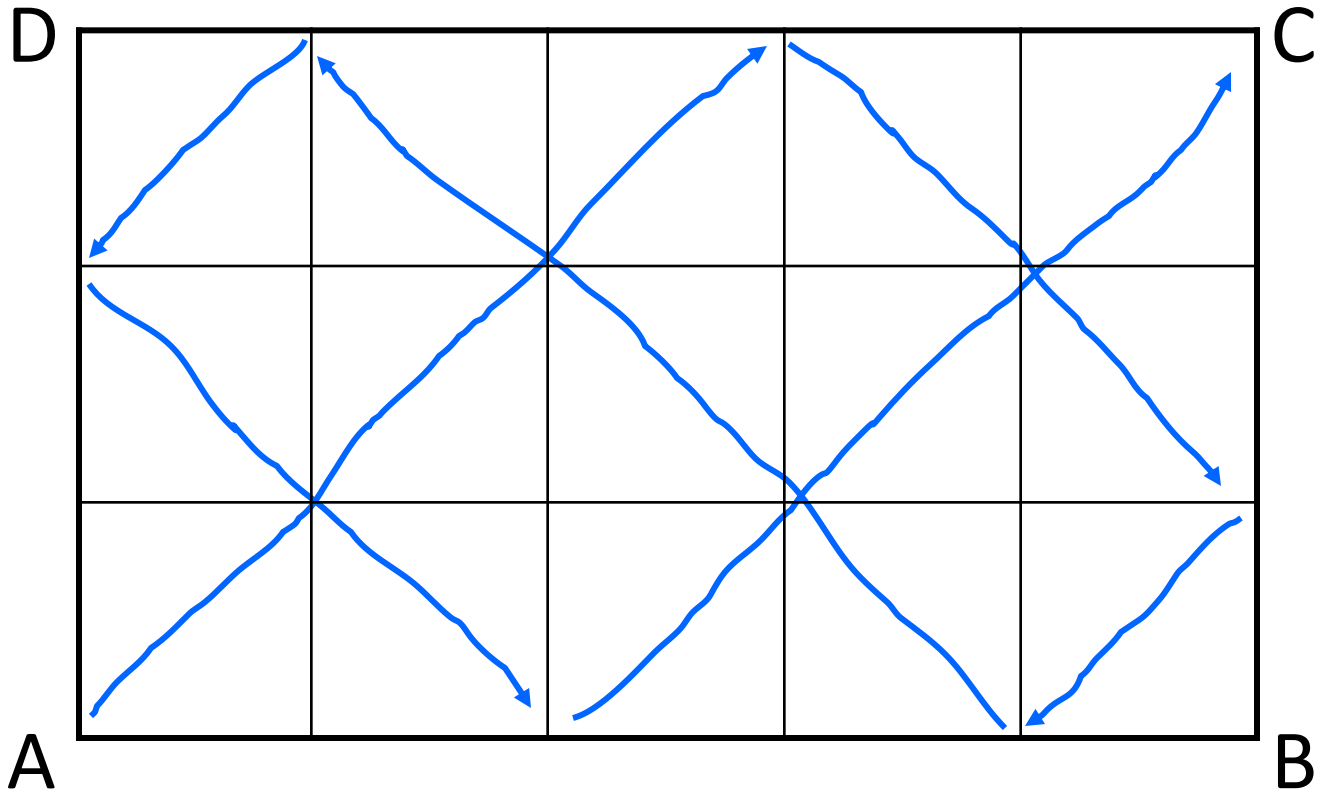# History

- ## Two teachers in Wyoming
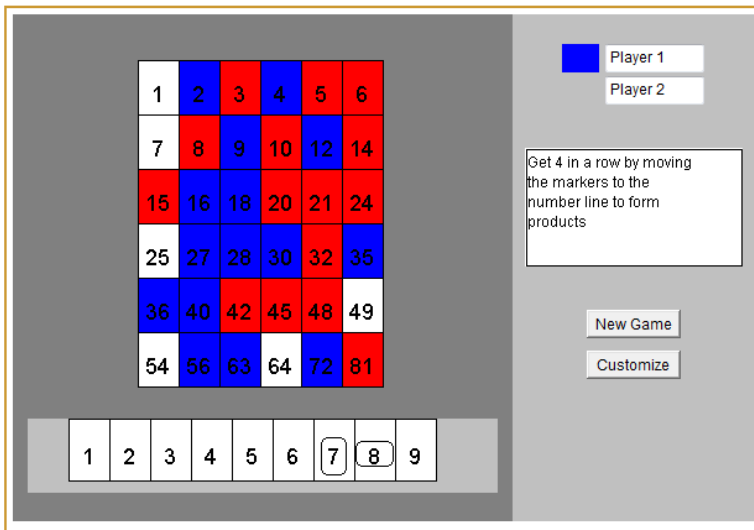
Paper Pool

How to Play Paper Pool

- The ball starts in corner A.

- The ball is **hit** with an imaginary stick so that it travels at a 45° diagonal across the grid.

- If the ball **hits** a side of the table, it bounces off at a 45° angle and continues its travel.

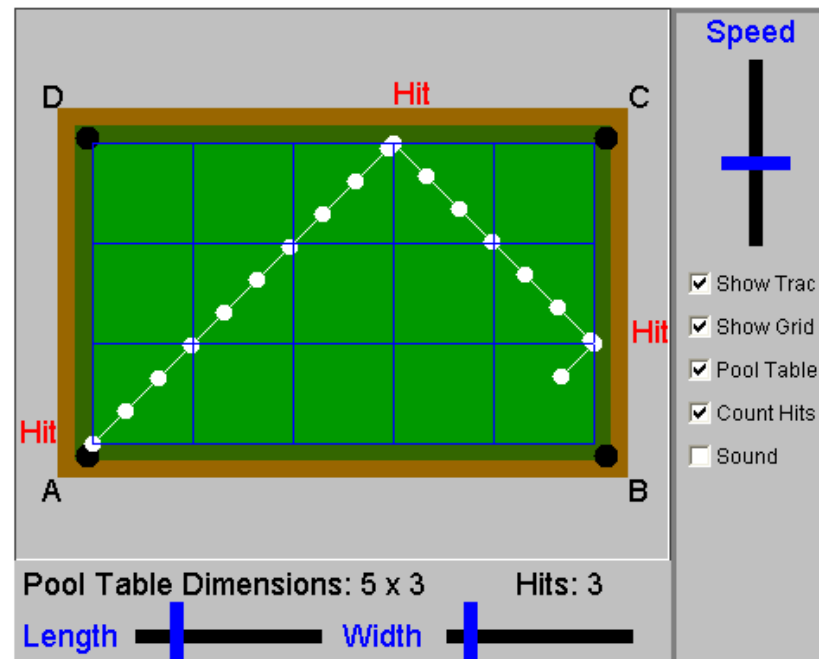- The ball continues to travel until it **hits** a pocket.

Paper Pool

From Paper Pool...

- Online Version of the Paper Pool Lesson
  http://illuminations.nctm.org/LessonDetail.aspx?ID=U165

…to Slam Ball

Game Design

- **Other Games:** Do the math, then you can do something fun.

- **Our Games:** Doing the math **IS** something fun.

http://calculationnation.nctm.org

Play a Game!

Click here: "Challenge Yourself"

Ker-Splash

- Choose an expression:

$$\textcolor{green}{17x} + \textcolor{blue}{29y} + \textcolor{red}{43}$$

$$\textcolor{green}{24x} + \textcolor{blue}{22y} + \textcolor{red}{39}$$

- The values of *x* and *y* are unknown... but you can choose to increase one of them by 1, and decrease the other by 1. Which would you like to increase and which to decrease?

- Now, here are the values: *x* = 6, *y* = 4

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Ker-Splash

Your Equation	$x + 1, y - 1$	$x - 1, y + 1$
17x + 29y + 43	249	273
24x + 22y + 39	273	269

Ker-Splash

Tips for Teaching with Games

- Do not show children how to play at a higher level. Instead, encourage them to do their own thinking.

- Do not reinforce "correct" behaviors or try to correct "wrong" ones.

- Play with individual children whenever possible.

Kamii, C. The Educational Value of Tic-Tac-Toe for Four-to Six-Year-Olds. *Teaching Children Mathematics,* May 2008.

Prime Time

Which is most likely to give an outcome of 4?

- Roll one die

- Roll two die, sum

- Roll two die, difference

- Spinner with four consecutive integers (your choice)

- Flipping *n* coins, number of heads

Prime Time

Roll one die

$$P(4) = 1/6$$

Prime Time

Roll two die, add

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Prime Time

Roll two die, subtract

	1	2	3	4	5	6
1	0	1	2	3	4	5
2	1	0	1	2	3	4
3	2	1	0	1	2	3
4	3	2	1	0	1	2
5	4	3	2	1	0	1
6	5	4	3	2	1	0

Prime Time

Spinner with four consecutive integers (your choice)
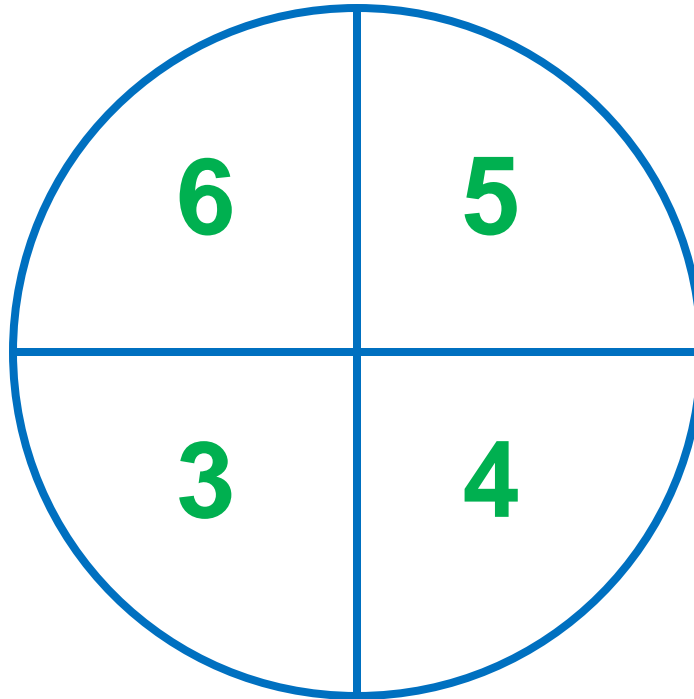
Prime Time

Flipping *n* coins, number of heads

n	P(exactly 4 heads)
1	0
2	0
3	0
4	1/16
5	5/32
6	15/64

n	P(exactly 4 heads)
7	35/128
8	70/256
9	126/512
10	210/1024
11	330/2048
12	495/4096

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Prime Time

Current Location: 19

Desirable Location: 23

illuminations.nctm.org

Illuminations

The web site currently contains…
- 607 Lessons
- 108 Interactive Tools

On average, 325,000 visitors per month
- August 2004 – 93,371
- March 2012 – 632,910

Illuminations

New in 2012…

- 1 new game for Calculation Nation®

- 10 new lessons, *based on Calc Nation games*

- 1 web app

- 3 mobile apps

Illuminations

www.thinkfinity.org

Thinkfinity

- Provides standards-based content and professional development for K–12 teachers

- Supported by the Verizon Foundation
  - NCTM received a three-year, $1.4M grant for Illuminations

- Consortium of content partners across all disciplines
  - science, arts, humanities, geography, economics, language arts, math, and history

NATIONAL COUNCIL OF
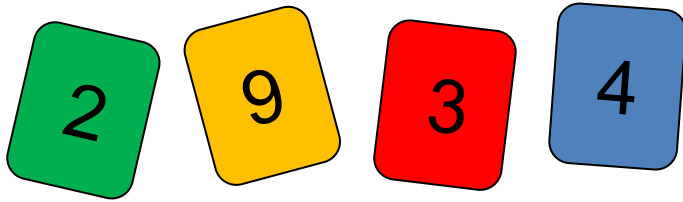TEACHERS OF MATHEMATICS

What a constraint!

Game of Nine Cards

- **Materials:** Nine cards numbered 1–9
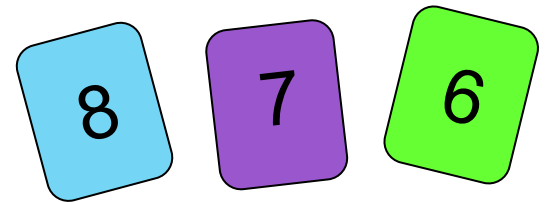- **Object:** To have any three cards in your hand that add up to 15
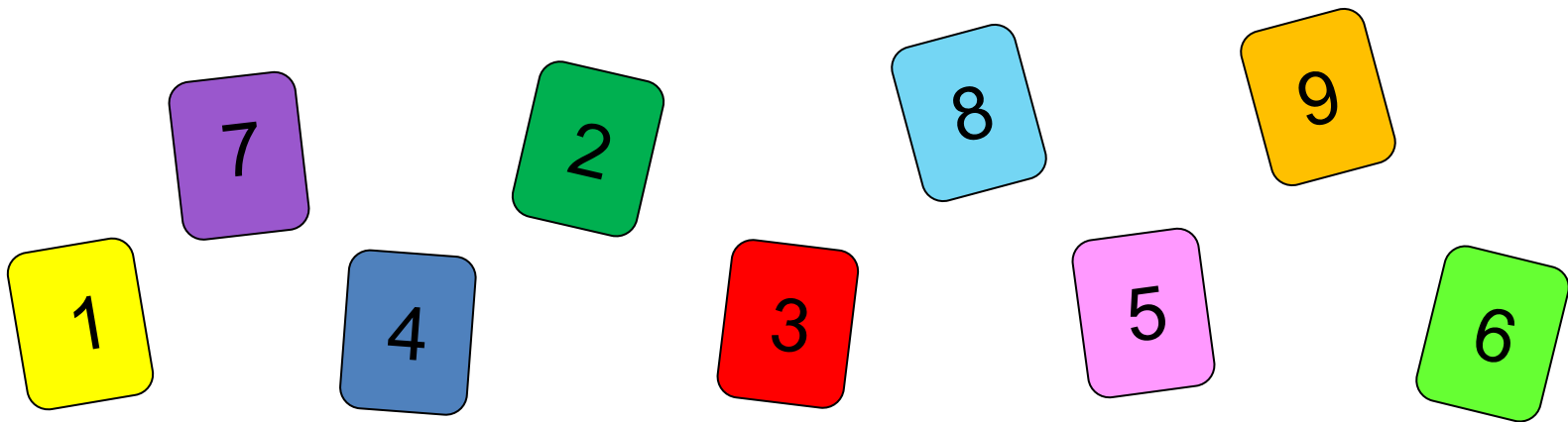
Game of Nine Cards

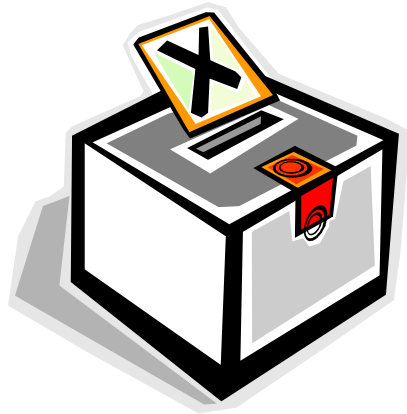- Sample Game:

Player 1

Player 2

Player 1 Wins: 2 + 9 + 4 = 15

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TEACHERS OF MATHEMATICS

Game of Nine Cards

Now what?

You Play!

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The Basics…

- Who is more likely to win — the first player or the second player? Why?

- Will someone always win? Lose?

- What can you do to ensure that you don't lose?

- Is there a "best" card to choose?

- Why do we use a sum of 15?

A *Winning* Strategy?

- You play first, **pick 8**.

- Your opponent then **chooses 3**.

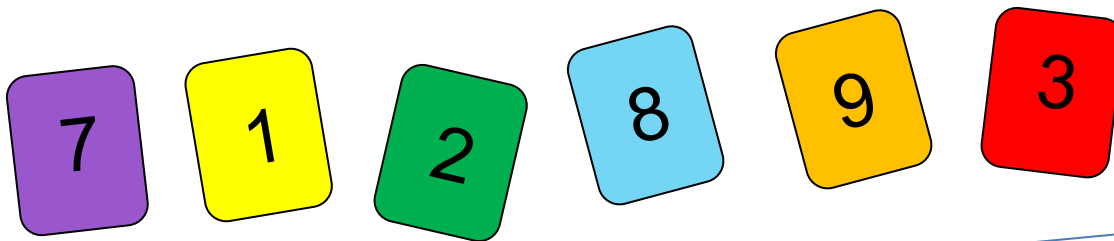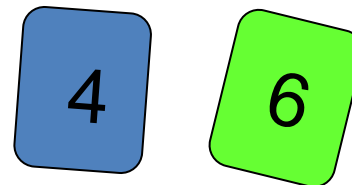- What are the **three numbers** that you can choose to ensure a win?

1 7 4 2 5 9 6

8

Yours

3
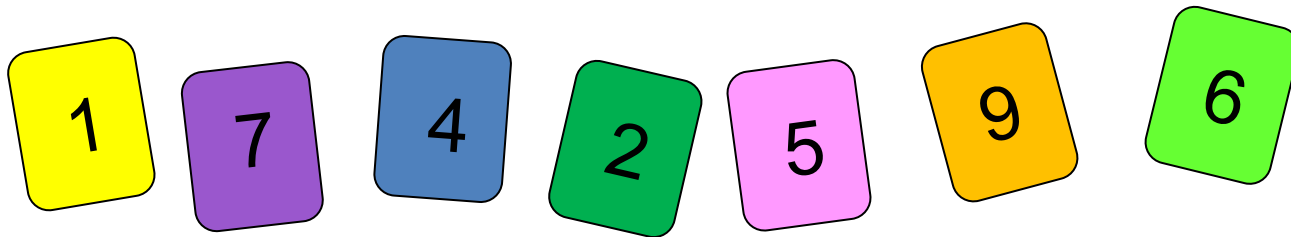
His or Hers

A *Winning* Strategy?

- Your opponent plays first, **picks 6**.

- You **choose 5**.

- Your opponent **picks 4**.
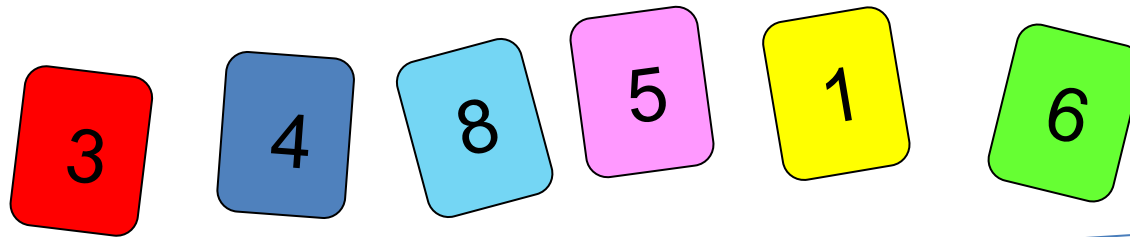
- Which **two numbers** should you *not* pick?

7 1 2 8 9 3

5

Yours

4 6

His or Hers

A *Winning* Strategy?

- Your opponent plays first, **picks 7**.

- Then you **choose 2**.

- Your opponent **picks 9**.

- Which **three numbers** should you *not* pick?

3 4 8 5 1 6

2

9 7

Yours

His or Hers

NATIONAL COUNCIL OF
TEACHERS OF MATHEMATICS

More Sophisticated Yet?

- If your opponent plays first and picks an **even** number, what number should you choose to **avoid a loss**?

Another App from Under the Sea

Deep Sea Duel

9 Card

Nice / Easy

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Game of Nine Cards

- **Deep Sea Duel** is online!
  - http://illuminations.nctm.org/deepseaduel
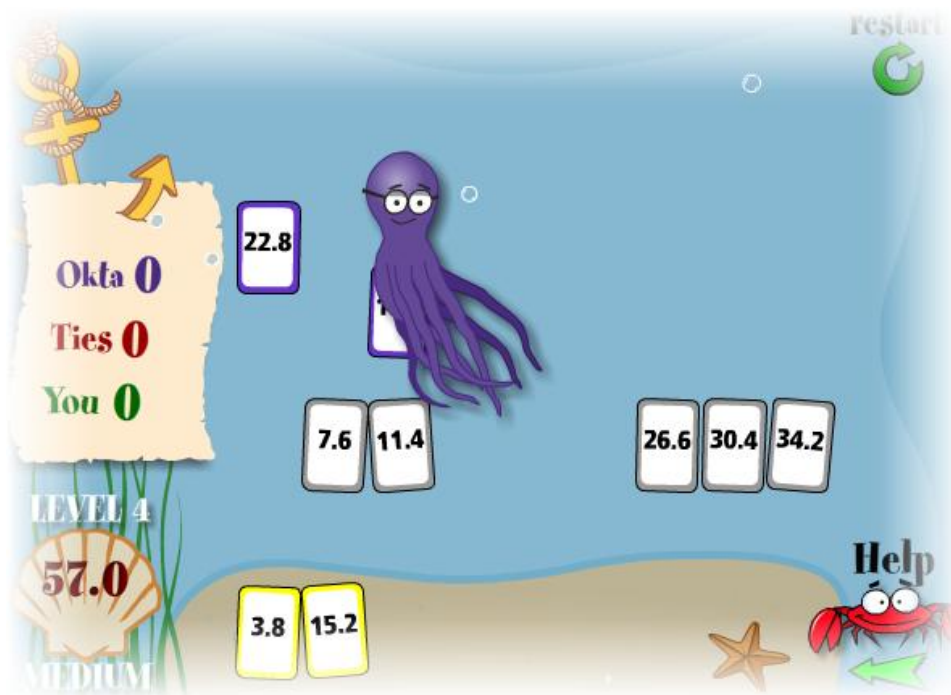
A Hint from Under the Sea
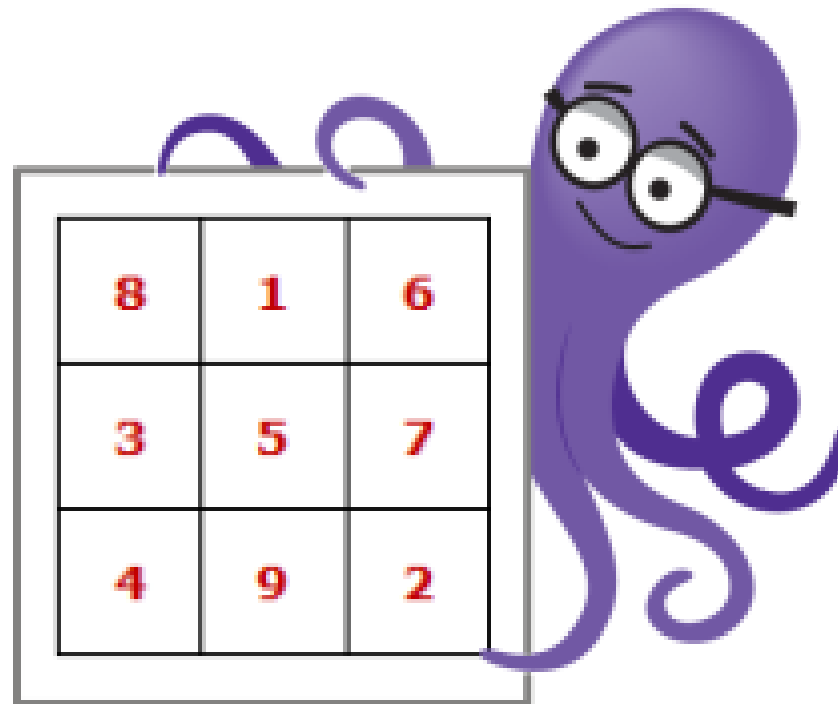
Modifying the Game of Nine Cards

- Label the nine cards as follows:
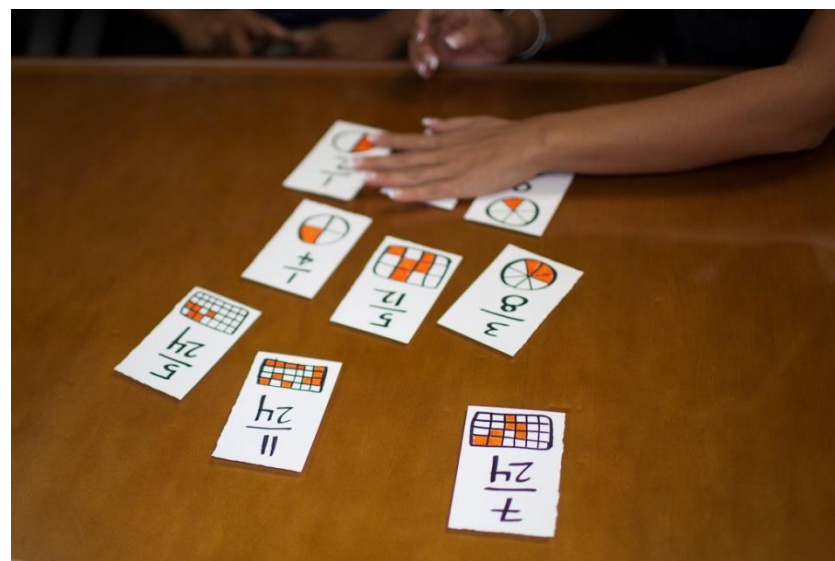
  5, 12, 19, 26, 33, 40, 47, 54, 61

  The winner must get three cards that total 99.

Mahoney, John. What Is the Name of This Game?
Mathematics Teaching in the Middle School, October 2005.

Modifying the Game of Nine Cards

- Label your nine cards with fractions:

    1/6, 5/24, 1/4, 7/24, 1/3, 3/8,

    5/12, 11/12, 1/2

The winner must get three cards that total 1.

Mahoney, John. What Is the Name of This Game?
Mathematics Teaching in the Middle School, October 2005.

Modifying the Game of Nine Cards

- Use words! Label the cards as follows:

  TIED, HOT, HEAR, TANK, WASP,

  WOES, SHIP, HORN, BRIM

The winner must get three cards that bear the same letter.

Mahoney, John. What Is the Name of This Game?
Mathematics Teaching in the Middle School, October 2005.
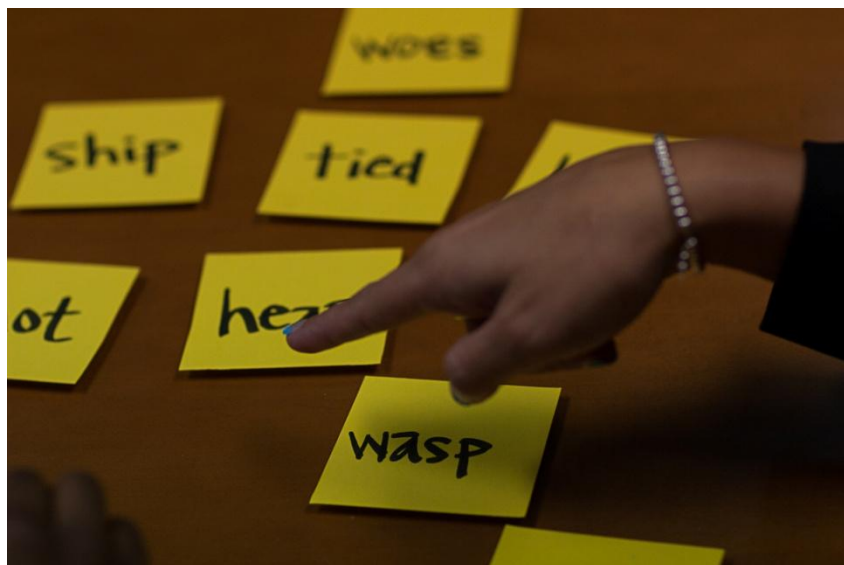
Modifying the Game of Nine Cards

- Use exponents!

- Label the nine cards as follows:

$$x, \ x^2, \ x^3, ..., x^9$$

The winner must the *product* get x^{15}.

Mahoney, John. What Is the Name of This Game?
Mathematics Teaching in the Middle School, October 2005.

What sum should the winner need to win?

From NINE to SIXTEEN

The winner would use the sum of *four* cards to win.

11 7 4 15 8 3 6

1 9

16 13 12 5 2 10

14

Mahoney, John. What Is the Name of This Game?
Mathematics Teaching in the Middle School, October 2005.
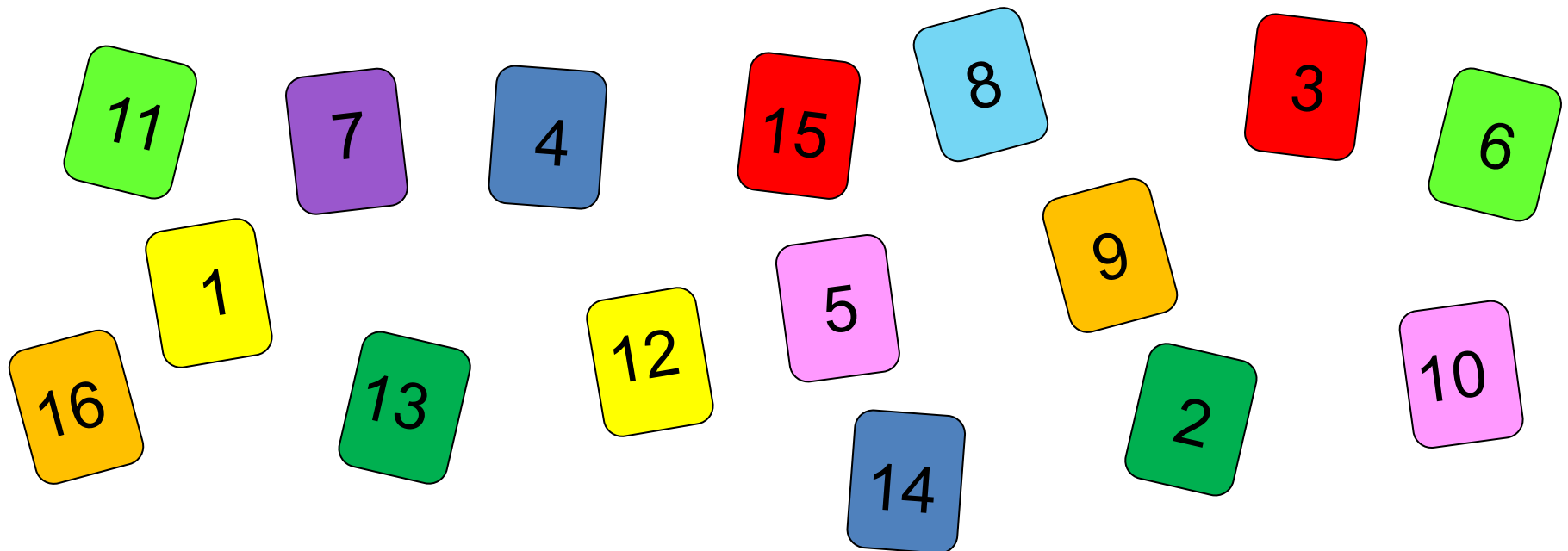
Another Extension

- The winner is the first player to obtain the sum of exactly 15 from any **TWO OR MORE** cards.

- Does your strategy change? How so?

Yeo, Joseph. [Title removed in order to not give away punch line of strategy.] *Mathematics Teacher,* August 2012.
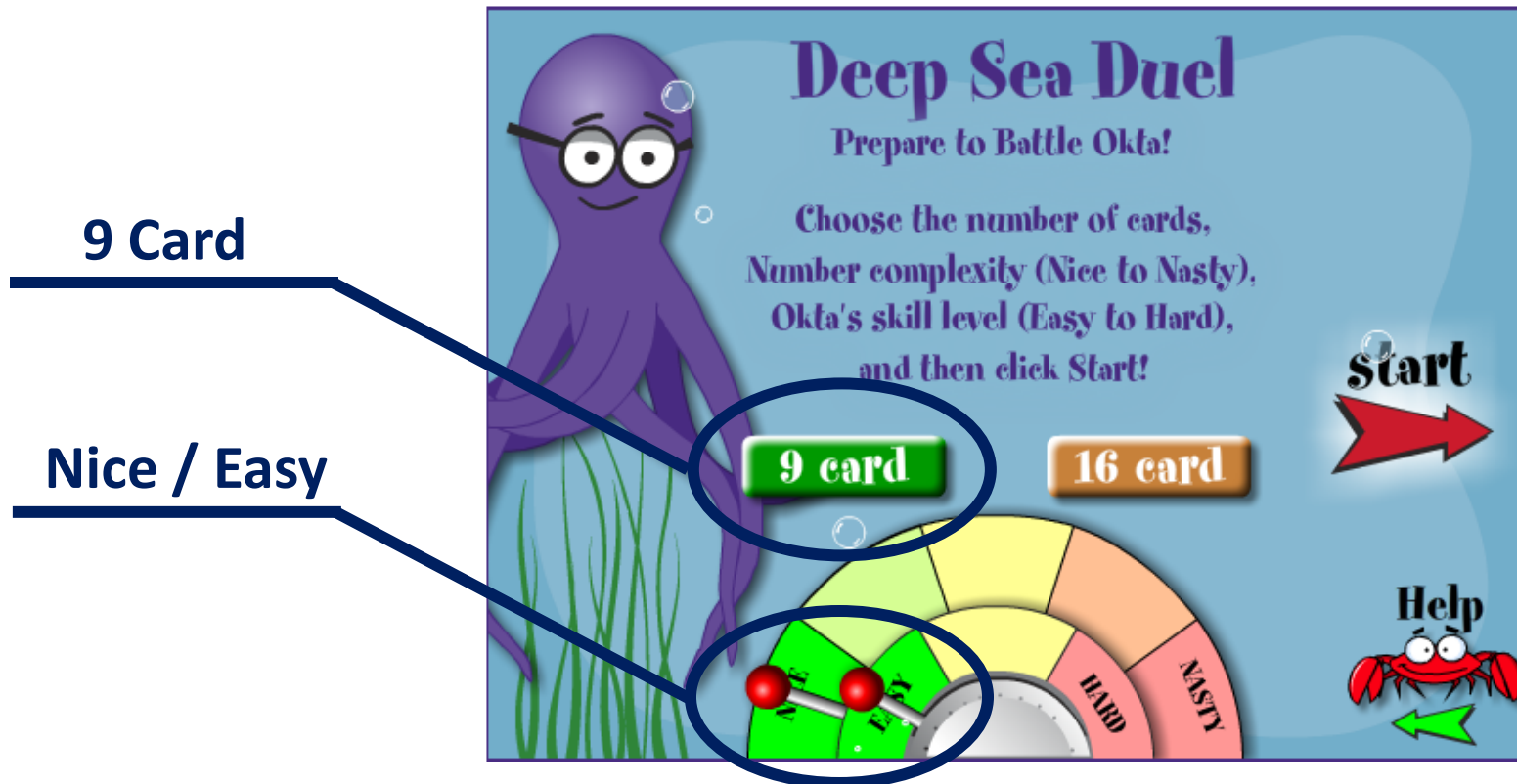
Reminder: What is the Goal?

- How does your strategy from the first version of the game of 9 cards compare to the strategy for these modifications?

- REFLECT: *How* did I come up with these other versions for the game of 9 cards?

Another App from Under the Sea

- http://illuminations.nctm.org/deepseaduel

9 Card

Nice / Easy

Challenge Okta to Deep Sea Duel on the web.

Options & Modifications in App

Learning is fun. Get addicted!

Deep Sea Duel is FREE online at Illuminations and Google Play and the App Store for phones and tablets.

KenKen®

illuminations.nctm.org/kenken

An Example

11Q – Q

Engaging and Free Online Resources for Teaching Operations and Fractions

Sarah DeLeeuw

sdeleeuw@nctm.org

nctm.org/games

illuminations.nctm.org

calculationnation.nctm.org

ILLUMINATIONS
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Resources for Teaching Math

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