Using Non-Routine Problems to Build Reasoning and Sense Making in Grade 4-8 Students

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Crazy Eights

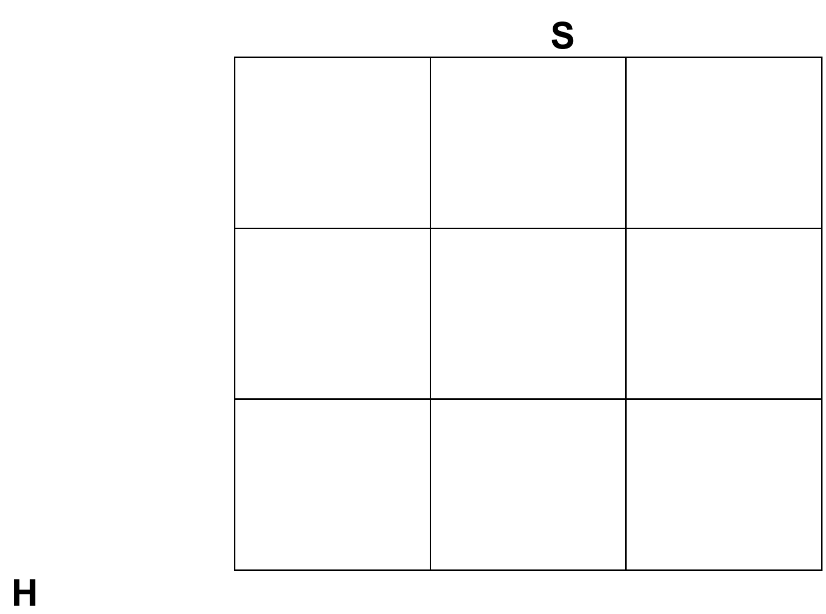
Problem: How many ways are there to use 8s strung together with plus (+) signs to equal 1000? Once you think you have an answer, can you provide proof that you are correct?

Here is **one** way to do it. You are asked **how many** ways there are to do it.

888 + 8 + 8 + 8 + 8 + 8 + 8 + 8 + 8 + 8 + 8 + 8 + 8 + 8 + 8 = 1000

Mathville

Downtown Mathville is laid out as a 3 x 3 square grid of streets (see diagram). Your home is located at the Southwest corner of downtown Mathville (see point H). Your school is located at the Northeast corner of downtown Mathville (see point S). You know that it is a 6 block walk to school and that there is no shorter path. Your curious classmate (we'll call her Curious Georgia) asks how many different paths (of exactly 6 blocks- you don't want to backtrack or go out of your way) could you take to get from your home to school. Can you solve Curious Georgia's math problem?



The Locker Problem

The Problem: In a certain school there are 50 lockers lining a long hallway. The lockers are numbered 1, 2, 3,…49, 50. **All of the lockers are closed**. Suppose that 50 students walk down the hall in single file, one after another. Suppose the first student (who we will call "Student 1") opens every locker. The second student ("student 2") comes along and closes every second locker (i.e. locker #2, 4, 6,…48, 50). Along comes Student #3 who **changes** the position of every third locker; if the locker is open, the student closes it, if it is closed this student opens it (i.e. the student closes locker #3, opens locker #6, closes locker #9, etc). Student #4 changes the "open or shut" position of every fourth locker, and so forth until the 50^{th} student changes the position of locker #50.

Which lockers are open at the end of this event?

Questions to Answer:

*How do you know your answer is correct? How could you prove it to someone else? (and could you do it without having to draw a picture or use a manipulative?)

*Could you extend this problem to figure out the answer if we had **100 lockers** and 100 students walked by and changed the door positions? Which lockers would be open?

*Challenge: Can you explain *why* these particular numbers are the numbers of the lockers that are open at the end of the event?

Crossing the River

A group of adults go on a camping trip with a group of 5th and 6th grade students. On the first day the campers come to a river. It's not a very wide river but it is too deep to wade across. Fortunately, the campers find a boat. Unfortunately, the boat is not very big. Even more unfortunately, the adults are rather big and only one adult can fit in the boat at one time. Fortunately, the students are quite small, small enough that the boat will hold any two of the students. Also fortunately, the students have experience boating and each can safely row across the river by themselves.

a) Suppose that there are four adults and two students on the camping trip. Is it possible to get the entire camping group across the river? If yes, how many one-way trips across the river will it take to get all six people across?

b) What if there were five adults and only one child on the trip? Is it possible to get the entire group across the river? If yes, how many one-way trips would it take?

c) What if there were five adults and two children?

d) What if there were 4 adults and 6 children?
e) How can this problem be generalized? Can you solve the generalized problem or if there is no generalized solution, can you solve any special cases?"

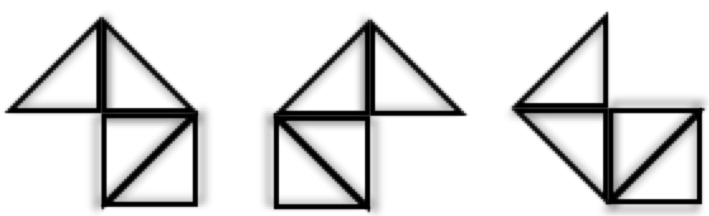
(Three) Four Weights

Using only 3 weights, is it possible to weight all of the unit amounts between 1 and 10 on a two-pan balance? For example, if we had weights of 7 and 8, we could put the 7 in the one pan and the 8 in the other, giving us a net weight of 1 (the difference between the total weights in the two pans is 1). You do not need to use all 3 weights to create a particular net weight. If it is possible, what are the 3 weights?

Extension**:** Could you create all of the unit amounts between 1 and 30 using 4 weights? Would the same 3 weights still work (and just add a 4th weight) or would the other weights have to change?

Four Triangles

Given 4 congruent isosceles right triangles, how many unique shapes can you make, using all 4 triangles each time? Two shapes are the same if some combination of slides, flips or turns will transform the 1st shape into the 2nd. For example, these shapes are not unique:



The sides of each triangle should fit *exactly* with the side of an attached triangle. The legs have to match up with the legs, the hypotenuse has to match up with the hypotenuse. For example, this would not be considered a legal shape:

The Water Jug Problem

Bruce Willis and Samuel Jackson have a 3-gallon jug and a 5-gallon jug, and an unlimited supply of water. If there are no markings on the jugs and they needed to get *exactly* four gallons of water to stop the bomb from going off, explain how they could do so. Try to use a few steps as possible- remember there is a limited amount of time before the bomb explodes!

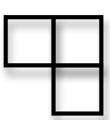
Questions to consider:
*Is it possible to measure exactly 4 gallons of water if there are no markings on the 3-gallon and 5-gallon jugs? Why do you think this?
*How can you use drawings and words to model your thinking and solution?
*How would you know if your solution (if one exists) uses the smallest number of steps possible (times the water is poured back and forth)? How could you prove it?

Extension: What if we were given a 5-gallon jug and a 7-gallon jug with no markings? Could we make exactly one gallon? Two? Three? How many different gallon measurements could we create using these two jugs?

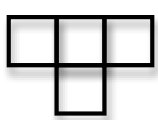
Pentaminos

A domino can be thought of as two squares joined along one side. Similarly, a triomino, might be a polygon formed by joining three squares together (in each case, two matching sides must fit together exactly). If we continue in this way, a **pentomino** can be created by joining five squares together to form a polygon. **Two pentominos are the same if one can be shifted, rotated and/or flipped to fit exactly onto one of the other pentominos**.

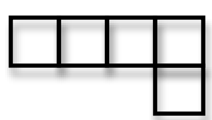
Triomino



Tetromino



Pentomino



How many pentominos are there? Can you convince yourself and your group that there are *exactly* that many, that they are all different, and that there are no more?

Four Fours

Use the symbol 4 exactly four times to write number sentences equivalent to the numbers 1 through 20. You may use any of the mathematical symbols listed below. A few examples are provided.

- Arithmetic operations: + - x ÷
- Parentheses: ( ) * Remember we do what is inside the parentheses **first**, parentheses mean that piece is a priority.
- Fractions: $\frac{4}{4}$ which is equal to one whole
- Exponents: 4^4 which equals 4 x 4 x 4 x 4, or you could try using the fraction $\frac{4}{4}^4$ which is really the same as 1^4 (and remember that would be 1 x 1 x 1 x 1)
- Square root: $\sqrt{4}$ which equals 2. *A square root is the opposite of a square number. 4 is a square because its factor 2 can be multiplied by itself to equal 4. If "two squared" equals 4, then the square root of 4 is 2.
- Factorial: 4! Which is equal to 4 x 3 x 2 x 1
- A multi-digit number: such as 44 or 444

Number	Four Fours Equation	Number	Four Fours Equation
1	$(\frac{4}{4})^{44} = 1$	11	
2		12	
3		13	
4		14	$\sqrt{4} + (4x4) - 4 = 14$
5		15	
6		16	
7		17	
8		18	
9		19	
10	$\frac{4!}{4} + \sqrt{4} + \sqrt{4} = 10$	20	

One Grain of Rice

The Indian raja promised to reward Rani the village girl for her good deed with anything she would like. She asks for a single grain of rice, then each day for 30 days double the number of grains she had received the day before. By the 30th day, how many grains of rice will Rani have received as her reward?

*Questions to answer (**in complete sentences**):*

1) How many total grains of rice will Rani receive by the end of 30 days? Show how you figured this total (either in words, a diagram or by showing mathematically)?
2) What type of transportation would Rani need to move this much rice? Could she collect it in a basket? A small cart? How many trips would it take to move it from the palace?
3) Where would Rani be able to store this much rice? Estimate how big of a container she would need.
4) Do you think the raja would have accepted Rani's proposal for a reward if he had done the math ahead of time? Why or why not?

Other Resources

Burns. M. (2007). *About teaching mathematics: A K-8 resource*. Sausalito: Math Solutions.
National Council of Teachers of Mathematics website www.nctm.org
Illustrative Mathematics www.illustrativemathematics.org

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