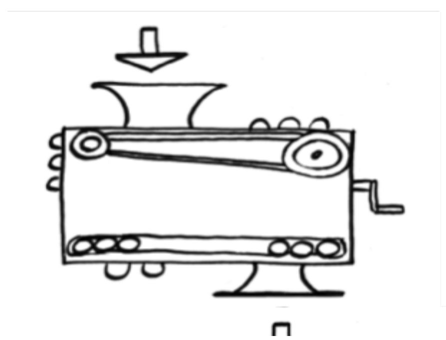
COLLEGE PREPARATORY MATHEMATICS

CPM®

visit our website at www.cpm.org

Algebra 2 Functions



For more information about the materials you find in this packet, contact:

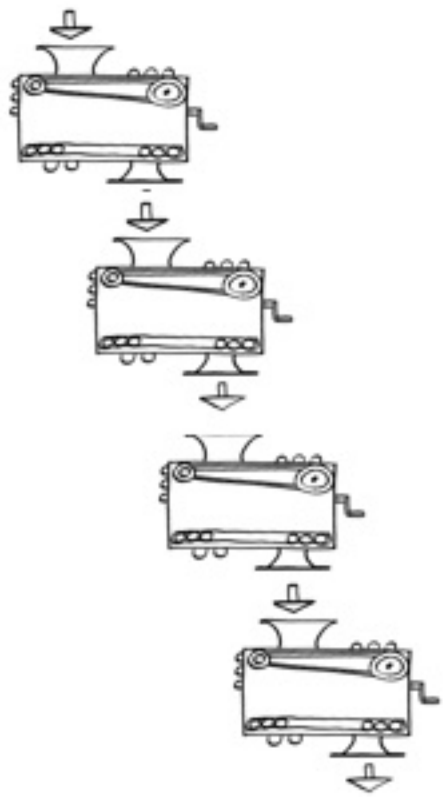
Chris Mikles
(888) 808-4276
mikles@cpm.org

1-2. FUNCTION MACHINES

Your teacher will give you a set of four function machines. Your team's job is to get a specific output by putting those machines in a particular order so that one machine's output becomes the next machine's input. As you work, discuss what you know about the kind of output each function produces to help you arrange the machines in an appropriate order. The four functions are reprinted below.

a. In what order should you stack the machines so that when **6** is dropped into the first machine, and all four machines have had their effect, the last machine's output is **11**

b. What order will result in a final output of 131,065 when the first input is 64?



$f(x)=\sqrt{x}$	$g(x)=-(x-2)^2$
$h(x)=2^x-7$	$k(x)=-\frac{x}{2}-1$

For the resource pages of the large function machines go to
http://www.cpm.org/pdfs/stuRes/A2C/chapter_01/1.1.1.pdf

Silent Board Games http://www.cpm.org/pdfs/stuRes/AC/chapter_03/3.1.2A.pdf

Taking a Function Walk: $f(x)=x+1$, $f(x)=x^2-5$, $f(x)=2^x$, $f(x)=\frac{1}{x}$ ...
The outdoor xy-coordinate system should be marked off before class. For the axes, some teachers use two pieces of rope wrapped with colored tape to mark the units; others use chalk to draw the axes on the pavement.
Start with the function $y=2^x$. Be sure nine students are standing on the x-axis with the mark that corresponds to their number (an integer, -4 to 4) between their feet and facing in the direction of the positive y-axis. Direct students to use their number as x, and calculate the value of y. Then when you say 'go,' they should take that many paces forward. Have the rest of the students record the function (equation) and as much detail as they can about the resulting graph. Three members of each team will have information about each function, but no one will have all four. Then have another group do $y=x+1$, the next do $y=x^2-5$, and then do $y=\frac{1}{x}$. While each team is "walking" the rest of the students should be recording the other three functions and their graphs.
Have students note the Domain as they stand on the x-axis. After they have moved into position have them make a 90˚ turn toward the y-axis and walk straight towards it. The space between 0 and 1 will become a bit crowded, but they should now be able to note the range.

What does it mean to describe a function completely?
In this lesson you will graph and **investigate** a family of functions with equations of the form $f(x)=\frac{1}{x-h}$. As you work with your team, keep the multiple representations of functions in mind.

Your task: a. Each team member should choose a different value of *h* and make a complete *x*, *y* table and graph for your new function.

b. Examine all of your team's functions. Together, generate a list of questions that you could ask about the functions your team created. Be as thorough as possible and be prepared to share your questions with the class

c. As your teacher records each team's questions, copy them into your Learning Log. Title this entry "Function Investigation Questions" and label it with today's date.

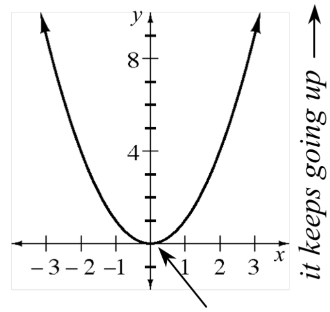
Discussion Points:
How can we be sure that our graph is complete?
How can we get output values that are greater than 1 or less than –1?

1-81. INVESTIGATING A FUNCTION, Part Two: SUMMARY STATEMENTS

Now you are ready for the most important part of your **investigation**: summary statements! Summary statements are a very important part of this course, so your team will practice making them. A summary statement is a statement about a function *along with thorough **justification***. A strong summary statement should be **justified** with multiple representations (*x*,*y* table, equation, graph, and situation, if applicable).

a. Read the example summary statement below, a summary statement about the range of the function $y=x^2$. Discuss it with your team and decide if it is **justified** completely.

Statement: The function $y = x^2$ has a range of all numbers greater than or equal to zero ($y \geq 0$).
First justification: You can see this when you look at the graph, because you can see that the lowest point on the graph is on the *x*-axis.
Second justification: Also, you can see this in the table, because none of the *y*-values are negative.



x	–3	–2	–1	0	1	2	3
y	9	4	1	0	1	4	9

they will keep getting higher ← → *they will keep getting higher*

this is the lowest output

Third justification: It makes sense with the equation, because if you square any number, the answer will be positive. For example, $(-2)^2 = 4$ and $3^2 = 9$.

b. Use your "Function Investigation Questions" Learning Log entry from problem 1-80 to help you make as many summary statements about your functions as you can. Remember to **justify** each summary statement in as many ways as possible.

1-82. SHARING SUMMARY STATEMENTS

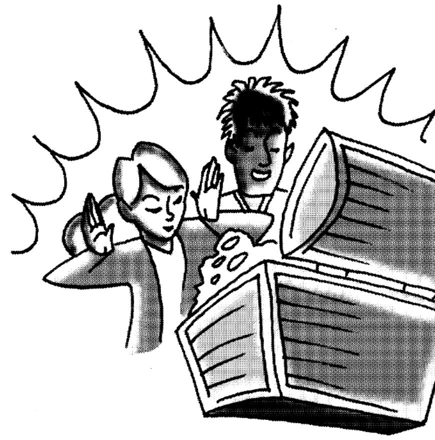
With your team, choose one summary statement that you wrote that you find particularly interesting. On an overhead transparency, write the summary statement along with its **justification**. Include sketches of graphs, *x,y* tables, equations, circles, arrows, colors, and any other tools that are helpful.

1-83. What will the graph of $\boldsymbol{f(x) = \frac{1}{x+25}}$ look like?

a. Discuss this question with your team and make a sketch of what you predict the graph will look like. Give as many reasons for your prediction as you can.
b. Use your graphing calculator to graph $\boldsymbol{f(x) = \frac{1}{x+25}}$. Do you see what you expected to see? Why or why not?
c. Adjust the viewing window if needed. When you see the full picture of your graph, make a sketch of the graph on your paper. Label any important points.
d. How close was your prediction?

1-112. In this activity you will **investigate** the function $f(x) = \frac{5}{x^2+1} - 1$.

11-98. TREASURE HUNT



Today your teacher will give you several descriptive clues about different relations. (This information is also available online at www.cpm.org.) For each clue, work with your team (or a partner) to find all the possible matches among the relations posted around the classroom or provided on the resource page. Remember that more than one relation may match each clue. Once you have decided which relation(s) match a given clue, defend your decision to your teacher and receive the next clue. Be sure to record your matches on paper.

Your goal is to find the match (or more than one match) for each of **eight** clues. Once you and your team (or partner) have finished, only one relation will be left unmatched. That relation is the treasure! **[The only relation with no match is $y=|x|$.]**

3-1. BEGINNING TO INVESTIGATE EXPONENTIALS

You already know that equations of the form $y=mx+b$ represent lines, and you know what effect changing the parameters m and b has on the graph. Today you will begin to learn more about exponential functions. In their simplest form, the equations of exponential functions look like $y=b^x$.

3-2. INVESTIGATING $y=b^x$ Part One

What types of graphs exist for equation of the form $y=b^x$?

Your task: With your team, **investigate** the family of functions of the form $y=b^x$. Decide as a team what different values of b to try so that you find as many different looking graphs as possible. Use the questions listed in the "Discussion Points" section below to help get you started.

Discussion Points:
What special values of b should we consider?
Are there any other values of b we should try?
How many different types of graphs can we find?
How do we know we have found all possible graphs?
For our value(s) of b, what does the graph look like?
Why does the graph look this way?
What values of b generate this type of graph?
What are the special qualities of this graph?

3-5. Exponential functions have some interesting characteristics. Consider functions of the form $y=b^x$ as you discuss the questions below

a. Exponential functions such as $y=b^x$ are defined only for $b>0$. Why do you think this is? That is, why would you not want to use negative values of b?

b. Can you consider $y=1^x$ or $y=0^x$ to be exponential functions? Why or why not? How are they different from other exponential function?

4-34. TRANSFORMING GRAPHS

Use your dynamic graphing tool to support a class discussion about the equation $y=a(x-h)^2+k$. Refer to the bulleted points below.

• Identify which **parameter** (a, h, or k) affects the orientation, vertical shift, horizontal shift, vertical stretch, and vertical compression of the graph compared to the graph of the parent function $y=x^2$.
• What values stretch the graph vertically? Compress the graph horizontally? Why do those values have these impacts?
• What values cause the graph to flip vertically?
• What values cause the graph to shift to the left? To the right? Why?
• What values cause the graph to shift up or down? Why?
• Are there points on your graph that connect to specific parameters in the equation? Explain.

4-59. In this **investigation** you will use what you have learned about transforming the graph of $y=x^2$ to transform four new parent graphs. In fact, your team will figure out how to use what you have learned to transform the graph of *any* function!

Your task: As a team, determine how you can make the graph of any function move left, right, up, and down and how you can stretch it vertically, compress it vertically, and flip it. Each team member should choose one of the following parent functions to **investigate**: $y=x^3$, $y= 1/x$, $y= \sqrt{x}$, and $y=b^x$. Remember that to **investigate** completely, you should sketch graphs, identify the domain and range, and label any important points or asymptotes. Then graph and write an equation to demonstrate each transformation you find. Finally, you will find a general equation for your family of graphs. (If you are **investigating** $y=b^x$, your teacher will give you a value to use for b.)

Discussion Points
How can we move a parabola?
How can we use our ideas about moving parabolas to move other functions?
What changes can we make to the equation?

For copies of the Parent Graph Tool Kit go to
http://www.cpm.org/pdfs/stuRes/A2C/chapter_04/4.2.2.pdf

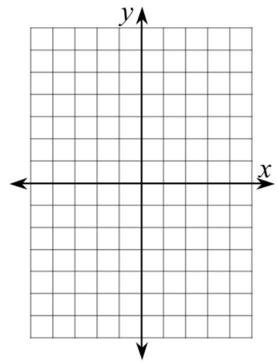
Parent Graph Tool Kit

Name:

Parent Equation:

General Equation:

x	y



Properties:

Domain:

Range:

Description of (h, k):

Carousel

- Write a different problem on poster sheets hung on the walls.
- Each team goes to a different poster, discusses the problem and decides what to write.
- Teams rotate to all of the posters, answering the question on their own paper.

Silent Board Game

IN	-6	2	½	10	-2	1	5	0	-1.5
OUT		2				-1	11		

Rule:

IN	9	-1	0	4	.5	20	-5	7	3
OUT		4		14			-4		

Rule:

IN	2	11	-3	-1/2	6	100	-8	5	0
OUT	1		11		-7				

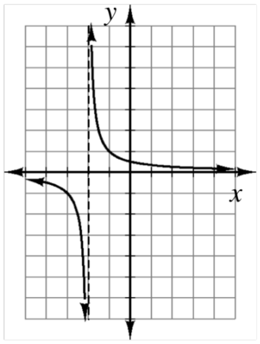
Rule:

IN									
OUT									

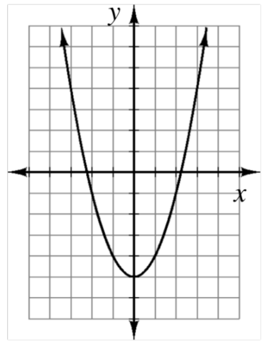
Rule:

4-78. Write a possible equation for each of these graphs. Assume that one mark on each axis is one unit. When you are in class, check your equations on a graphing calculator and compare your results with your teammates. **[a: $y=\frac{1}{x+2}$; b: $y=x^2-5$; c: $y=(x-3)^3$; d: $y=2^x-3$; e: $y=3x-6$; f: $y=(x+2)^3+3$; g: $y=(x+3)^2-6$; h: $y=-(x-3)^2+6$; i: $y=(x+3)^3-2$]**

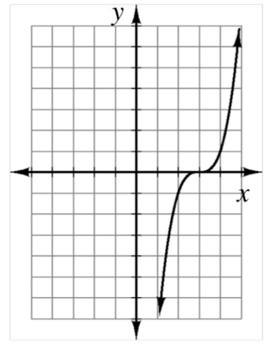
a.



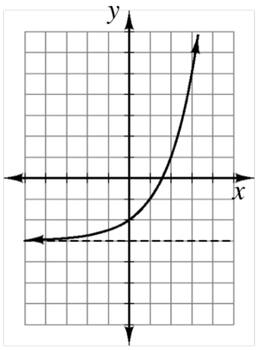
b.



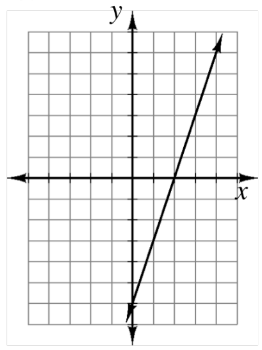
c.



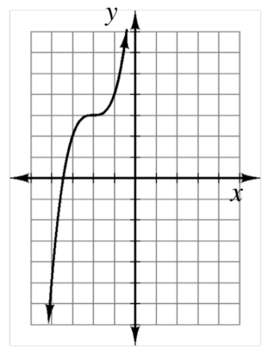
d.



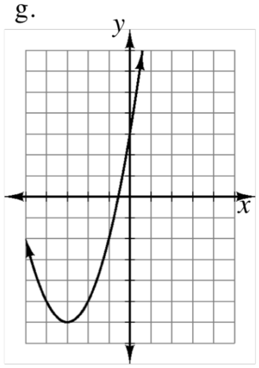
e.



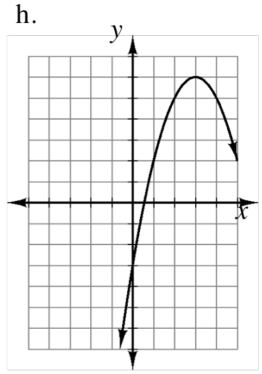
f.



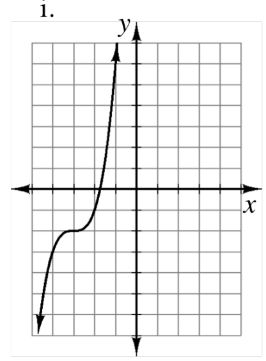
g.



h.



i.



Clue Cards

Treasure-Hunt Clues A Find one (or more) relation(s) that: A1. Has no symmetry. A2. Has a domain of all numbers except $x \neq 3$. A3. Has x-intercepts (2, 0) and (8, 0). A4. Is not a function. A5. Has a range of all numbers less than or equal to 3. A6. Is a linear relation. A7. Has a range of all numbers. A8. Has a y-intercept at (0, –1).	**Treasure-Hunt Clues C** Find one (or more) relation(s) that: C1. Is a linear relation. C2. Has a range of all numbers. C3. Is not a function. C4. Has x-intercepts (2, 0) and (8, 0). C5. Has a domain of all numbers except $x \neq 3$. C6. Has a y-intercept at (0, –1). C7. Has a range of all numbers less than or equal to 3. C8. Has no symmetry.
Treasure-Hunt Clues B Find one (or more) relation(s) that: B1. Is not a function. B2. Has no symmetry. B3. Has a range of all numbers less than or equal to 3. B4. Has a range of all numbers. B5. Has a y-intercept at (0, –1). B6. Has a domain of all numbers except $x \neq 3$. B7. Has x-intercepts (2, 0) and (8, 0). B8. Is a linear relation.	**Treasure-Hunt Clues D** Find one (or more) relation(s) that: D1. Has a range of all numbers less than or equal to 3. D2. Has x-intercepts (2, 0) and (8, 0). D3. Has a domain of all numbers except $x \neq 3$. D4. Is a linear relation. D5. Has no symmetry. D6. Has a range of all numbers. D7. Has a y-intercept at (0, –1). D8. Is not a function.

You can find the function cards for the carousel, the answers and also the clues at
http://www.cpm.org/pdfs/stuRes/AC/chapter_11/11.3.1B.pdf and
http://www.cpm.org/pdfs/stuRes/AC/chapter_11/11.3.1A.pdf and
http://www.cpm.org/pdfs/stuRes/AC/chapter_11/11.3.1C.pdf and
http://www.cpm.org/pdfs/stuRes/AC/chapter_11/11.3.1D.pdf

"NAILS"

Neat

*nice & neat
*straight line

Arrows

*on axes
*on graphed line

Intervals

*x-axis scaled correctly
*y-axis scaled correctly

Labels

*x- & y-axis labeled
*rule/equation

Special points

*x- & y-intercept(s)
*min & max