

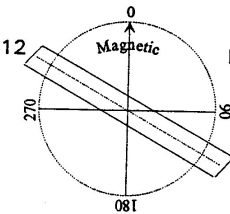
*Using Applications And
Activities to Motivate
Middle Grades Mathematics*

RICK BILLSTEIN
MATH DEPT
THE UNIVERSITY OF MONTANA
rickb@mso.umd.edu

Examples taken from
Middle Grades MATH*Thematics*
McDougal Littell Publishing Co.

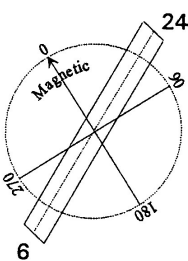
Houghton Mifflin Harcourt

17. Determine the compass readings of a plane landing on the runways pictured below. Enter them on a piece of paper as indicated below.

a.  Runway 12-30

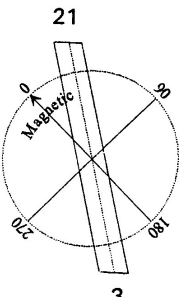
approaching from the northwest _____

approaching from the southeast _____

b.  Runway 6-24

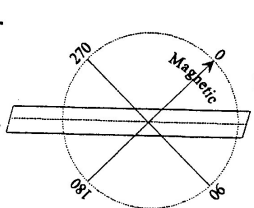
approaching from the southwest _____

approaching from the northeast _____

c.  Runway 3-21

approaching from the northeast _____

approaching from the southwest _____

d.  Runway 4-22

approaching from the southwest _____

approaching from the northeast _____

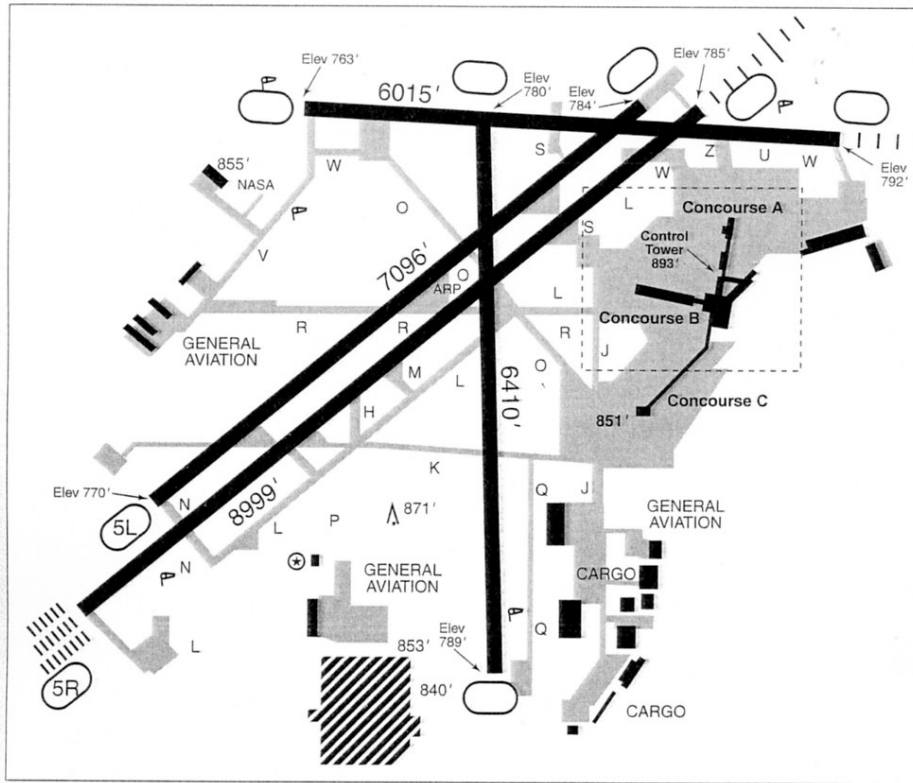


Fig. 6 Parallel runways 5L and 5R

A follow-up visit by a pilot was also very interesting because students had a chance to ask questions about runways and airplanes and to find out how much mathematics pilots needed to know to fly airplanes.

Students had practiced with goniometers in previous modules. Students enjoy using goniometers and had a much better feeling for an angle as an amount of rotation as a result of their experiences with them. One difficulty that students had was drawing angles of fewer than 25 degrees. They had to figure ways to mark the amount of the opening. The goniometer is a valuable and popular device in the middle school. It is the best instrument I have found for measuring the angles on pattern blocks.

Many extensions to this activity are possible. Some examples follow.

- When airports are being designed, the direction of runways is determined by the local prevailing winds. How do you think winds might influence the direction in which the runways are built?
- Are a Runway 0 and a Runway 36 possible? Why or why not?

- How are true north and magnetic north related? Why do pilots use magnetic north for headings?
- Do you think that runway numbers ever have to be changed? Why?
- Is water navigation done in a similar manner? That is, do sailors use compass headings to find their way?
- How are compasses used in orienteering? In astronomy? How is this use similar to what you learned about how runways are named?
- Goniometers could be used to examine optimum angles of descent from pattern altitudes to airport runways at different distances from the airport.

A version of this activity is among the many examples of real-world explorations that have intrigued students in the *Six through Eight Mathematics (STEM)* project's middle school mathematics curriculum materials (in press). The STEM project was developed at the University of Montana and funded by the National Science Foundation in 1992. Thematic modules and relevant applications are an integral part of mathematics in the STEM curricu-

Thinking about Naming Runways

ONCE STUDENTS KNOW HOW TO FIND headings, have them determine the headings of an airplane landing on each runway in **figure 5**, entering the headings in the indicated spaces. Then they are ready to use **figures 1, 5, and 6** to answer the following questions.

- 1a. If an airplane approaching a runway has a heading of 22 degrees, what is the heading of an airplane approaching the same runway from the opposite direction? Why?
 - b. If an airplane has a heading of n degrees, what is the heading of an airplane approaching the same runway from the opposite direction? Explain.
2. On the basis of your findings about headings and runway numbers in **figure 5**, how do you think runways are named?

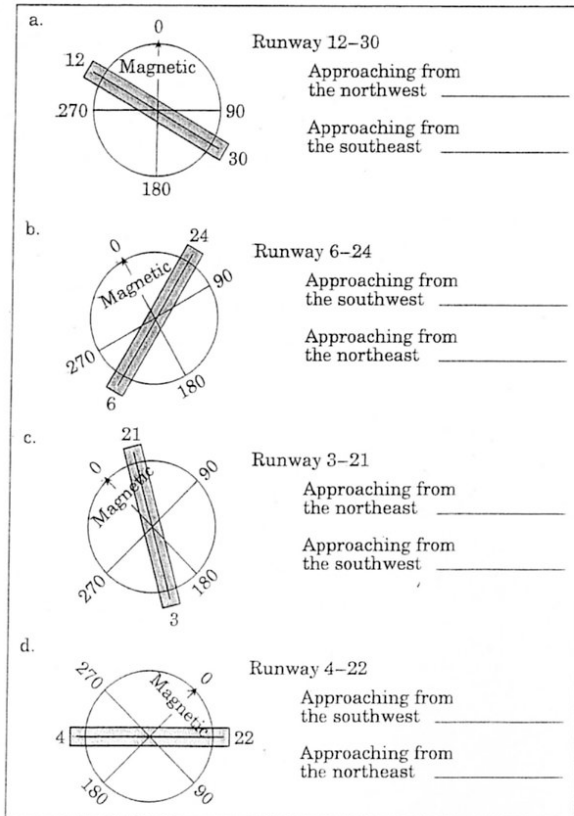


Fig. 5 Determining the headings for airplanes

3. If an airplane with a heading of 320 degrees lands on a runway, what is the runway number?
4. Does your conjecture for naming runways in question 2 seem to work for the runways labeled in **figure 1**? Explain.
- 5a. In **figure 6**, how are runways 5R and 5L related?
 b. What do you think the R and L mean when used with the runway numbers?
 c. Name the runways opposite 5L and 5R.
6. Why would two parallel runways at the same airport have the same number?
- 7a. If two different runways have the same runway numbers (disregard letters), do they necessarily have to be parallel? Why?
 b. Does your answer change if we are talking about real-world runways instead of runways that are parallel in the mathematical sense?
 c. What are some reasons that airports are designed with runways that are not parallel?
- 8a. Some major airports have three runways that are parallel. Design a system for naming them, and explain why your system works.
 b. Does your system work for four, five, or six parallel runways? If not, design a system to handle up to six parallel runways.

Comments on the Activity

FIXED BASE OPERATORS AT LOCAL AIRPORTS ARE usually happy to dispose of outdated maps of runways and airports at little or no cost. The questions in this activity could easily be adapted to local airport configurations to enhance relevance.

This activity worked well in groups of four in which students had a chance to compare headings and conjectures. They disproved many conjectures within a group and had some marvelous discussions. Groups had a chance to explain how they thought runways were named, and then other students had a chance to discuss whether they thought the group was correct. Students in groups could also help one another read the goniometers.

The activity furnished an excellent setting for discussing rounding rules and, in particular, what to do if the digit to the right of the place to be rounded was equal to 5.

The discussion of lines' being parallel and segments' being parallel in the real world and in the mathematical world proved very interesting. The activity also gave us a chance to talk about if . . . then . . . statements and logic.

Section 3

A Problem Solving Approach

IN THIS SECTION

- EXPLORATION 1
• Understand the Problem
- EXPLORATION 2
• Make a Plan
- EXPLORATION 3
• Carry Out the Plan and Look Back

Planning Tools

Setting the Stage

SET UP Work as a class. You will need 9 index cards numbered 1–9.

One tool for success in mathematics and in life is knowing how to solve a problem. As you play a game called *Card Swappers*, you'll learn a 4-step approach that will help you become a better problem solver.

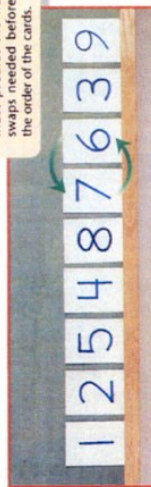
- ▶ As a class, first practice swapping by putting these cards in order from 1 to 9. Try to use as few swaps as possible. Record your swaps.

CARD SWAPPERS

The object of the game is to put nine cards in order from least to greatest in the fewest swaps possible.

A swap is made by exchanging the positions of two cards.

The challenge is that you must predict the number of swaps needed before you see the order of the cards.



Think About It

- 1 Do you think the class put the cards in order in the fewest swaps possible? Try swapping another way to see.
- 2 Do you think the number of swaps needed for different arrangements of the cards will be the same? Try it and see.

GOAL

- LEARN HOW TO...
• develop an understanding of a problem
- AS YOU...
• play and think about the Card Swappers game

Exploration 1

Understand the Problem

SET UP Work first as a class and then in a group of four. You will need 9 index cards numbered 1–9 for each group.

- ▶ You have had some experience swapping cards. Now you'll use your experience to play the game *Card Swappers*.

You'll need two teams. At the start of each game, the cards should be shuffled and placed facing away from both teams.



First

Each team bids the number of swaps they think it may take to put the cards in order.



Next

The cards are turned over so that the numbers can be seen.



Then

The lowest bidding team puts the cards in order from least to greatest.

If the lowest bidding team uses no more than the number of swaps they bid, then they win. Otherwise the other team wins.

- ▶ One way to understand the game better is to play it several times.

- 3 Play *Card Swappers* two times with the whole class divided into two teams.

- 4 **Discussion** When you practiced swapping cards on page 29, you just needed to put the cards in order from least to greatest in the fewest swaps possible. What new challenges did you face when you played the game?

Exploration 2

The LAST CARD Problem

SET UP Work in a group. You will need 15 index cards numbered from 1 to 15.

One of the steps in the 4-step approach is to make a plan to solve the problem. Making a plan often involves choosing a problem-solving strategy such as look for a pattern or use a model. You might use one of these strategies as you explore the Last Card Problem.

GOAL

- LEARN HOW TO...
 - apply the 4-step approach to problem solving
- AS YOU...
 - predict outcomes in a card problem

► **The Last Card Problem** Start with a stack of cards in numerical order from top to bottom. Then do as follows:



Put the top card face up on the table.

Put the next card on the bottom of the stack. cards are face up.

Continue until all the cards are face up.

► Can you predict what the last card will be if you know how many cards you start with?

10 Suppose you start the Last Card Problem with six cards.

a. Which card do you think will be last?

b. Try the experiment with your cards. Were you right?

11 Suppose you start the Last Card Problem with nine cards. What will be the number on the last card?

12 Suppose you start with 16 cards or 20 cards. What will be the number on the last card in each case? Explain your answers.

HOMEWORK EXERCISES See Exs. 4–7 on pp. 45–46.

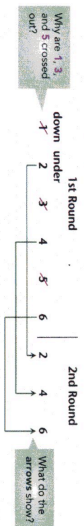
► Here is how Kim's group approached the Last Card Problem.

First we used the strategy "Try a simpler problem." We found that with 3 cards the number on the last card was 2. Next, we decided to try the strategies "make a table" and "look for a pattern" to see if we could predict the last card for 16 and 20 cards. We used our 15 numbered cards to complete our table, but we could not find a pattern.

Number of cards	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Number on last card	1	2	2	4	2	4	6	8	2	4	6	8	10	2	4

- 5 **Discussion** If Kim's group kept without finding a solution, how should they score their work on the problem-solving scale? Why?
- 6 Describe any patterns you see in the table made by Kim's group.

► When your approach to a problem does not help you find a solution, you may need to try a different method. As a new approach, Kim's group modeled each move for 6 cards.



7 a. Copy and complete the model. What number is crossed out next? What will be the last card?

b. Use this method to solve the Last Card Problem for 16 cards and for 20 cards.

8 **CHECKPOINTS** Suppose Kim's group uses the method above and finds the last card for 16 cards and for 20 cards. How should they score their work on the problem-solving scale? Why?

HOMEWORK EXERCISES See Exs. 1–3 on p. 56.

Last Card Problem

1 -----

2 -----

3 -----

4 -----

5 -----

6 -----

7 -----

8 -----

9 -----

10 -----

11 -----

12 -----

13 -----

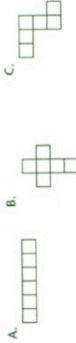
14 -----

15 -----

Section 5

Practice & Application Exercises

1. **Visual Thinking** Choose the letter(s) of the pattern(s) that can be folded into a cube. Then draw two other patterns that will work.



2. Suppose next semester you want to take music, advanced science, creative writing, and basketball. Music is held periods 1, 2, and 4 only. Advanced science is offered periods 2, 3, and 4. Creative writing meets periods 1, 2, and 3, while basketball is offered periods 1, 3, and 4. List all the possible ways you could take these four classes.

3. **Use Labsheet 5B.** Assess your work in Exercises 1 and 2 using the problem solving scale.

4. Suppose you have 10 cards numbered from 1 to 10. How would you order the cards so that when you follow the procedure below, the cards are in numerical order in a pile on a table, with the 1 at the bottom and the 10 at the top? (Note: Your solution should explain what you did and why you did it.)

- Place the top card face up on a table.
- Put the second card on the bottom of the stack.
- Place the third card face up on the table.
- Put the fourth card on the bottom of the stack, and so on, until all cards are on the table.



GOAL

LEARN HOW TO...
 • find geometric probabilities
 • use probability to predict

AS YOU...
 • describe the chance of an event occurring in a particular area

KEY TERM
 • geometric probability

Exploration 1

GEOMETRIC Probability

SET UP Work as a class. You will need an inflatable globe.

- ▶ A meteorite has an equally likely chance of hitting anywhere on Earth. To find the probability that a meteorite will hit land you will conduct an experiment with a globe.

- 2 **Try This as a Class** Follow the steps below to simulate a meteorite falling to Earth.



First Make a table like the one shown to record the results of your experiment.

Next Carefully toss an inflated globe from one person to another. Each time the globe is caught, make a tally mark to record whether the left index finger is touching land or touching water before tossing it again. Record the results of 30 tosses.

Then Complete your table by calculating the percent of the tosses that hit land and the percent that hit water.



Tally		Number of Tosses	Percent of Tosses
Land			
Water			
Total		30	

FOR HELP
 with finding probabilities, see **MODULE 4**, p. 242



A PHONE CHAIN

- 55 people
- 30 sec/call
- all have
- 2 calls each

The Situation

When a search and rescue team is needed, all members of the search and rescue team must be notified as quickly as possible. This is often accomplished through a phone chain. In a phone chain, the first person calls two people. Each of those people call two more people. Each of those people call two more people and so on, until everyone in the search and rescue team is notified of the emergency.

The Problem

Estimate the least amount of time it would take using a phone chain like the one described above to notify all the members in a 55-person search and rescue team.

Something to Think About

- How might a table or drawing help you solve this problem?
- Could you modify the procedure to shorten the total amount of time involved? If so, how?
- What are some assumptions you might need to make about phone calls?



Present Your Results

Describe what you did to solve the problem. Explain why you solved it this way. Show any drawings, diagrams, charts, or tables that you used to solve the problem. Why do you think your solution is accurate?



Mystery State

The Situation

Try the following puzzle. Is mystery or mathematics at work?

- Pick an integer between 1 and 10.
- Multiply your number by 6.
- Add 12.
- Divide by 3.
- Subtract 4.
- Divide by your original number.
- Add 4.
- Match the number with the corresponding letter of the alphabet (1 = A, 2 = B, etc.).
- Think of a state in the United States that begins with that letter.
- Look at the third letter of the name of the state. Think of a fruit that begins with that letter and grows in that state.
- Turn your book upside-down and look at the bottom of the page to complete the mystery.

The Problem

Find out why the mystery puzzle works. Then create one of your own.

Something to Think About

- How might examining a mystery puzzle with fewer steps help you?
- Does this puzzle work for any positive integer? Would it work for negative integers? decimals? fractions?

Present Your Results

Write your puzzle on a sheet of paper. Include the solution on the back. Explain why the puzzle above works, and why your mystery puzzle works.

You thought of oranges in Florida.

$$\begin{aligned}
 & n \\
 & 6n \\
 & 6n + 12 \\
 & 2n + y \rightarrow 2n \rightarrow 2 \rightarrow 6 \rightarrow F \rightarrow \text{Florida} \rightarrow \text{oranges}
 \end{aligned}$$



FOR ASSESSMENT AND PORTFOLIOS



Wet Paint

The Situation

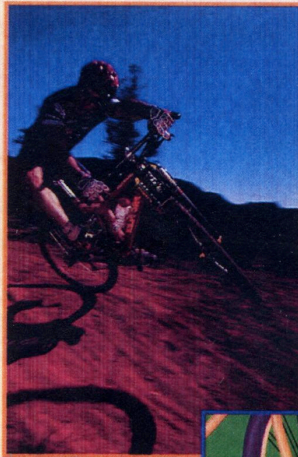
While riding your bike, you accidentally rode across some wet paint.

The Problem

Suppose the paint spill was 4 in. wide where you rode across it. If you continue to ride in a straight line, what will the track left by the bike's tires look like? Your bike tire has a diameter of 26 in. (You can ignore the pattern of the tire tread.)

Something to Think About

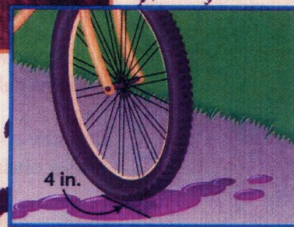
- ◆ How much of the bike's tires will be covered with paint?
- ◆ What are some assumptions you can make about wet paint on a moving object?
- ◆ Which problem solving strategies can you use to help solve this problem?
- ◆ How can you test your conclusions?



Present Your Results

Describe what you did to solve the problem. Show any problem solving strategies you used. Explain why you think your answer is correct.

For help with assessing your work on this and other Extended Explorations (E^2), see the Toolbox, pages 607–608.



4 in.

A Clever Twist

Setting the Stage

You will need:

- strips cut from adding machine tape
- colored pencils or markers
- scissors
- transparent tape

Paul Bunyan Versus the Conveyor Belt

by William Hazlett Upton

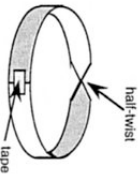
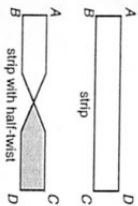
One of Paul Bunyan's most brilliant successes came about not because of brilliant thinking, but because of Paul's caution and carefulness. This was the famous affair of the conveyor belt.

Paul and his mechanic, Ford Fordsen, had started to work a uranium mine in Colorado. The ore was brought out on an endless belt which ran half a mile going into the mine and another half mile coming [back] out—giving it a total length of one mile. It was four feet wide. It ran on a series of rollers, and was driven by a pulley mounted on the transmission of Paul's big blue truck. "Babe." The manufacturers of the belt had made it all in one piece, without any splice or lacing, and they had put a [single] half-twist in the return part so that the wear would be the same on both sides.

Think About It

1. a. Color each side of your strip a different color. How many sides does it have?

- b. Use the colored strip to make a model of the conveyor belt.



The belt you made is called a *Möbius strip*.

2. a. According to the story, putting a single half-twist in the conveyor belt will make it wear the same on both sides. Do you think this is true? Explain.
- b. How many sides does the conveyor belt actually have?

1

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Activity

The Amazing Möbius Strip

The story continues...

After several months' operation, the mine gallery had become twice as long, but the amount of material coming out was less. Paul decided he needed a belt twice as long and half as wide. He told Ford Fordsen to take his chain saw and cut the belt in two lengthwise.

Charlie "Loud Mouth" Johnson bets Ford Fordsen \$1000 that if he cuts the belt in two lengthwise, he won't end up with one belt twice as long as the original belt. Instead, he will have two belts, each the same length as the original belt.

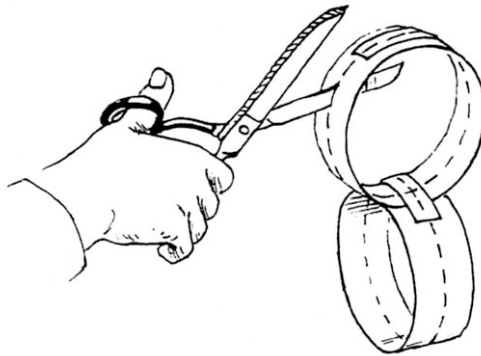
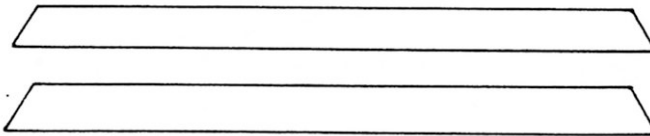
3. a. Who do you think won the bet?
- b. Cut your model lengthwise. What happens?
- c. How many half-twists does the new belt have? how many sides?
4. Later in the story, the conveyor belt again needs to be made twice as long. "Loud Mouth" Johnson bets Paul Bunyan \$1000 that if he cuts the belt in two lengthwise again, he will have a single belt twice as long as the first. Paul says the result will be two belts, each with the same length as the original belt.
 - a. Cut your model lengthwise. What happens? Who won the bet?
 - b. What do you think would happen if you made a lengthwise cut on a belt with 3 half-twists in it?
5. Experiment with different models to determine the following.
 - a. How does the number of sides a belt has relate to the number of half-twists in the belt?
 - b. Suppose you are going to cut a belt lengthwise. How does the number of new belts formed relate to the number of half-twists in the original belt?

2

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Construction: Circular Strips

Cut two strips of paper, each one about 12 inches by 1 inch. Fold and tape each strip so that it forms a separate circle. The overlap should be about $\frac{1}{2}$ inch. Tape on both sides of the overlap. Tape the two circular strips together as shown.



What happens if you cut completely around the middle of each circular strip as above? Guess first, then cut to see if you guessed correctly!

guess #1 _____

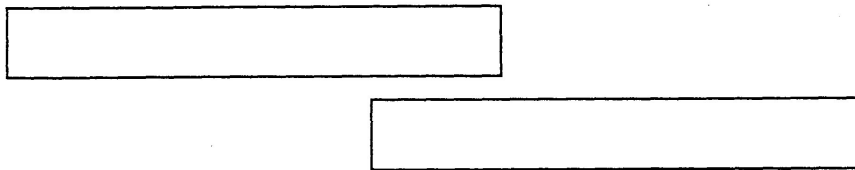
guess #2 _____

guess #3 _____

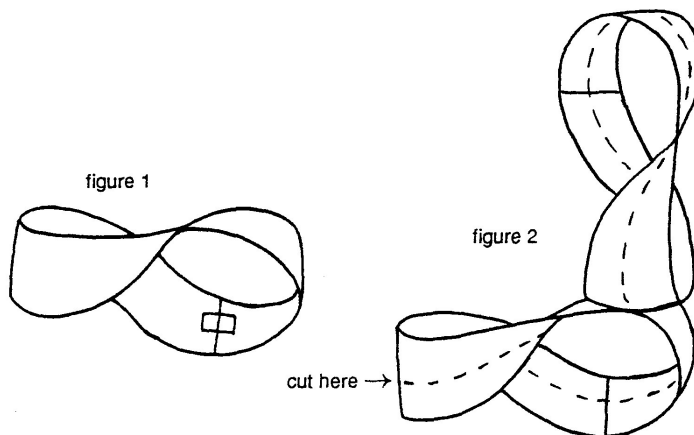
Challenge What other shapes can you create by taping two or more circular strips in different ways? Explain and illustrate.

Construction: What Will It Be?

Use two strips of paper about 5 centimeters wide and 28 centimeters long.



Hold one paper strip the long way and twist it once *toward* you. Then tape the two ends together on both sides as in figure 1. Take the second strip and twist it once in the *opposite* direction. Then tape the ends. Finally, tape the two strips together securely as in figure 2. Guess what results when you carefully cut along the middle of each strip. Then use a scissors to cut and find out.



Be careful when cutting where the two strips are joined.

Exploration 2

Theoretical Probability

SET UP Work with a partner. You will need: • LabSheet 1A
• die or numbered cube

The object of the game *Never a Six* is to be the first player to score a total of 50 or more points.

Never a Six

- Players alternate turns.
- On your turn, roll the die. If the result is not a 6, record the number rolled. This is your point score for that roll.
- If you roll a 6, your turn is over. Any points rolled during the turn cannot be added to your total score.
- You may continue to roll the die and record points until you decide to stop or until you roll a 6.
- If you decide to stop, total the points you rolled during your turn and add them to your total score.



GOAL

LEARN HOW TO...

- find theoretical probabilities
- compare theoretical and experimental probabilities
- find probabilities for an event

AT YOU

- play the game *Never a Six*

KEY TERMS

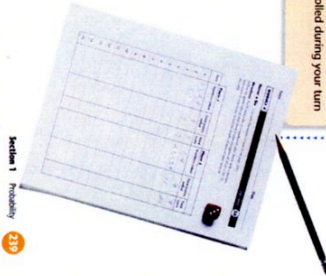
- event
- theoretical probability
- experimental probability
- impossible event
- certain event

Use LabSheet 1A for Questions 15 and 16.

15 With a partner, play one game of *Never a Six*. Record in the table your rolls and the resulting score for each turn.

16 a. List all the possible outcomes for a single roll of a die.

b. Look at the numbers you rolled during the game of *Never a Six*. Do all the numbers seem to have an equal chance of occurring? Explain.



Section 1 Probability 239

Name _____

Date _____

MODULE 4

LABSHEET 1A

Never a Six (Use with Questions 15 and 16 on page 239.)

Directions You'll need a die or a numbered cube. Work with a partner, follow the game rules in your book to play one game of *Never a Six*. To keep track of your score, record the results of each of your turns in the table.

Turn	Player 1		Player 2	
	Numbers rolled	Total points rolled	Numbers rolled	Total points rolled
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				
11				
12				
13				
14				
15				

Section 4

Mean, Median, Mode

Animal Averages

IN THIS SECTION

- EXPLORATION 1
- Finding Mean, Median, and Mode
- EXPLORATION 2
- Appropriate Averages

Selling the Stage

In *The Phantom Tollbooth*, by Norton Juster, Milo is surprised to see what seems to be half of a child.

"Pardon me for staring," said Milo, after he had been staring for some time. "But I've never seen half a child before."

"It's 58 to be precise," replied the child from the left side of his mouth (which happened to be the only side of his mouth).

"I beg your pardon?" said Milo.

"It's 58," he repeated. "It's a little bit more than a half."

"Have you always been that way?" asked Milo impatiently for he felt that that was a needlessly fine distinction.

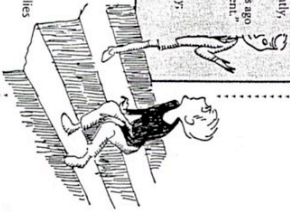
"My goodness, no," the child assured him. "A few years ago I was just 42 and believe me, that was terribly inconvenient."

"What is the rest of your family like?" said Milo, this time a bit more sympathetically.

"Oh, we're just the average family," he said thoughtfully: "mother, father, and 2.58 children—and, as I explained, I'm the 58."

Think About It

- 1 a. What do you think the word *average* means?
b. How do you think the average number of children for families in your class is 2.58? Why or why not?
- 2



Section 4 Mean, Median, Mode 99

Section 9

Using Ratios

BODY RATIOS

IN THIS SECTION

- EXPLORATION 1
- Comparing Ratios
- EXPLORATION 2
- Estimating Ratios
- EXPLORATION 3
- Predicting with a Graph

Selling the Stage

In the Jonathan Swift classic *Gulliver's Travels*, Lemuel Gulliver is shipwrecked and swims to the island of Lilliput, where the people have an average height of slightly less than six inches. Since Gulliver's only clothes were those he was wearing, the Lilliputians had to make new clothing for him.

GULLIVER'S TRAVELS by Jonathan Swift

The seamstresses took my measure as I lay on the ground, one standing at my neck, and another at my mid-leg, with a strong cord extended, the length of the cord with a rule of an inch long. Then they measured my right thumb, and desired no more for by a mathematical computation, that twice round the thumb is once round the wrist, and so on to the neck and the waist; and by the help of my old skin, which I displayed on the ground before them for a pattern, they fitted me exactly.



Think About It

- 1 The height of a Lilliputian is about what fraction of your height?
- 2 What two measurements did the Lilliputians take in order to make a shirt for Gulliver?
- 3 a. What do you think Gulliver meant by "twice round the thumb is once round the wrist"?
b. What do you think he meant by "and so on to the neck and the waist"?

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