



Spaghetti and Meatballs in Korea, Cyprus, and the US

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3-5; Pre-Service and In-Service;
and Research Session
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The Presenter

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**This semester teaching Math Methods for Grades 2-4;
Math Content for Elementary and Middle Grades
Teachers; and Probability and Statistics service course**



Copy of Presentation

I will enter this presentation on the NCTM
Conference website.

Teaching the Lesson

- I have collaborated with a 5th grade teacher on this and with many other math activities. She taught this lesson several times and continues to teach it. She also taught it to her students as part of a Lesson Study group at IASD, with several teachers observing her.
- I taught 3-day lessons with students in Korea, Cyprus, and the fifth grade teacher in the US on perimeter and area during my sabbatical in the spring of 2011.
- I have also taught this lesson with teachers in graduate classes and workshops; with children in grades 3-5; and with IUP students.

Ms. Bungo (US) had 18 fifth graders



46 students in Cyprus

- I worked with a former IUP graduate assistant, Irene Nicolaidou, who is now a school administrator in Cyprus.
- The fifth graders were in two classes.
- This was a secular private English language school.





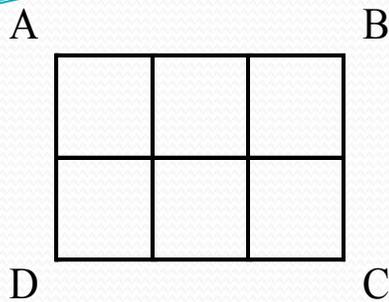
22 “third graders”

- My Korean contact was Dr. Woo Whang, former graduate assistant and visiting professor at IUP, now a Professor at Korea University.
- Their school year started in March, just a couple of weeks before I was there. Determination of ages and cut-off dates for starting school are very different.
- This was a public school in an upscale area of Seoul. Many of the students had travelled to the US or UK. Many were studying English and other subjects after school. I had translators but didn't always need them.

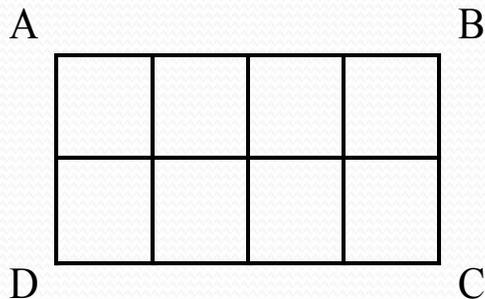




**Please answer the following
question, then share your
answers with others.**



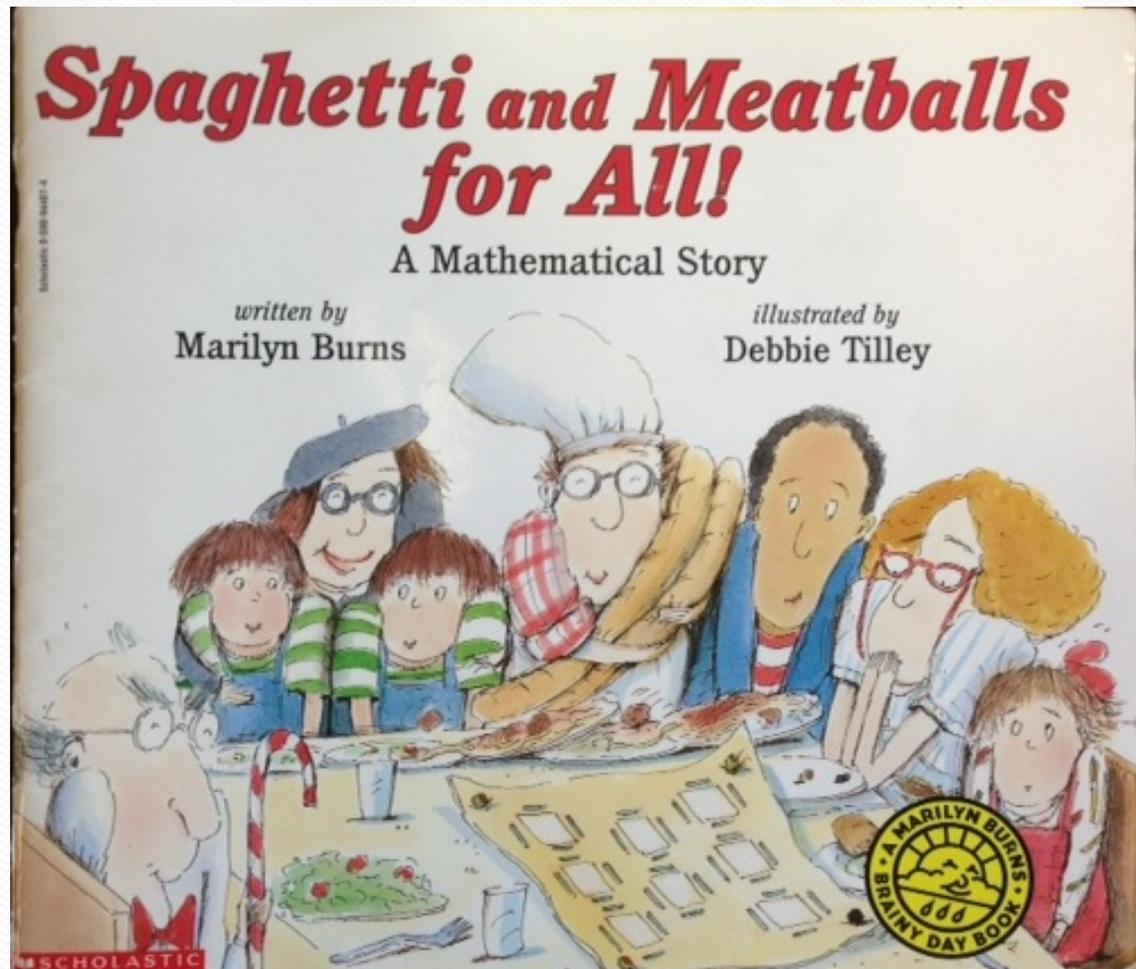
This flat candy bar has 6 pieces. An ant starts at point A, moves 3 units to point B, then 2 units to point C, then 3 units to point D, then 2 units to A. The ant traveled a total of 10 units.



This flat candy bar has 8 pieces. An ant starts at point A, moves 4 units to point B, then 2 units to point C, then 4 units to point D, then 2 units to A. The ant traveled a total of 12 units.

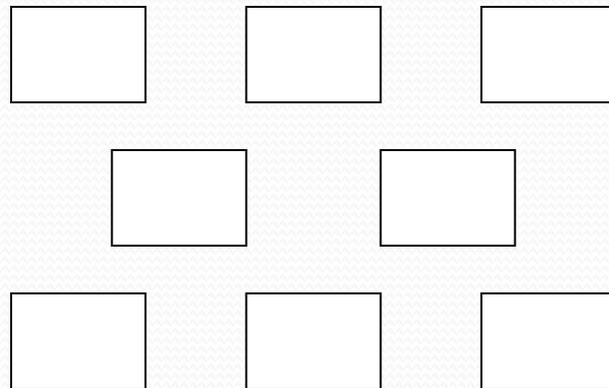
John thinks that whenever an ant travels more units all the way around a candy bar, there will always be more pieces in the candy bar. Do you think John is right? Why or why not?

This lesson had as a goal to help students with that question.



At the beginning of the story

- Mr. and Mrs. Comfort invite 32 family members and friends for a reunion.
- They set eight square tables to seat four people at each, one to a side.
- The area (tables) is 8 and the perimeter (people) is 32.

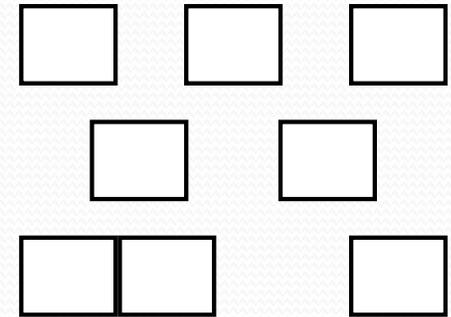


Change 1

$2 + 4 = 6$ people

(Only the 2×1 is used.

The other tables are not used yet) \rightarrow



Perimeter = 1 rectangle $\times 6 = 6$

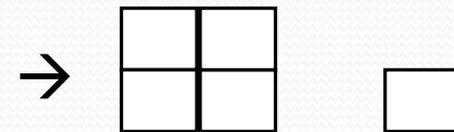
Area = 2

The students used square tiles to model all the changes. They discovered many other ways to get the same number of people.

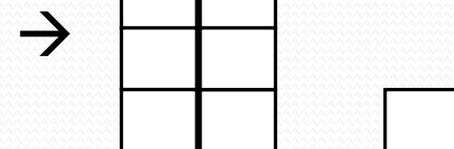
Change 2

Then there are 8 people, then 2 more come, then another 2.
They get rearranged as follows.

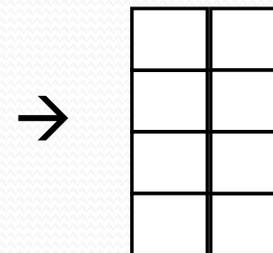
2a. Perimeter = 1 square = 8
Area = 4



2b. Perimeter = 1 rectangle (3x2) = 10
Area = 6

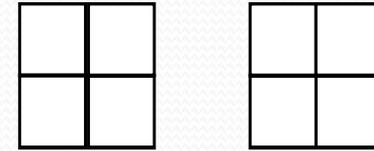


2c. Perimeter = 1 rectangle (4x2) = 12
Area = 6



Change 3

$$12 + 4 = 16 \text{ people}$$



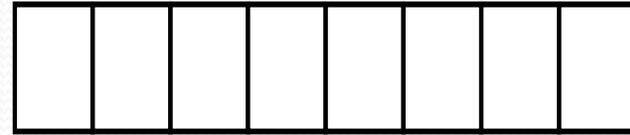
$$\text{Perimeter} = 2 \text{ squares} \times 8 = 16$$

$$\text{Area} = 8$$

Mrs. Comfort is upset because putting the tables together means there will not be enough seats for the 32 guests to fit with only 8 tables.

Change 4

$$16 + 2 = 18 \text{ people}$$



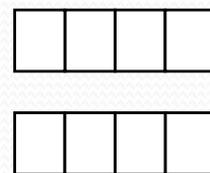
$$\text{Perimeter} = 1 \text{ rectangle} \times 18 = 18$$

$$\text{Area} = 8$$

How else could you arrange 18 people?

Change 5

$$18 + 2 = 20 \text{ people}$$



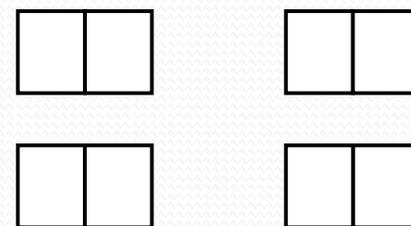
$$\text{Perimeter} = 2 \text{ rectangles} \times 10 = 20$$

$$\text{Area} = 8$$

The children moved from the square tiles at first to graph paper when they were ready.

Change 6

$$20 + 4 = 24 \text{ people}$$



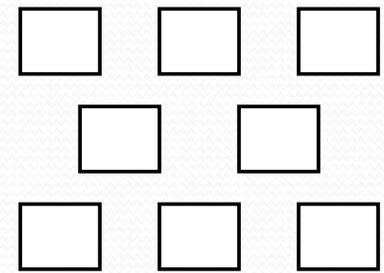
$$\text{Perimeter} = 4 \text{ rectangles} \times 6 = 24$$

$$\text{Area} = 4 \times 2 = 8$$

Change 7

Back to the original

$$24 + 8 = 32 \text{ people}$$



$$\text{Perimeter} = 8 \text{ squares} \times 4 = 32$$

$$\text{Area} = 8$$



“Perimeter” Introduced Gradually

- Gradually, the teacher switches from “number of people” to “perimeter”
- The concept of perimeter is described as the distance along the edge of the figure.
- There are the same number of people as the perimeter.



“Area” Introduced Gradually

- Also, gradually, the teacher switches from “number of tables” to “area”
- The area is the same as the number of tables.

A Dilemma

The following dilemma was given to the students.

- Dr. Feldman says the “size” of this shape is 8
- Mrs. Bungo says the “size” of this shape is 18.
- Who is right?
- After discussion they realized both were correct.

The perimeter is 18, the area is 8.

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Fixed Area, Variable Perimeter Rectangles

The children were told to work with exactly 8 of their square tiles (tables). They were asked to figure out how many people they could invite. All chairs will be filled.

Please do this on the graph paper with rectangles only at first.

Fixed area (8), variable perimeter

Fixed Area (8), Variable Perimeter Rectangles Only

Next children created non-rectangular shapes with no tiles off to the side (an “L” shape for the 8 tiles is OK but you can’t have any tiles sitting by themselves – there has to be a single polygon).

This was explained that “Kids tables” are not allowed.

Share your polygons with others.



Area of 8, All Possible Perimeters

With polygons of area 8, the possible perimeters are: 12, 14, 16, and 18.

The children were able to discover all of these possible perimeters and many different configurations with each perimeter.

Variable Area, Fixed Perimeter (10)

What are all the possible single polygons you can find for exactly 10 people? Find all the possible areas.

No “kids tables”.

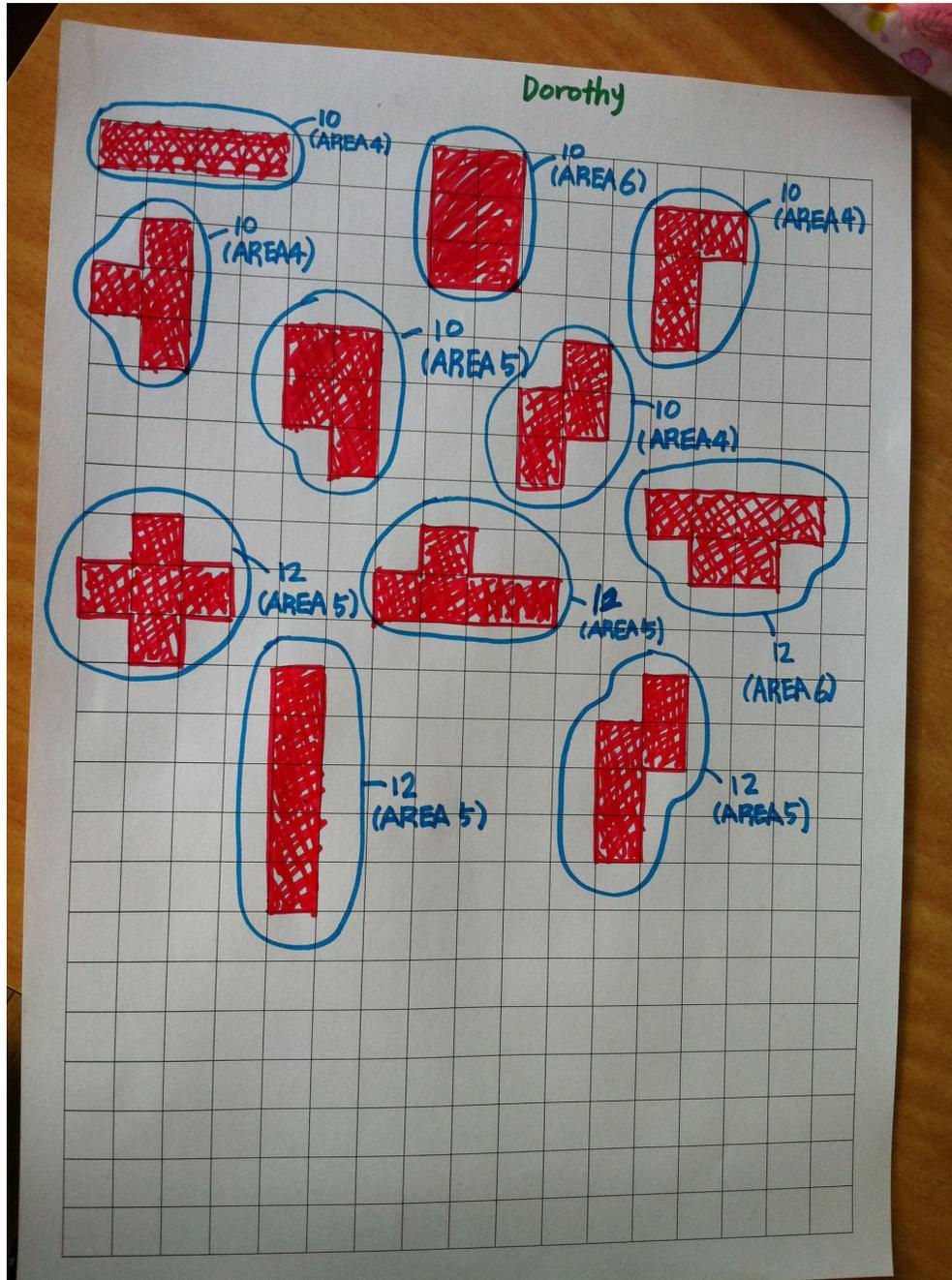


All Possible Areas, Perimeter of 10

All the possible areas for polygons with perimeter of 8 are 4, 5, and 6.

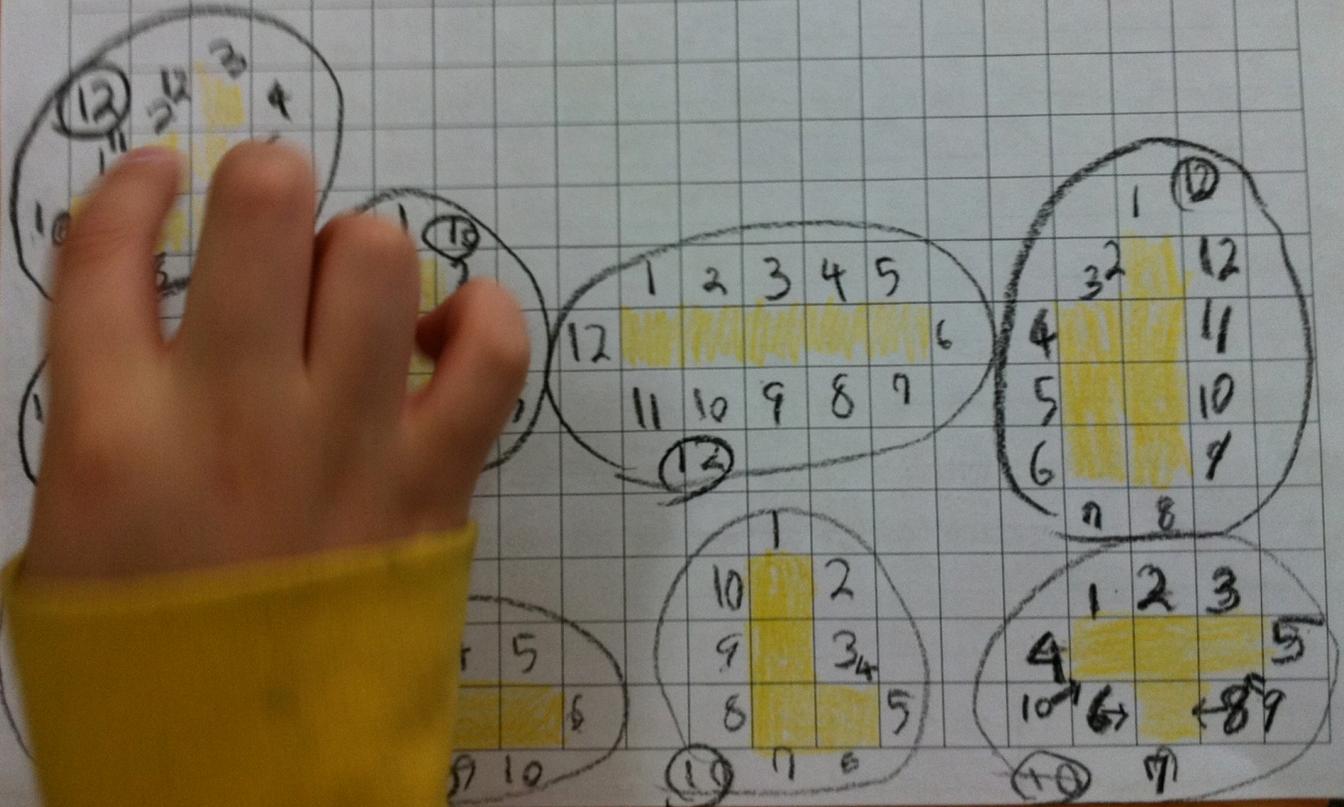
Again, children discovered all the possible areas and many different configurations with each area.

K
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K
o
r
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Tae Young





4 Big Ideas Were Explored

1. The meaning of area.
2. The meaning of perimeter.
3. The difference between area and perimeter.
4. Larger perimeters might lead to larger areas but not always.

This activity relates to a chapter from Liping Ma's book
(1999, Second Edition 2010)

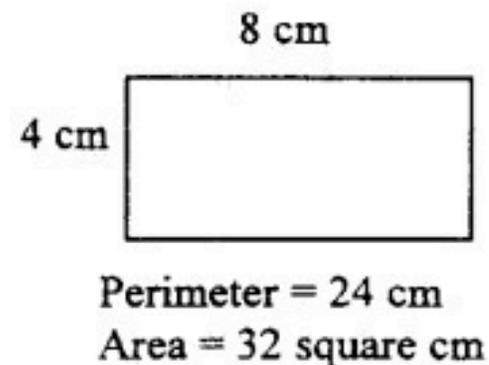
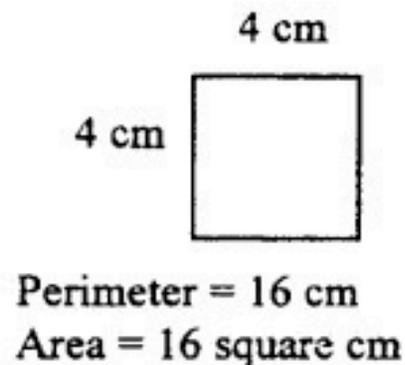
KNOWING AND TEACHING ELEMENTARY MATHEMATICS

TEACHERS' UNDERSTANDING OF
FUNDAMENTAL MATHEMATICS IN
CHINA AND THE UNITED STATES

Liping Ma asked Chinese and US teachers this question.

Scenario

Imagine that one of your students comes to class very excited. She tells you that she has figured out a theory that you never told the class. She explains that she has discovered that as the perimeter of a closed figure¹ increases, the area also increases. She shows you this picture to prove what she is doing:



How would you respond to this student?



What do you think?

How do you think you and / or teachers you know would respond to a question like this coming from a student?



Responses from Chinese and US Teachers

1. Simply accepted the claim (9% Chinese, 8% US) It looks to be true so why question it?
2. Not sure, no explanation (0% Chinese, 78% US) This is broken down to 2 sub-categories.
 - 2a. Look it up somewhere (0% Chinese, 22% US) Some teachers may have been unsure of how to compute area or perimeter.
 - 2b. Calling for more examples (0% Chinese, 57% US) They simply looked for more examples without coming to mathematical principles or conclusions based on patterns.



Responses from Chinese and US Teachers

3. Explored, problematic strategy (22% Chinese, 9% US)

They searched for counterexamples but did not find any.

4. Explored, correct strategy (69% Chinese, 4% US) Liping

Ma identified many levels of correct strategies.

4a. Counterexample (19% Chinese, 4% US) They found a counterexample, which is an adequate proof that the conjecture is false but they did not go further.

4b. Higher levels (50% Chinese, 0% US) Here they created categories for the relationships between area and perimeter and showed when the conjecture will always be true.



Responses from Chinese and US Teachers

Note that this is not a study of random samples of US and Chinese teachers. It can be used to categorize types of thinking and to help set goals that teachers can aspire to.

Responses from Chinese and US Teachers Ma (p. 92)

92

CHAPTER 4

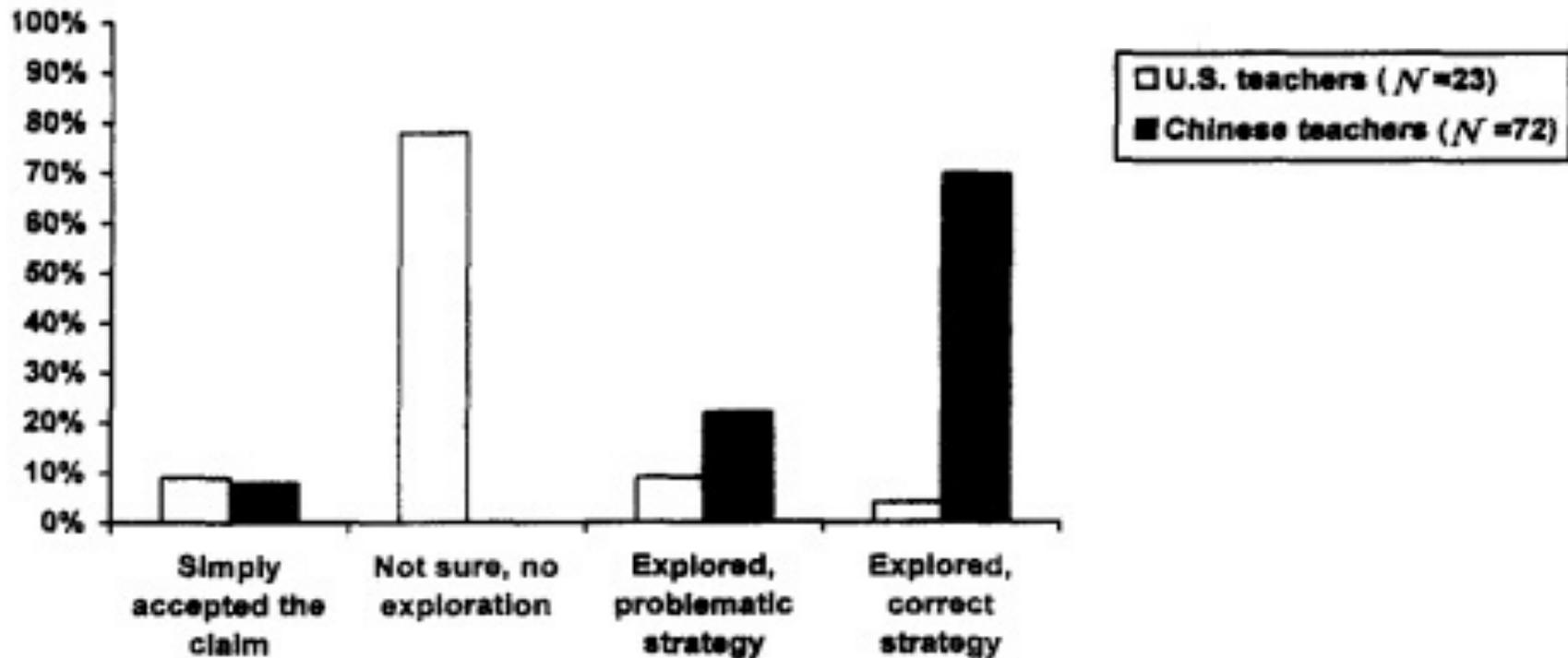


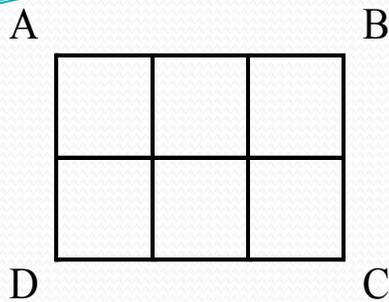
FIG. 4.1. A comparison of teachers' reactions to the student's claim.



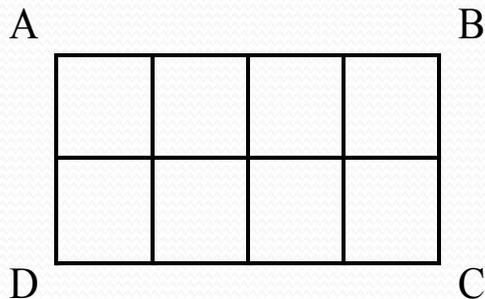
Back to the children

After the 3 day lesson, the three groups of children were given the question given to you earlier.

How do you think they responded?



This flat candy bar has 6 pieces. An ant starts at point A, moves 3 units to point B, then 2 units to point C, then 3 units to point D, then 2 units to A. The ant traveled a total of 10 units.



This flat candy bar has 8 pieces. An ant starts at point A, moves 4 units to point B, then 2 units to point C, then 4 units to point D, then 2 units to A. The ant traveled a total of 12 units.

John thinks that whenever an ant travels more units all the way around a candy bar, there will always be more pieces in the candy bar. Do you think John is right? Why or why not?

Categories for the 3 groups of children US, Cyprus, and Korea

1. Understanding Not Evident

(Cyprus 37%, US 44%, Korea 27%)

- a. Blank or unintelligible
- b. Incorrect Answer, incomplete or incorrect explanation
- c. Correct answer, incomplete or incorrect explanation

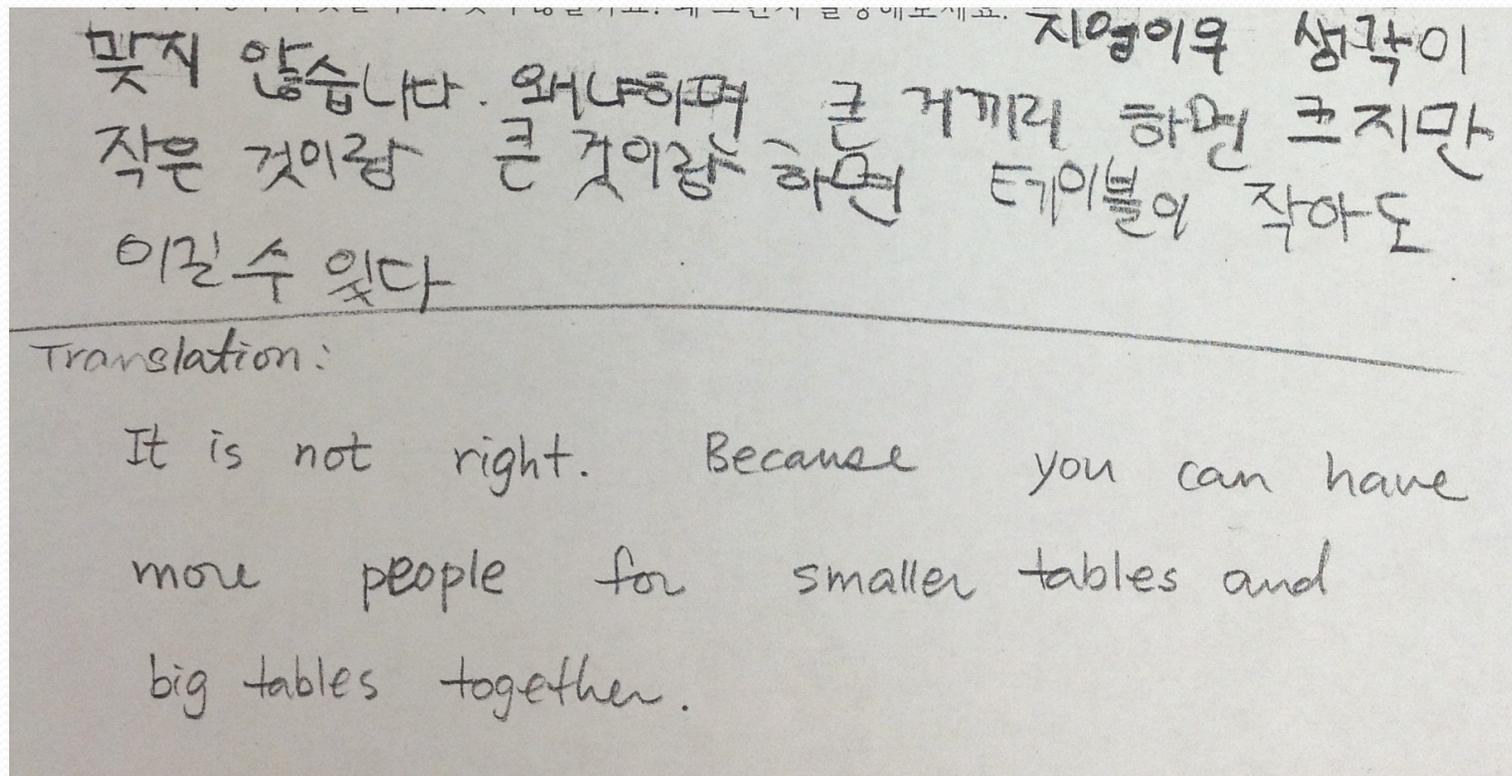
1b.

**Incorrect Answer, incomplete or incorrect explanation
Cyprus**

Yes, I think that John is right because well
if there are going to be more pieces that means
there will be more units. More pieces, more units

1c

**Correct answer, incomplete or incorrect explanation
Korea**



“Smaller tables and big tables together”



Categories for the 3 groups of children US, Cyprus, and Korea continued

2. Partially Developed Understanding
(Cyprus 57%, US 39%, Korea 17%)

- a. Incorrect answer, developing explanation
- b. Correct answer, developing explanation

2a
Incorrect answer, developing explanation
USA

I think John is write because if the ant travels more units all the way around, the ant will need more pieces to go all the way around the candy bar

2a

Incorrect answer, developing explanation

USA

be more pieces in the candy bar. Do you think John is right? Why or why not?

I think John is right because the first candy bar had 6 pieces and had 10 units. Then, the second candy bar had 8 pieces and with 12 units. So I think the third chocolate bar would have 10 pieces and with 14 units. That is why I think John is right.

I think this would happen and I think John is right.

Good explanation but only looked at one type of rectangle.

2b

**Correct answer, developing explanation
Cyprus**

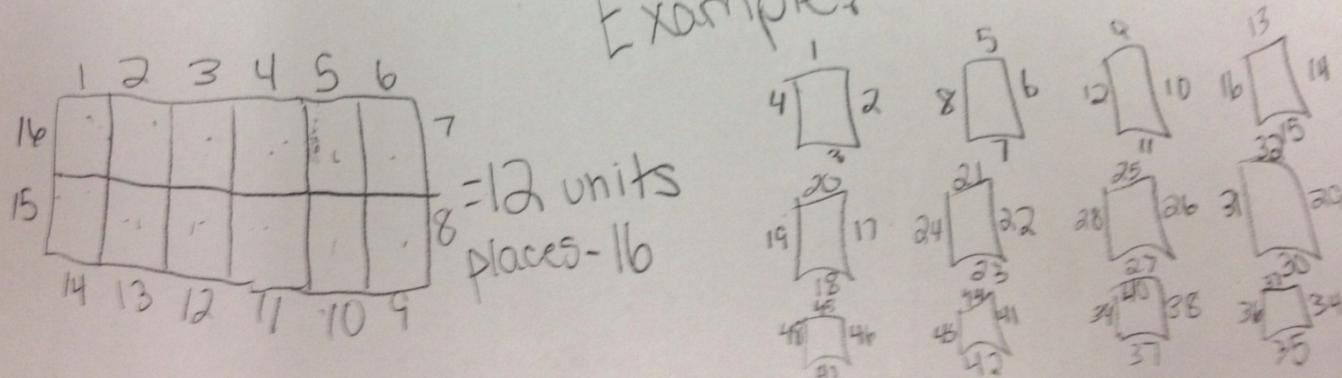
I think John is not right, sometimes there will be more and sometimes there will be less. It won't always be less because the numbers go up sometimes and sometimes it will go less.

2b

Correct answer, developing explanation USA

I think John is wrong because it depends on how you put the the units together. If you would have 12 units and all of them were seperated, then there would be more places. If you had 12 units and all of them were put together, then you will have less places.

Example:





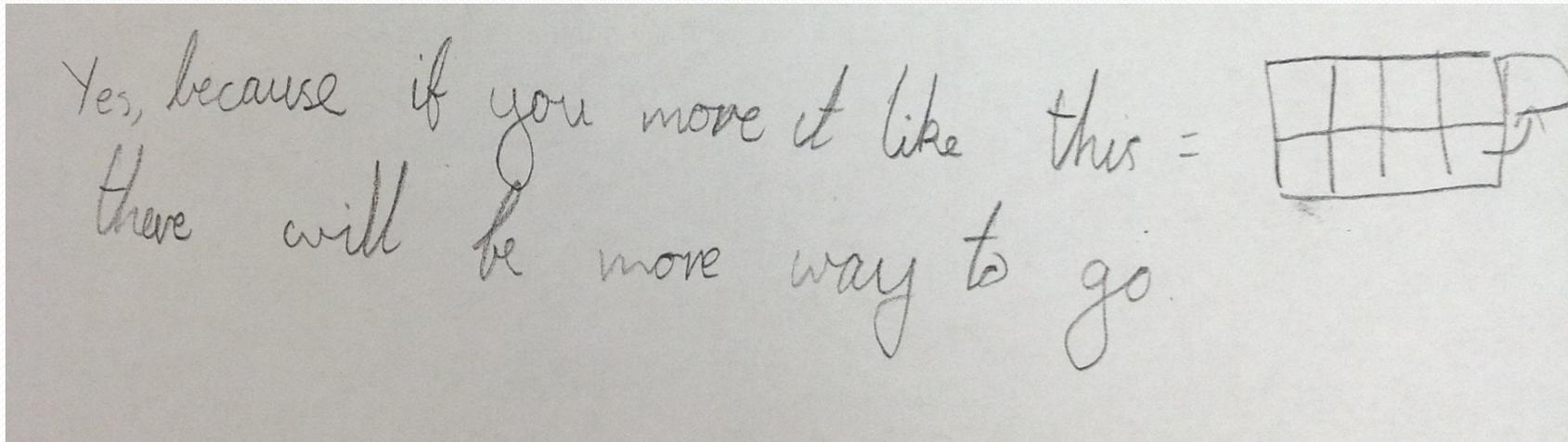
Categories for the 3 groups of children (US, Cyprus, and Korea) continued

3. Developed Understanding
(Cyprus 7%, US 17%, Korea 23%)
 - a. Correct answer, a correct counterexample without an effective explanation
 - b. Correct answer, with an effective explanation (with or without a correct counterexample)

3a

**Correct answer, a correct counterexample
without an effective explanation**

Cyprus



Original $P=12$, $A=8$

Changed $P=14$, $A=8$

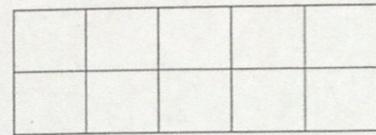
Perimeter goes up but Area stays the same

3a

Correct answer, a correct counterexample without an effective explanation Korea

옆 그림에는 10 개의 테이블이 있습니다.
그리고 이 테이블에는 14 명이 앉을 수
있습니다.

→

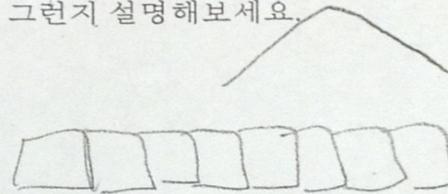


14명

지영이는 책상이 더 많으면 항상 더 많은 사람들이 앉을 수 있다고 생각합니다.

지영이의 생각이 맞을까요? 맞지 않을까요? 왜 그런지 설명해보세요.

No



18명

Translation:

No, The number of people is 14

"

is 18.

3b

**Correct answer, with an effective explanation
(with or without a correct counterexample)**

USA

No, because if you have something like this

	4	5	6	7
3	2	3	4	7
2	1	4	5	6

it has 7 peices and 14 units but

if you have something like this

	1	2	3	4	5
12	1	2	3	4	5
11	5	6	7	8	6
	10	9	8	7	

it has 8 peices, but it only has 12 units.

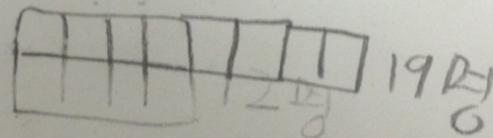
11 10
↓
14. unit

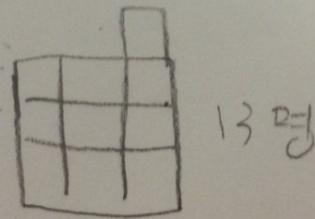
3b: Correct answer, with an effective explanation (with or without a correct counterexample)

Korea

Off by one
(13 not 14)
and
(19 not 20),
but good
explanation

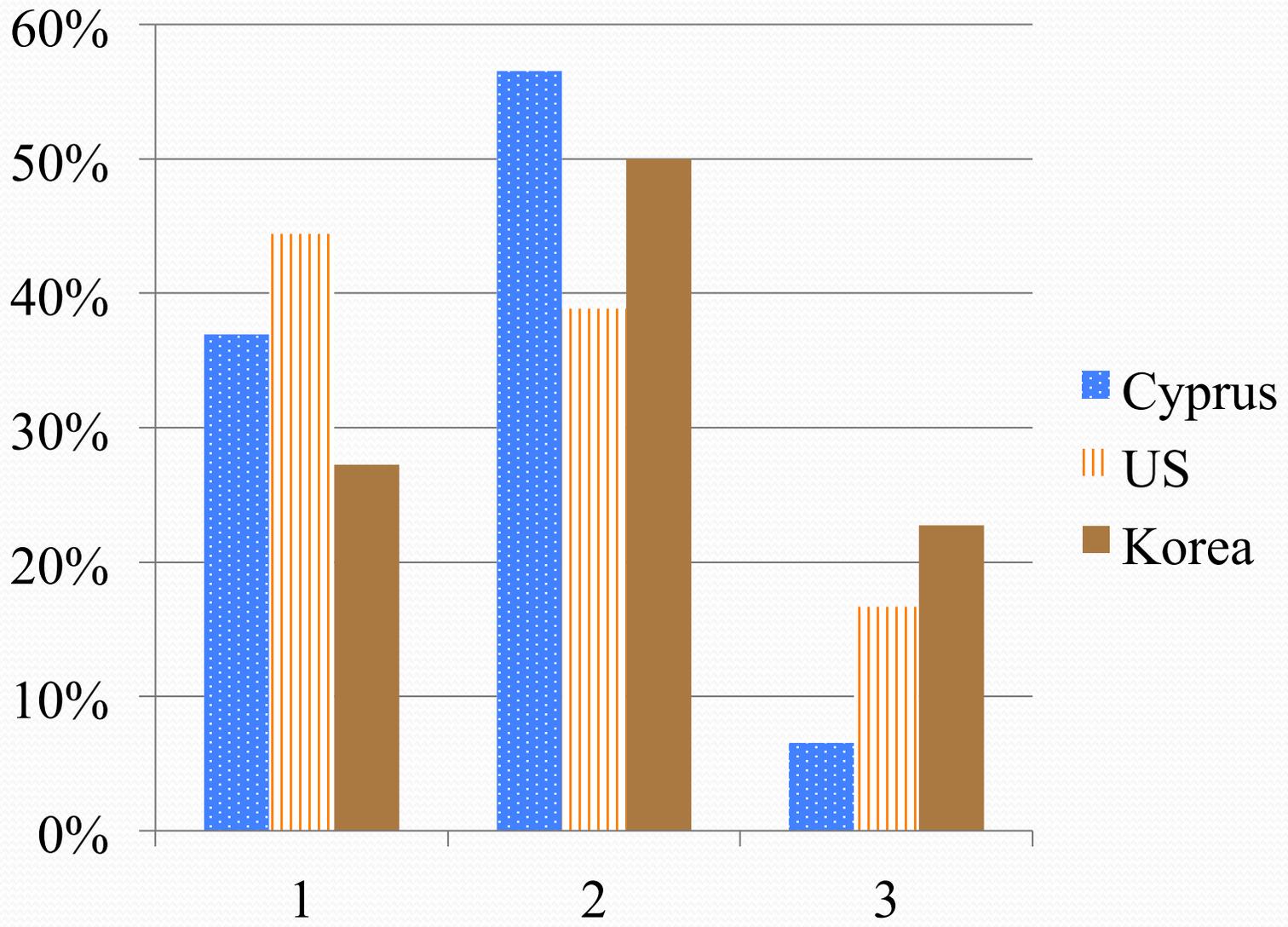
아니요.
테이블을 어떻게 붙이느냐에
따라 다릅니다.

8개의 테이블 →  19명

10개의 테이블 →  13명

Translation: No, because it depends on the arrangement of the tables.

8 tables → 19 people
10 tables → 13 people





Statistical Comparison

The p-value from the Pearson Chi-Square Test for Homogeneity is .522, showing no significant differences across the three groups.

Although the Third Grade Koreans may have done slightly better than the Fifth Graders in Cyprus and the US, the difference was not significant enough to generalize. The sample size was very small.



Summary

- These children were excited by this mathematical exploration. At first the interesting context provided the interest but then the mathematical exploration was the main motivation.
- All students were able to explore these four big ideas in a problem-solving environment.
 - The meaning of area.
 - The meaning of perimeter.
 - The difference between area and perimeter.
 - Larger perimeters might lead to larger areas but not always.



Summary (cont.)

- Many students were not successful with coming up with a general statement about why the claim was not true or an effective counterexample to the claim.
- However, many of these young students were able to go beyond the explorations and come to relatively sophisticated conclusions.

Implications

- The 86% of US teachers who were in Liping Ma's category 1 or category 2 would not likely be able to teach a lesson such as this one. If they lack the mathematical curiosity or knowledge to begin the process of exploring this situation on their own, it is unlikely they would be able to teach it.
- The 13% of US teachers in Liping Ma's category 3 or category 4 at least have the mathematical disposition and at least partially the knowledge to be able to teach a lesson such as this one. There is no guarantee that they would be able to teach such a lesson but at least they have a foundation.



Implications (cont.)

- Liping Ma highlights throughout the book the importance of teachers having requisite levels of mathematical knowledge.
- She highlights that with this perimeter and area situation the lack of interest and self-confidence was perhaps even more crucial.
- Teachers had at least rudimentary knowledge of perimeter and area but most did not significantly explore the situation.



Implications (cont.)

- Opportunities for significant mathematical explorations such as this one will be lost with ineffective teacher preparation in developing interest and self-confidence along with mathematical knowledge.
- Students are ready for activities that implement the Common Core Standards for Mathematical Practice but many teachers are not.
- What can we do to improve this situation?
- Let's all make an effort to investigate a rich mathematical situation for our students and then teach it to them.



Questions?



Please feel free to contact me

Don't forget to get the handout
on the NCTM Conference site.

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THANKS FOR COMING!