

Proportional Reasoning

Jean McGivney-Burelle

John Tapper

University of Hartford

Nicholas Balisciano

CCAT

Plan for this session

- ◉ Some context for our work
- ◉ Some proportion reasoning problems
- ◉ A look at how PR develops
- ◉ Research on student strategies
- ◉ Questions

Proportional Reasoning and PRISM

- A collaboration between CCAT, the University of Hartford and local middle and high schools
- Aimed at improving content knowledge and pedagogical skill
- A year-long program with a summer intensive and follow-up sessions during the school year
- Focused on proportional reasoning, and the power of inquiry and deep conceptual understanding

Problems

To feed 2 butterflies the zoo needs 5 drops of nectar per day. How many drops would they need each day for 12 butterflies? Show all of your working and explain your answer in as much detail as possible.

Problems

A recipe for a sticky mess needs 4 cups of sugar and 10 cups of flour. You decide to make a larger amount of the recipe, and have 6 cups of sugar. How many cups of flour will you need for the recipe to work?

Problems

Three cups have different amounts of water. The first is full, the second one half full, and the third is one third full. The first cup has three spoons of sugar stirred in, the second has two spoons, and the third has one spoon of sugar. Which is the sweetest?



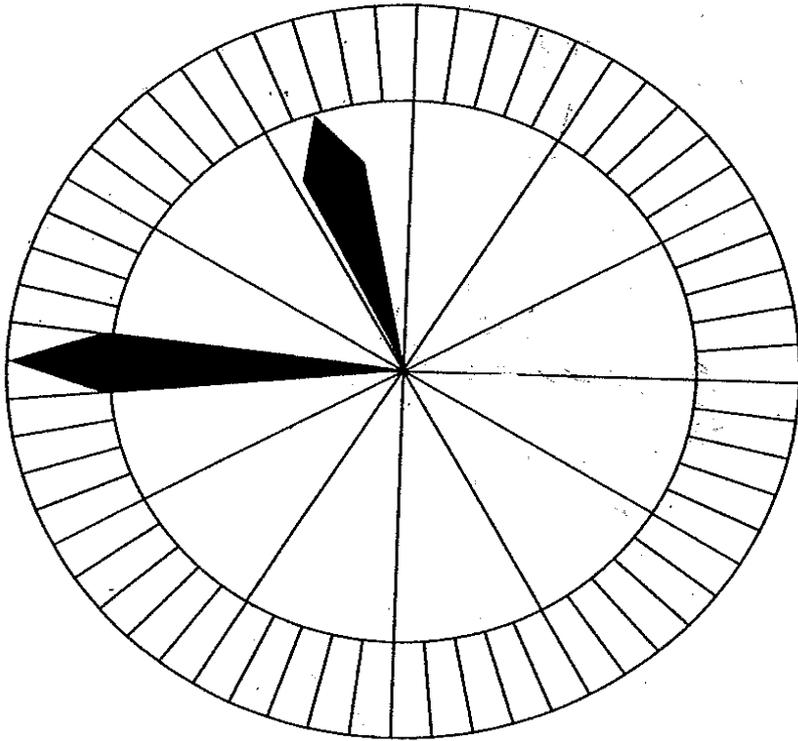
Problems

Six people can build a garage in 3 days. Assuming that all of the people work at the same rate, how many people would it take to build the garage in 1 day?

Problem Results

Grade	% Correct on Butterfly Problem	% Correct on Sticky Mess Problem	% Correct on Three Cups Problem	% Correct on Garage Building Problem
Answer	30	15	Cup B	18
5	47.4	3.8	13.5	61.7
6	54.1	9.8	19.7	72.1
7	65.7	16.1	29.9	61.3
8	67.4	25.1	27.6	73.5
9	89.4	36.2	29.8	83

Clock Problem



Clock Problem

- ◉ Strategies and solution?
- ◉ How is this proportional reasoning?

Overview

- Over the past twenty-five years research has consistently shown that
 - Relatively few middle school students of average ability use proportional reasoning in a consistent fashion (Post, Lesh & Behr, 1988)
 - The topic remains problematic for many college students (Lawton, 1993)
 - A large segment of our society never acquires fluency in proportional thinking (Hoffer, 1988)

Research Findings

- Researchers have

- Investigated the development of proportional reasoning in students
- Categorized proportional reasoning problems into three main types
- Identified the strategies students use in solving proportional reasoning problems

Development of Proportional Reasoning

1. Qualitative: Young students generally possess a good deal of knowledge about quantity that permits them to answer questions about more or less (e.g., which drink is sweeter?)
 - Example: If Nick mixed less lemonade mix with more water than he did yesterday, his lemonade drink would taste a) Stronger b) Weaker c) Exactly the same or) Not enough information to tell.

Development of Proportional Reasoning

- 2. Early Attempts at Quantifying: Early attempts at quantifying often involve constant additive differences (i.e., $a-b=c-d$) rather than multiplicative relationships. Still rely on counting up or counting down

- Example: A recipe calls for 4 c of sugar and 6 c of flour. Nick wants to make a bigger recipe so he uses 6c of sugar. How many cups of flour will he need?

Sugar		Flour
4	(+2)	6
6	(+2)	8

Development of Proportional Reasoning

- 3. Multiplicative Reasoning/Pre-Proportionality: Students employ a sort of “pattern recognition and replication” strategy, which some have called a “build up” strategy.

- Example: A recipe calls for 4 c of sugar and 6 c of flour. Nick wants to make a bigger recipe so he uses 12c of sugar. How many cups of flour will he need?

<u>Sugar</u>	<u>Flour</u>
4	6
8	12
12	18

Development of Proportional Reasoning

4. Proportional Reasoning: Students recognize the multiplicative relationship between two terms and this relationship is then applied to two other terms.

- Example: A recipe calls for 4 c of sugar and 6 c of flour. Nick wants to make a bigger recipe so he uses 6c of sugar. How many cups of flour will he need?
- Since 6 is 1.5 times 4, he will need 1.5 times 6 or 9 cups of flour.

Strategies Students Use to Solve PR Problems

Two shirts cost \$8. How much will 7 shirts cost?

Unit-rate

- As the name implies this is a “how many for one?” strategy

Unit -rate

\$8/2 shirts = \$4 per shirt

So 7 shirts will cost $7 \times 4 = \$28$

Factor-of-change

- This is a “times as many” strategy
- Ease of this strategy is related to the numerical aspects of the problem
- Students less apt to use this if the factor to be used is not an integer

Factor-of-change

<u>Shirts</u>		<u>Cost</u>
2	($\times 4$)	\$8
7	($\times 4$)	\$28

Strategies Students Use to Solve PR Problems

Two shirts cost \$8. How much will 8 shirts cost?

- Fraction

- When the labels are dropped in a rate problem and students apply the fraction rule for finding equivalent fractions



- Fraction

$$\frac{8}{2} = \frac{16}{4} = \frac{32}{8}$$

- Cross Product

- Extremely efficient but devoid of meaning in the real world



- Cross Product

$$\frac{8}{2} = \frac{x}{8}$$

$$2x = 64$$

$$x = 32$$

Research on PR Strategies

- The unit-rate strategy and factor-of-change appear to be more intuitive for students
- On non-proportional problems students who haven't learned the cross product algorithm were more successful than those who had

Research-Based Recommendations

- Students should be exposed to a variety of problem types
- Students should be exposed to multiple strategies, including unit rate, factor of change, fractions, and the cross-product algorithm.
- Teachers should begin instruction with more intuitive strategies, such as the unit rate and factor of change.

Research-Based Recommendations

- The cross product algorithm should only be taught after students have explored problems using more intuitive strategies.
- As textbooks often focus on procedural knowledge, teachers have to go beyond the content of textbooks to offer meaningful instruction for this important domain.

Questions?