Seeing Patterns and Structure: Connecting Elementary and Middle Grades Mathematics

Pamela J. Wells
Grand Valley State University
wellsp@gvsu.edu

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What do mathematicians do?

- Mathematicians make and test conjectures.
- Mathematicians look for and generalize patterns.
- Mathematicians justify and explain why their answers are reasonable.
- Mathematicians look for short cuts that they can justify and that work in many cases.

What do the Standards for Mathematical Practice ask our students to do?

- **Look for and make use of structure. (SMP #7)**

Mathematically proficient students look closely to discern a pattern or structure.

- **Look for and express regularity in repeated reasoning (SMP #8)**

Mathematically proficient students notice if calculations are repeated and look both for general methods and for shortcuts…As they work to solve a problem, mathematically proficient students… continually evaluate the reasonableness of their intermediate results.

Problem Strings

- Start with easily solved problems (using prior knowledge or building off the meaning of the operation)
- Progress to related but more challenging problems:
  - ❑ students can use the answer to the previous problem to help them solve the harder problem OR
  - ❑ students can use a similar reasoning to solve the problem.
- End with a discussion of patterns noticed or short cuts for computation found.

Problem String—Multiplying by powers of 10

- 5.NBT.2 Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.
- 8.EE.3 Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other.

Multiplying by Powers of 10

- 8 x 5
- 80 x 5
- 80 x 50
- 8 x 500
- 80 x 500

Patterns noticed?

Why do those patterns make sense?

Multiplying by powers of 10

- 8 x 1000 Patterns noticed?
- 8 x 100
- 8 x10 Why do those patterns make sense?
- 8 x 1
- 8 x 0.1
- 8 x 0.01
- 8 x 0.001

Multiplying by powers of 10

- 362.57 x 10 Patterns noticed?
- 362.57 x 100
- 362.57 x 1000 Why does the pattern make sense?
- 362.57 x 0.1
- 362.57 x 0.01
- 362.57 x 0.001

Positive and negative integer exponents

- 6.EE.1 Write and evaluate numerical expressions involving whole-number exponents.
- 8.EE.1 Know and apply the properties of integer exponents to generate equivalent numerical expressions.

Integer Exponents

- $2^5 = 32$
- $2^4 = 16$
- $2^3 = 8$
- $2^2 = 4$
- $2^1 = 2$

What do you notice about the first 5 problems? What problem do you think I will write next? What answer would make sense for it?

Integer Exponents

- $2^0 = 1$ What patterns do you notice?
- $2^{-1} = ½$
- $2^{-2} = ¼ = 1/2^2$ How are positive and negative
- $2^{-3} = 1/8 = 1/2^3$ integer exponents related?
- $2^{-4} = 1/16 = 1/2^4$
- $2^{-5} = 1/32 = 1/2^5$

Multiplying and Dividing Integers

- 7.NS. 2a Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (–1)(–1) = 1 and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.

Multiplying and Dividing Integers

- 5 x 5 = 25 What patterns do you notice?
- 5 x 4 = 20
- 5 x 3 = 15 What problem should come next?
- 5 x 2 = 10
- 5 x 1 = 5 What product would make sense for it?
- 5 x 0 = 0

Multiplying and Dividing Integers

- 5 x -1 = -5 What patterns do you notice?
- 5 x -2 = -10
- 5 x -3 = -15 Write one sentence that describes
- 5 x -4 = -20 how to multiply a positive integer
- 5 x -5 = -25 by a negative integer.

What do you predict is the answer to 5 x -10?

Multiplying two negative integers

- -5 x 5 = -25 What patterns do you notice?
- -5 x 4 = -20
- -5 x 3 = -15 What problem should come next?
- -5 x 2 = -10
- -5 x 1 = -5 What should its product be?
- -5 x 0 = 0

Multiplying two negative integers

- -5 x -1 = 5 What patterns do you notice?
- -5 x -2 = 10
- -5 x -3 = 15 Write one sentence that describes
- -5 x -4 = 20 how to multiply a negative integer
- -5 x -5 = 25 by a negative integer.

What do you predict is the answer to -5 x -10?

Other uses for number strings

- Subtracting a number is the same as adding its opposite
- Dividing Fractions
- Finding 10% of a number
- Finding any percent of a number

Use number strings in addition to context, physical models, visual models etc…

Our goal is to see a pattern and use the pattern to develop an efficient rule for computation.

Searching for Patterns to generalize percent word problems

Kendra's mom was pleased that she had found a summer babysitting job. She told Kendra that she needed to put 10% of all her earnings in the bank toward college expenses.

1. How much money would Kendra need to put in the bank if she earned $50? Clearly explain how you found the answer using some combination of words, numbers, and pictures.

Percent Word Problems

- How much money would Kendra need to put in the bank if she earned $120? Does the same approach you used in #1 still work?
- How much money would Kendra need to put in the bank if she earned $275? How did you determine the amount?
- Write a one-sentence description that clearly explains how to find 10% of a number and then use your description to find 10% of $452.

Percent word problems

Claudia wants to save money for her 8th grade class trip. She decided she will save 30% of all her earnings from mowing lawns in the summer.

- How much money should Claudia save if she earns $230 mowing lawns during the summer? How is the approach you took to solve this problem similar to or different from the approach you took to finding 10% of a number on the previous page?
- How much money should Claudia save if she earns $346 mowing lawns during the summer?

Percent word problems

- Kaden wants to put 65% of the money he gets for his birthday into his savings account. How much money will he put into his account if he gets $80 for his birthday? What if he gets $125 for his birthday?
- How did the approach you used to answer this question compare to the way you answered earlier questions?

Ratio Word Problems

Jon lives in an old house and the faucet has been leaking! He called the plumber but the plumber can't come until tomorrow and Jon is concerned that a lot of water will be wasted because he has noticed that the faucet leaked 6 ounces of water in 8 minutes.

If the faucet continues to leak at this same rate…

1. How much water will have leaked in 40 minutes?
2. How much water will have leaked in 20 minutes?
3. How much water will have leaked in 60 minutes?
4. How long will it take for the faucet to leak 9 oz?
5. How long will it take for the faucet to leak 18 oz?

In Summary

- Students need to see the importance of recognizing and generalizing patterns.
- Looking for patterns allows students to make sense of many typical middle school rules and algorithms.
- Being comfortable with patterns allows students to reproduce rules or algorithms when they are forgotten.
- Looking for patterns reinforces mental math skills and helps students check the reasonableness of their answers.