CAN CHILDREN REASON ALGEBRAICALLY?

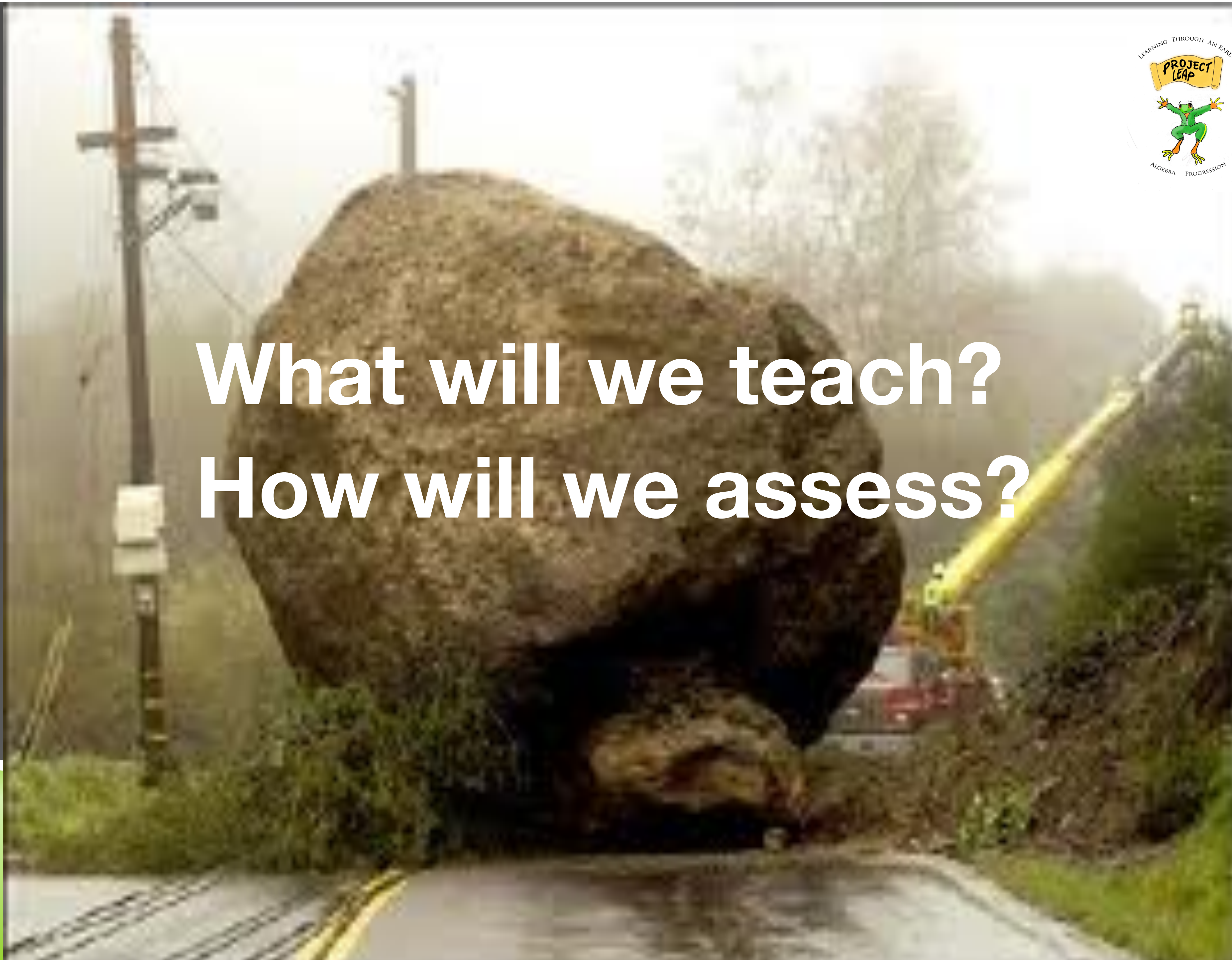
CAN CHILDREN REASON ALGEBRAICALLY?

Results of a One-year, Grades 3-5 Early Algebra Intervention

Maria Blanton
TERC

The research reported here was supported in part by the National Science Foundation under DRK-12 Award #1207945. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect the views of the National Science Foundation.

What is the impact of a sustained, comprehensive early algebra intervention on children's algebra understanding?


What will we teach?
How will we assess?

CONTENT OF OUR INTERVENTION AND ASSESSMENT

- Developed as part of our construction of an Early Algebra Learning Progression and based on our synthesis of
  1. research on teaching and learning algebra across grades K-9;
  2. state and national curricular standards and frameworks;
  3. elementary and middle grades mathematics curricula; and
  4. mathematical perspectives on the sequencing of algebra content.

CONTENT OF OUR INTERVENTION AND ASSESSMENT

- Integrates 5 core content domains – "Big Ideas" – associated with early algebraic thinking:
  1. generalized arithmetic
  2. equivalence, expressions, equations, inequalities
  3. functional thinking
  4. variable
  5. proportional reasoning

WHO, WHEN, WHERE, WHAT

- Participants were 300 students in grades 3-5 from one school district; 10 control classrooms, 6 experimental classrooms
- 3rd-grade: 38 from 2 experimental classrooms; 71 from four control classrooms
- ~23 early algebra lessons* (one hour per week) taught by one member of the research team (a former elementary school teacher) using a problem-based approach
- Same (grade 3 level) lesson taught at each grade
- One-hour assessment* given in September as a pre-test, prior to the intervention; same assessment administered in May as a post-test, after the intervention.

*Available at http://algebra.wceruw.org/

WITHIN-GRADE RESULTS AT GRADES 3-5

- At pre-test, there were no significant differences in performance between experimental and control at each of grades 3-5.
- At post-test, experimental students significantly outperformed control students at each of grades 3-5 (most significant at grade 3).
- Grade level did not significantly impact post-test performance for experimental students, although it did for control students.
    - That is, third-grade experimental students were able to "match" the performance of grades 4-5 experimental students, suggesting that the intervention addressed algebra ideas appropriate for grade 3.
- Grade 3 experimentals significantly outperformed grade 3 controls on ALL items on the assessment except for #10e and #11. For both of these, experimentals outperformed controls, but not significantly.

A CLOSER LOOK AT THIRD-GRADE STUDENTS' PERFORMANCE AND STRATEGY USE

RELATIONAL THINKING

EEEI: ITEMS 1A AND 2B

- **Item 1a**

Fill in the blank with the value that makes the following number sentence true. How did you get your answer?

7 + 3 = ____ + 4 Why?

- **Item 2b**

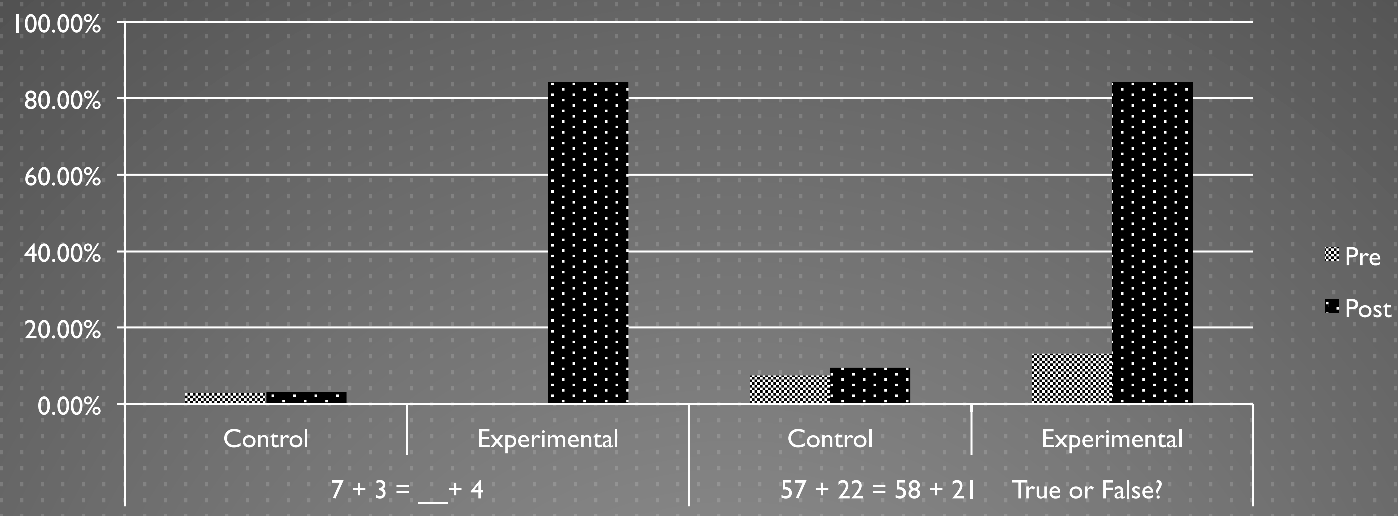
Circle True or False and explain your choice.

57 + 22 = 58 + 21 True False

How do you know?

3RD-GRADE STUDENTS' PERFORMANCE – ITEMS 1A, 2B

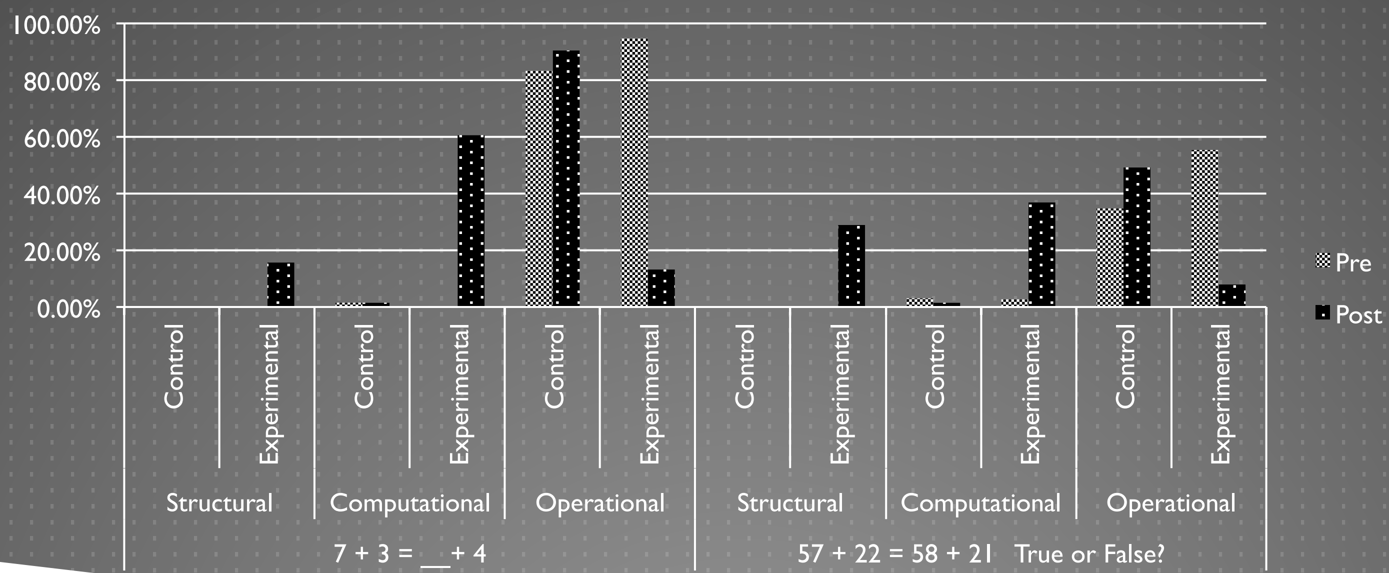
- PRE-TEST: No significant differences between experimental and control on either item.
- POST-TEST: Experimental students significantly outperformed controls on both items ($p<0.001$ for both items).



3RD-GRADE STUDENTS' STRATEGIES – ITEMS 1A, 2B

PRE-TEST:

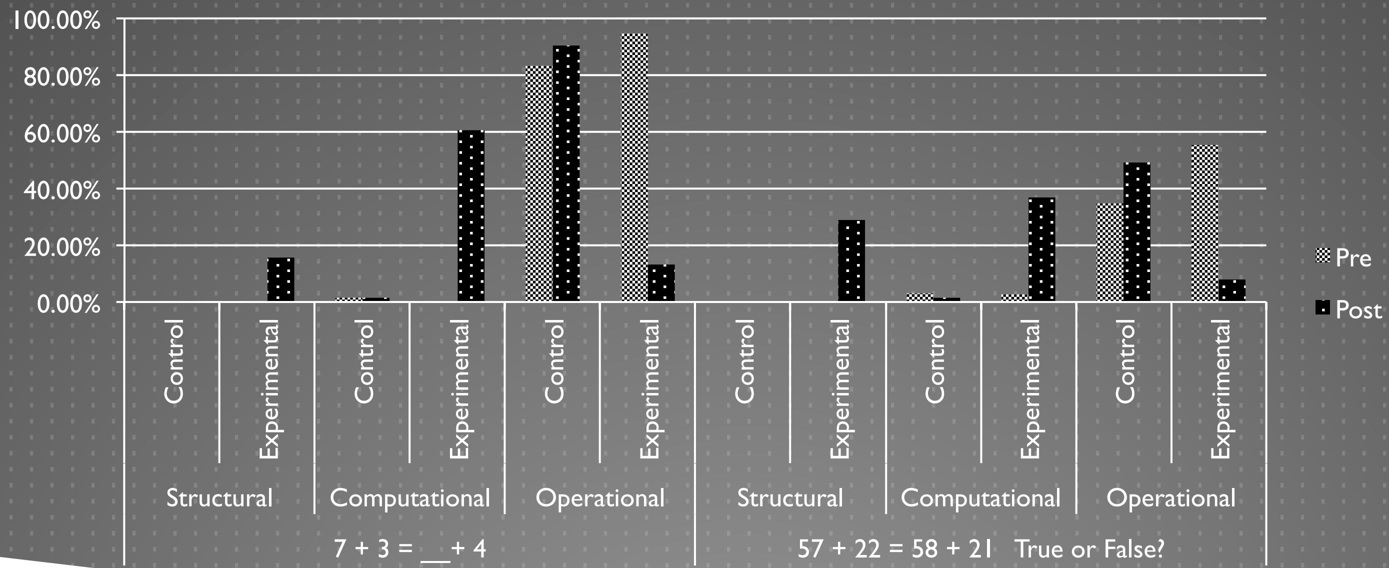
- Both controls and experimentals used primarily an operational strategy at pre-test (95%, 83% for 1a; 55%, 35% for 2b);
- NO students used a structural strategy at pre-test and almost no students used a computational strategy.



3RD-GRADE STUDENTS' STRATEGIES – ITEMS 1A, 2B

POST-TEST:

- For item 1a, 77% of experimentals used either a structural or computational strategy (13% used operational), while only 2 % of controls used a computational strategy and no controls used a structural strategy (90% of controls used an operational strategy).
- For item 2b, 66% of experimentals used either a structural or computational strategy (8% used operational), while only 2% of controls used a computational strategy, and no controls used a structural strategy (50% of controls used an operational strategy).



REPRESENTING UNKNOWN QUANTITIES

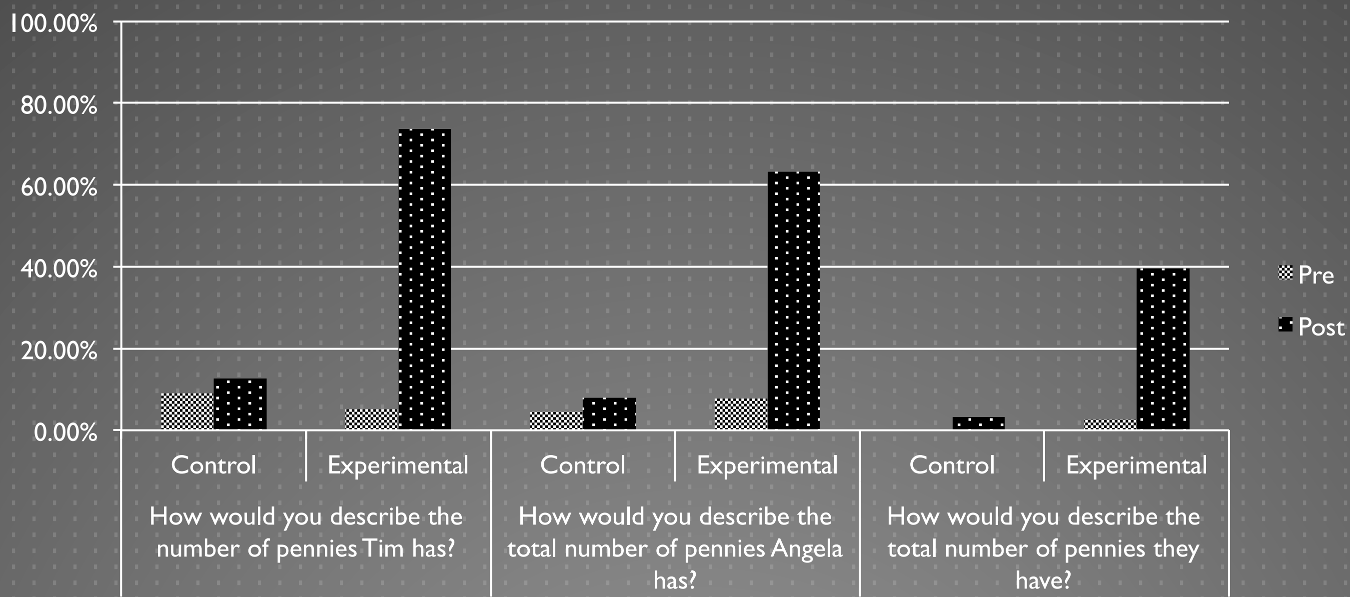
EEEI: ITEM 7

7. Tim and Angela each have a piggy bank. They know that their piggy banks each contain the same number of pennies, but they don't know how many. Angela also has 8 pennies in her hand.

a) How would you describe the number of pennies Tim has?
b) How would you describe the total number of pennies Angela has?
c) Angela and Tim combine all of their pennies to buy some candy. How would you describe the total number of pennies they have?

3RD-GRADE STUDENTS' PERFORMANCE – ITEM 7

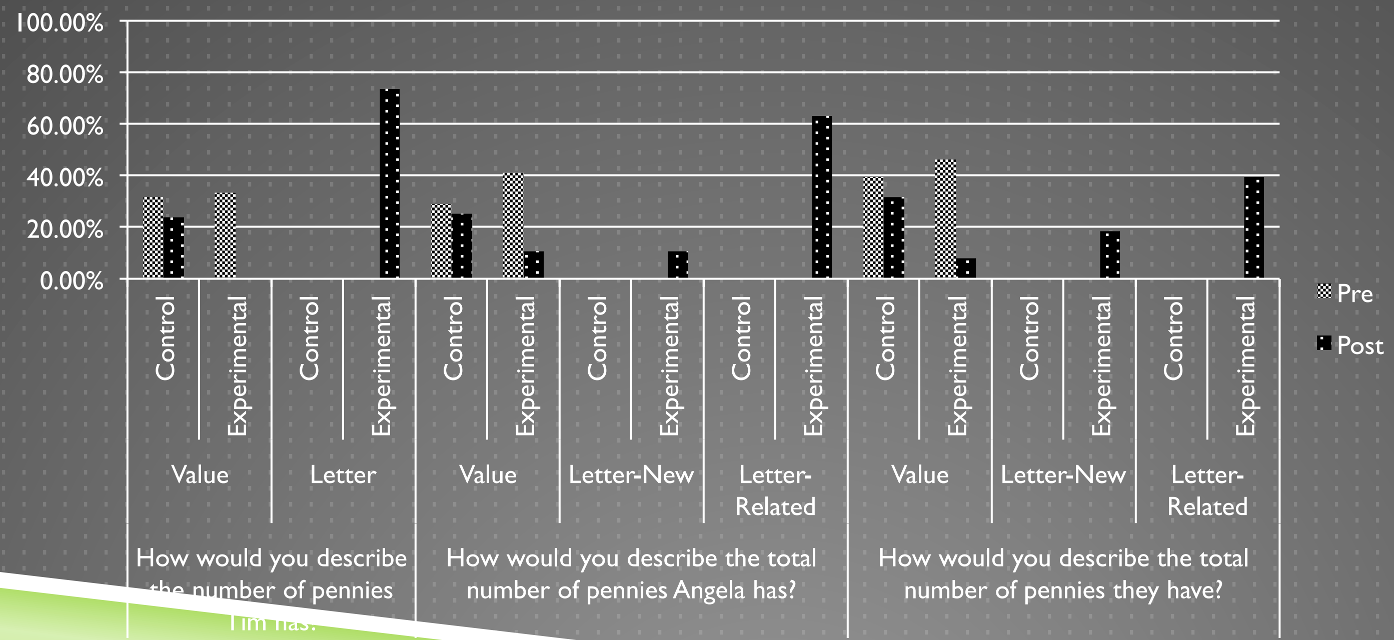
- PRE-TEST: No significant differences between experimental and control for each part of item 7
- POST-TEST: Experimental students significantly outperformed controls at post-test ($p<0.001$ for all 3 items).



3RD-GRADE STUDENTS' STRATEGIES – ITEM 7

PRE-TEST:

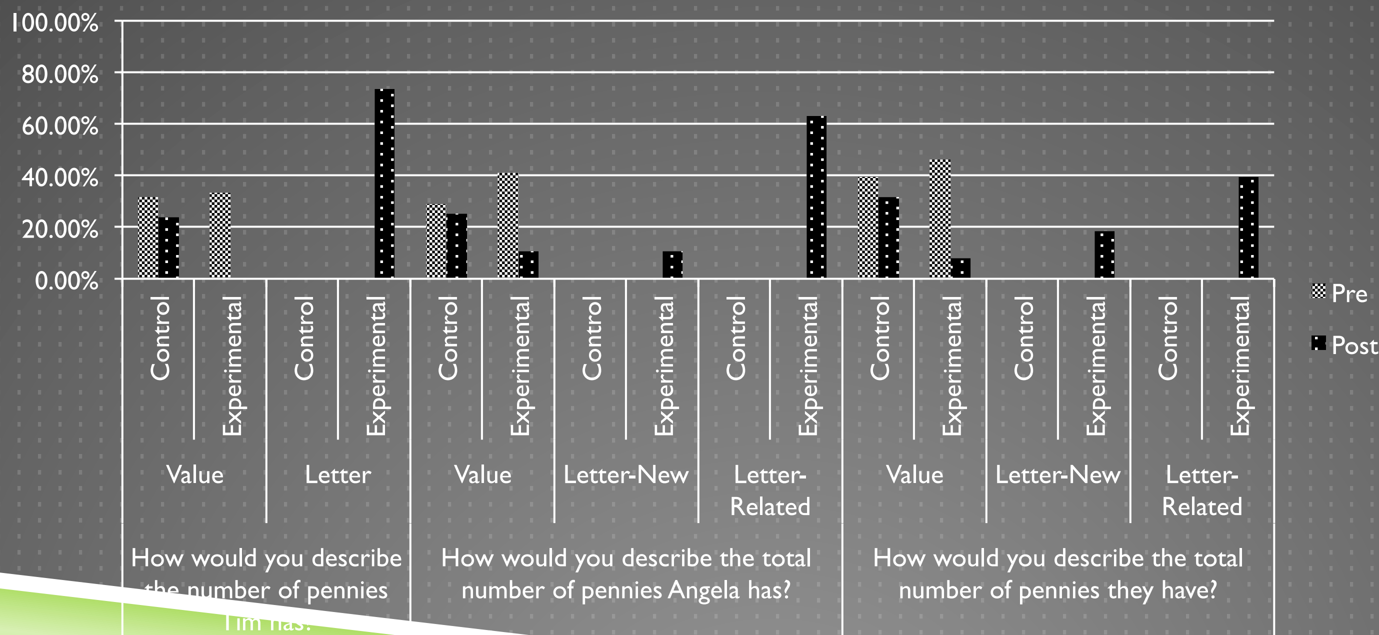
- Neither experimentals nor controls were able to use variables to represent expressions.
- All students could only assign specific numerical values to the quantities.



3RD-GRADE STUDENTS' STRATEGIES – ITEM 7

POST-TEST:

- 7a: No experimental assigned a numerical value and 74% used a variable to represent the quantity; no controls could use variables and could still only assign a numerical value (24%).
- 7b: 74% of experimentals used variable expressions, with 63% related variables; no controls could use variables and could still only assign numerical values (25%)
- 7c: 58% of experimentals used variable expressions, with 40% related variables; no controls could use variables and 32% still assigned numerical values



RECOGNIZING & REPRESENTING ARITHMETIC STRUCTURE

GA – ITEM 4

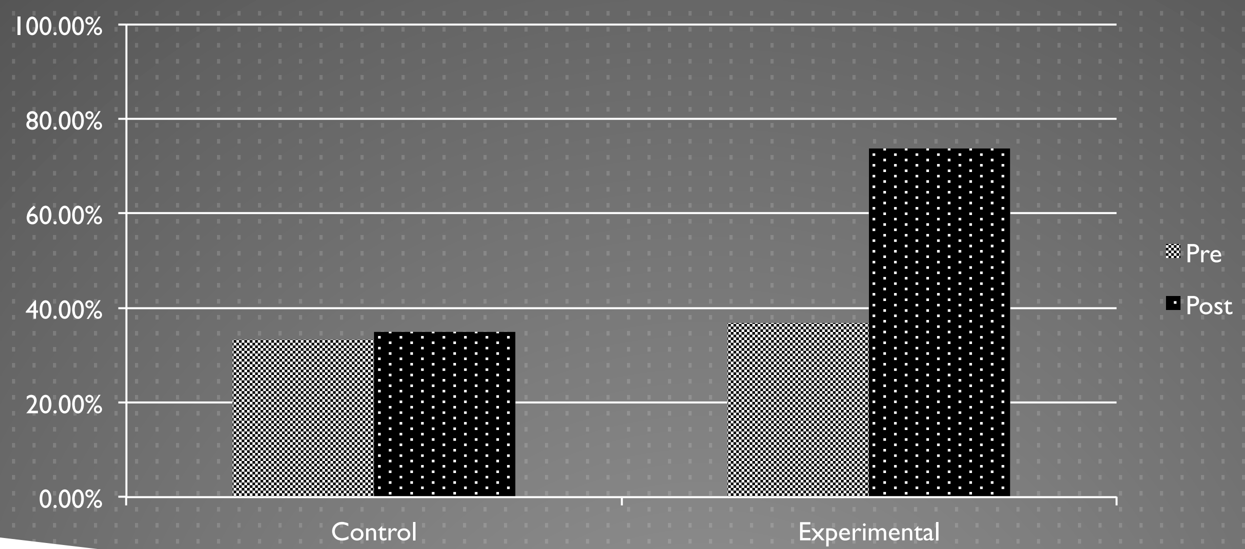
Marcy's teacher asks her to figure out "23 + 15." She adds the two numbers and gets 38. The teacher then asks her to figure out "15 + 23." Marcy already knows the answer.

a) How does she know?

b) Do you think this will work for all numbers?

3RD-GRADE STUDENTS' PERFORMANCE - ITEM 4B

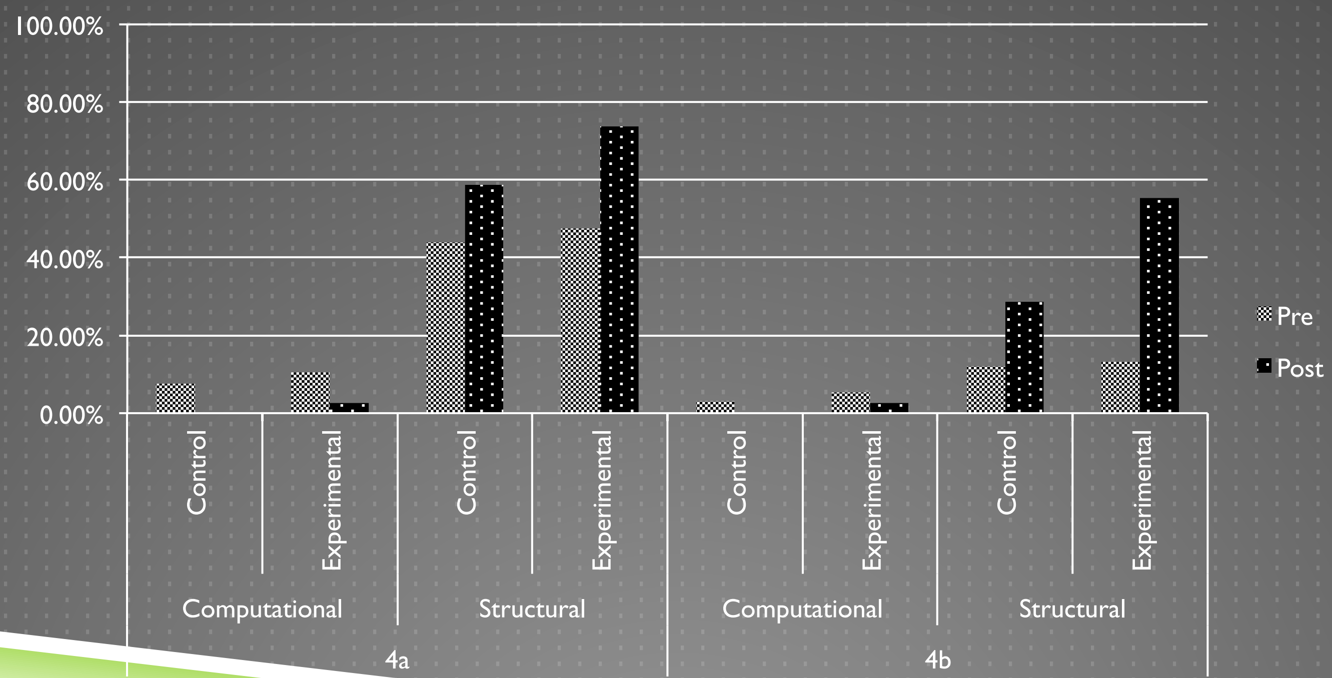
- PRE-TEST: No significant differences between experimental and control on 4b.
- POST-TEST: Experimental students significantly outperformed controls at ($p<0.001$).



3RD-GRADE STUDENTS' STRATEGIES - ITEM 4

POST-TEST:

Experimentals were more likely than controls to use a structural approach, where they recognized the underlying structure (property), as the basis for their argument.



GA – ITEM 6

6. Evelyn computes the following:

8 – 8 = ___   12 – 12 =___

She gets an answer of 0 each time. She starts to thinks that anytime you subtract a number from itself, the answer is 0. Which of the following best describes her thinking? Circle your answer.

a) $a + 0 = 0$

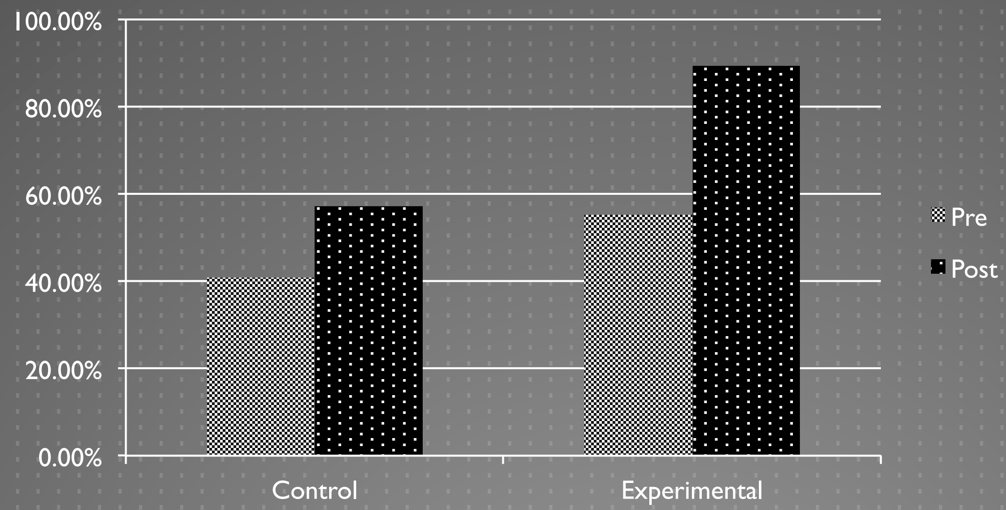
b) $a = b + a + b$

c) $a - a = 0$

d) $a \times 0 = 0$

3^{RD}-GRADE STUDENTS' PERFORMANCE – ITEM 6

- PRE-TEST: No significant differences between experimental and control.
- POST-TEST: Experimental students significantly outperformed controls ($p<0.001$).
- In addition to recognizing the structure of a fundamental property, this suggests that experimentals were better able to interpret equations with variable expressions and navigate between representations (natural language and symbolic notation)

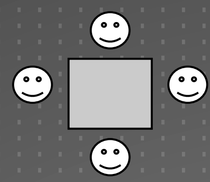


RECOGNIZING AND REPRESENTING CO-VARYING RELATIONSHIPS

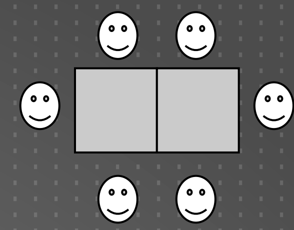
FT-ITEM 10

10. Brady is having his friends over for a birthday party. He wants to make sure he has a seat for everyone. He has square tables.

He can seat 4 people at one square table in the following way:



If he joins another square table to the first one, he can seat 6 people:



a) If Brady keeps joining square tables in this way, how many people can sit at:

3 tables? 4 tables? 5 tables? Record your responses in the table below and fill in any missing information:

b) Do you see any patterns in the table? Describe them.

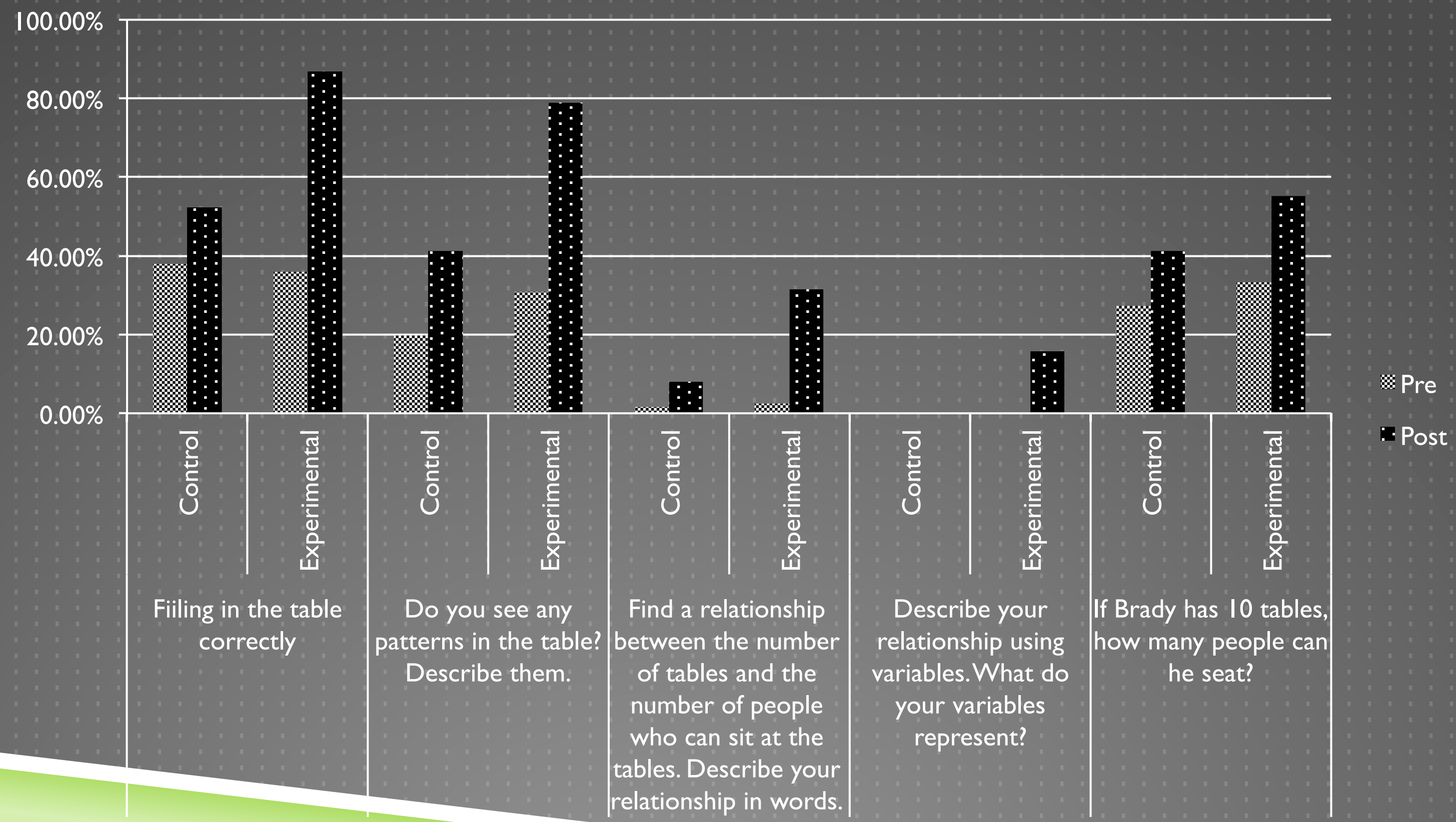
c) Find a rule that describes the relationship between the number of tables and the number of people who can sit at the tables. Describe your rule in words.

d) Describe your relationship using variables. What do your variables represent?

e) If Brady has 10 tables, how many people can he seat? Show how you got your answer.

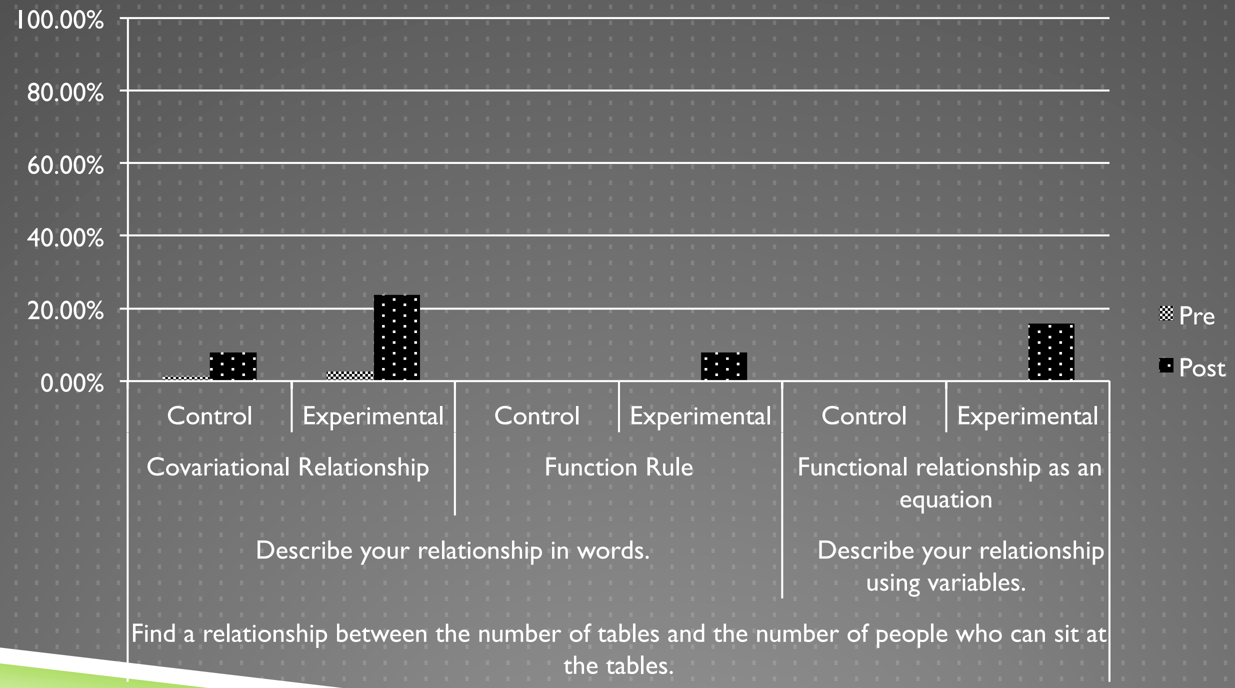
3RD-GRADE STUDENTS' PERFORMANCE – ITEM 10

- PRE-TEST: No significant differences between experimental and control.
- POST-TEST: Experimental students <u>significantly</u> outperformed controls on items 10a-d (resp., $p<0.001$, $p<0.001$, $p<0.01$, and $p<0.05$); outperformed controls on 10e, but not significantly.



3RD-GRADE STUDENTS' STRATEGIES – ITEMS 10C, 10D

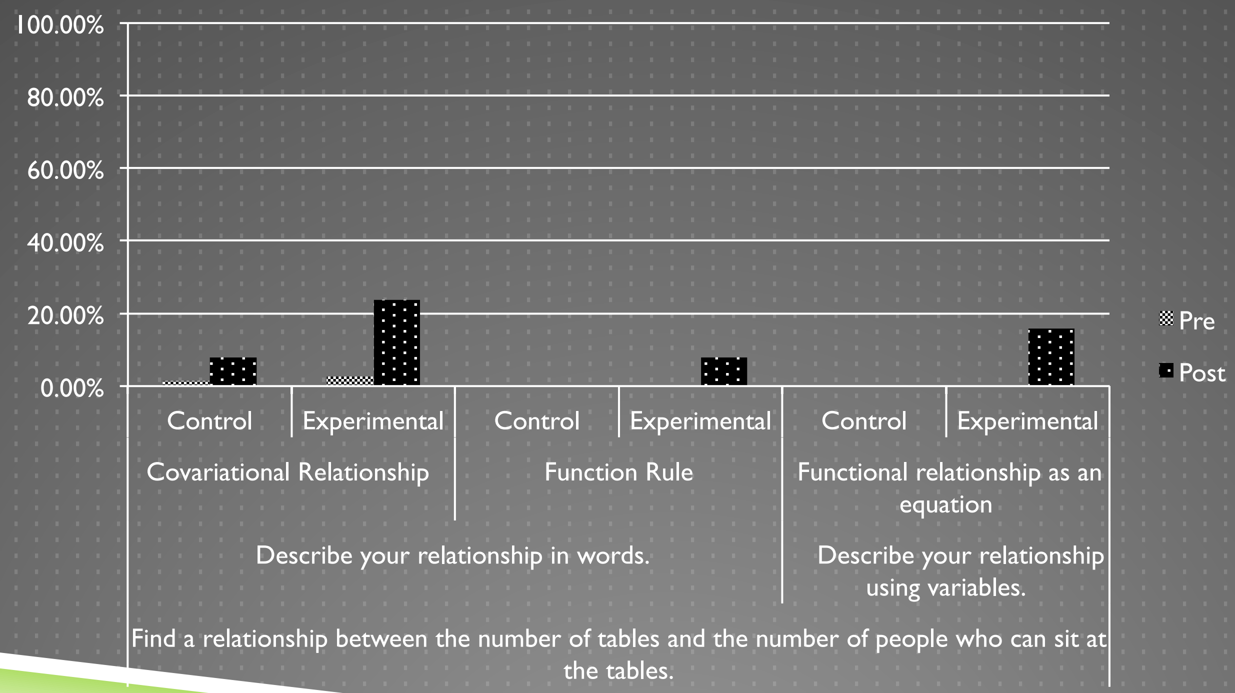
- PRE-TEST: Only 3% of experimentals and 2% of controls could describe a co-varying relationship; no students could identify a function rule in words or symbols;



3RD-GRADE STUDENTS' STRATEGIES – ITEMS 10C, 10D

POST-TEST:

- 10c: 24% of experimentals could provide a co-varying relationship and 8% could provide a function rule in words; 8% of controls could provide a co-varying relationship and no controls could describe a functional rule in words.
- 10d: 16% of experimentals could describe a function rule in variables; no controls could.



SOME CONCLUSIONS…

- Significant gains in experimentals' ability to correctly interpret and to think relationally about the equal sign. Controls continued to think primarily operationally and showed no evidence of the development of a relational understanding of the equal sign.
- Significant gains in experimentals' ability to represent unknown quantities with variable expressions. Moreover, the majority of experimentals were able use variables in meaningful ways to represent different unknown quantities. No controls were able to use variables in any way pre or post.
- Experimentals were more likely to recognize the underlying structure of fundamental properties and use this as a basis for justifying generalizations on a domain of numbers.

SOME CONCLUSIONS…

- Experimentals were more likely to understand that a generalization might hold over a broad domain of numbers, not just a particular instance.
- Experimentals were better able to interpret equations with variable expressions and navigate between representations (natural language and symbolic).
- Not only did experimentals significantly outperform controls in their ability to identify and describe function rules in words or variables, they were more likely to choose variables to represent their rule.
- Experimental students increasingly used strategies that reflected more algebraic (structural) approaches, while control students did not.

SOME CONCLUSIONS…

In one year, in less than one day of a students' life, we can statistically significantly improve children's algebraic understanding

GROWING TRAIN

WHAT DO YOU THINK?

- In terms of what you think we should expect of children?
- In terms of what mathematical ideas are important to development in children's thinking (e.g., variable, co-varying relationships) and <u>when</u> those ideas should be developed?

QUESTIONS?

- Maria_Blanton@terc.edu