

Teaching and Assessing Students' Ability to Create and Evaluate Representations

November 30, 2012

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Two Rivers Public Charter School

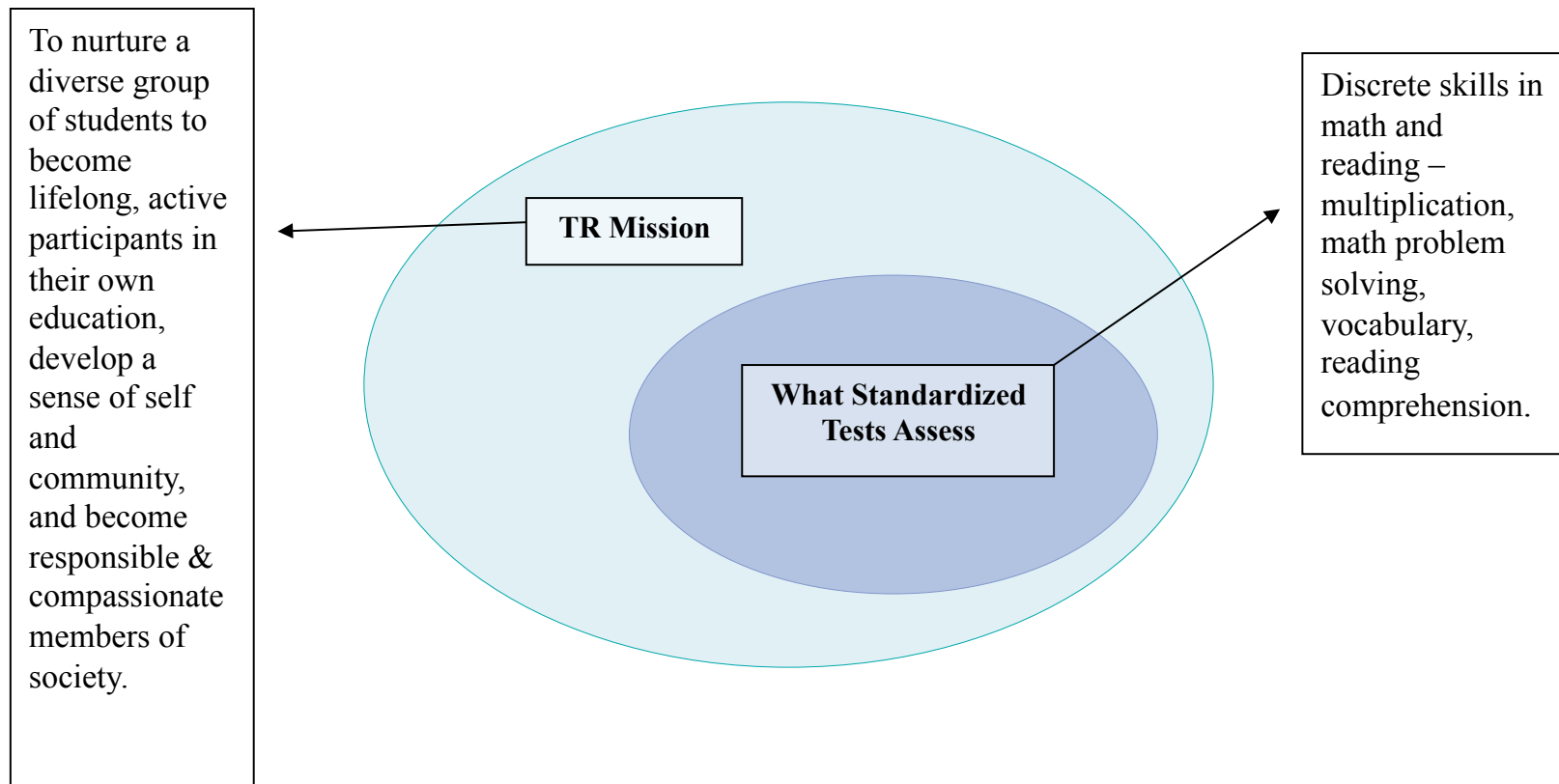
Washington, DC

Our Mission

- To nurture a diverse group of students to become lifelong, active participants in their own education, develop a sense of self and community, and become responsible and compassionate members of society.



What does our data NOT tell us?



An Expanded Vision of Data

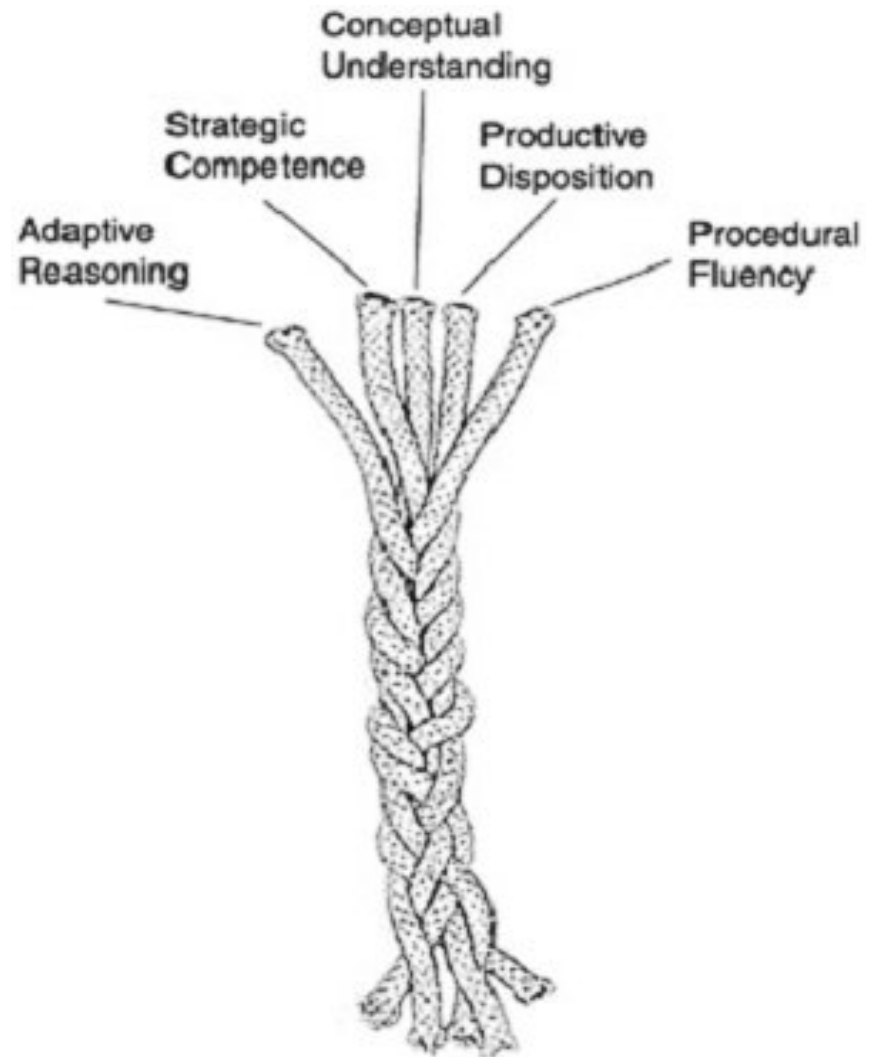
- Data is any evidence that can be collected that can be used to determine progress towards goals.
- Data includes:
 - Standardized Test Data (i.e. ANET, MAP, DC CAS, Dibels etc.)
 - Classroom Assessments (i.e. tests, quizzes, exit tickets)
 - Anecdotal Notes
 - Student Work

An Expanded View of Educational Outcomes

What does math look like at Two Rivers?

- Mathematics at Two Rivers emphasizes *computational and procedural skills, conceptual understanding, and problem solving.*
- We place a special emphasis on developing expert thinking and complex communication skills that go beyond basic algorithmic competency in math.

Intertwined Strands of Proficiency from *Adding + It Up: Helping Children Learn Mathematics* (2001) from the National Research Council



Expert Thinking

- **Strong Fund of Knowledge**-facts and discrete basic skills (factual knowledge)
- **Schema** – mental structures that represent concepts by organizing and linking various facts and knowledge (conceptual knowledge)
- **Pattern Recognition** – experts recognize meaningful patterns of information that are not noticed by novices (procedural knowledge)
- **Metacognition** – Reflecting on a problem- solving process and deciding when to change course (metacognitive knowledge)

Goals of Our Mathematics Program

- Develop conceptual understanding of the “big ideas” in each strand of mathematics
- Develop critical thinking skills and habits of mind that can be applied to any problem in and outside of mathematics
- Concisely and precisely communicate mathematical ideas
- Develop efficiency with number and computation

How do we get our students to
achieve these expanded
outcomes in our classrooms?

Problem-Based Tasks

“Most, if not all, important mathematics concepts and procedures can best be taught through problem solving.”

- John Van de Walle

Teaching Student-Centered Mathematics

Problem Solving from Student Centered Mathematics

- A problem is defined here as any task or activity for which the students have no prescribed or memorized rules, nor is there a perception by students that there is a specific correct solution method (Hiebert et al., 1997)
- Problems must begin where the students are.
- The problematic or engaging aspect of the problem must be due to the mathematics that the students are to learn.
- The problem must require justifications and explanations for answers and methods.

COMMON EXPECTATIONS

- **Math is fun:** Time is built into lessons to play with math.
- **All students can learn math:** Every student has the capacity to learn deeper conceptual knowledge in mathematics.
- **A common approach to problem solving:** We use the K-W-I to teach steps to problem solving in and outside of math.
- **Representations:** We value the multiple ways that ideas can be modeled or demonstrated and encourage making connections between various representations.
- **Concise and precise communication:** We reinforce precision and brevity during class discussion.
- **Reasoning and proof:** Mathematical arguments are weighed on the merits of their logic NOT on the status of the speaker or beauty of the language.

What is critique?

Defining Critique: A Reading from Ron Berger's *An Ethic of Excellence*

- According to the author, what does he mean by critique in a classroom?
- Why is it critical to define the areas of focus with the students?
- What norms must be present for the critique protocol to be effective in your classroom?

Steps of the Whole Group Exemplary Critique

- Before the critique session:
 - Teacher(s) determines 2-3 areas of focus for critique ahead of time.
 - Teacher(s) preps a 2-3 column chart to record brief definition of the areas of focus and to collect student ideas
 - Teacher(s) selects high-quality exemplars (1-2)

Steps of the Whole Group Exemplary Critique

- During the critique session:
 - Introduce the goals of the critique and review rules for critique
 - Put the 2-3 areas of focus on the board in columns
 - Teacher gives brief definition of the areas of focus
 - Students individually read the exemplars
 - Students individually fill out the columns with what they see as noteworthy in the exemplars
 - Whole class engages in discussion about what they see as noteworthy in the exemplars related to the areas of focus

Steps of the Whole Group Exemplary Critique

- After the critique session:
 - Teacher organizes and formats the student-generated criteria for future use by students (poster, checklist, rubric, etc.)
 - Teacher uses student ideas to add to the rubric for more specificity

An Example of An Exemplary Critique

The Problem

- Girls on Track signed up to run a relay race with Coach Erica. Sayla ran $\frac{2}{3}$ of a mile, Cassidy ran $\frac{5}{6}$ of a mile, and Mia ran $\frac{1}{4}$ of a mile. As a team, how far did they run?
- Solve the problem and create a representation that other 5th graders will be able to use to understand your solution to the problem.

Student #1

Name ~~XXXXXXXXXX~~

Date 3/29/12

Fraction Math Task #3- Revision

Learning Targets: I can add and subtract fractions with unlike denominators. I can clearly communicate my thinking and explain a solution to a math problem.

Problem:

Girls on Track signed up to run a relay race with Coach Erica. Sayla ran $\frac{2}{3}$ of a mile, Cassidy ran $\frac{5}{6}$ of a mile, and Mia ran $\frac{1}{4}$ of a mile. As a team, how far did they run? In the space below, solve the problem and create a model that explains the solution.

$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$
$\frac{1}{3}$		$\frac{1}{3}$		$\frac{1}{3}$	
$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$		

$\boxed{E} = \frac{5}{12} = (\text{LCM}=12) \frac{3}{12} + \frac{4}{12} + \frac{10}{12} = \frac{17}{12}$

K-I know that I have $\frac{5}{6}, \frac{1}{4}, \frac{2}{3}$. I need to add them all together.
W-The answer to $\frac{5}{6} + \frac{2}{3} + \frac{1}{4}$
I-Add them together using the LCM method

Student #2

Name ██████████ Date 3-28-12

Fraction Math Task #3- Revision

Learning Targets: I can add and subtract fractions with unlike denominators. I can clearly communicate my thinking and explain a solution to a math problem.

Problem:

Girls on Track signed up to run a relay race with Coach Erica. Sayla ran $\frac{2}{3}$ of a mile, Cassidy ran $\frac{5}{6}$ of a mile, and Mia ran $\frac{1}{4}$ of a mile. As a team, how far did they run? In the space below, solve the problem and create a model that explains the solution.

Sayla	$\frac{1}{12}$	$\frac{2}{12}$	$\frac{3}{12}$	$\frac{4}{12}$	$\frac{5}{12}$	$\frac{6}{12}$	$\frac{7}{12}$	$\frac{8}{12}$	$\frac{9}{12}$	$\frac{10}{12}$	$\frac{11}{12}$	$\frac{12}{12}$
Cassidy	████	████	████	████	████	████	████	████	████	$\frac{11}{12}$	$\frac{12}{12}$	
Mia	████	████	████	$\frac{4}{12}$	$\frac{5}{12}$	$\frac{6}{12}$	$\frac{7}{12}$	$\frac{8}{12}$	$\frac{9}{12}$	$\frac{10}{12}$	$\frac{11}{12}$	$\frac{12}{12}$

$\frac{2}{3} + \frac{5}{6} + \frac{1}{4} =$
 $\frac{8}{12} + \frac{10}{12} + \frac{3}{12} = \frac{21}{12}$
 \downarrow
 $1 \frac{9}{12}$
 \downarrow
 $1 \frac{3}{4}$

Student #3

Name [REDACTED] Date March 28, 2012

Fraction Math Task #3- Revision

Learning Targets: I can add and subtract fractions with unlike denominators. I can clearly communicate my thinking and explain a solution to a math problem.

Problem:

Girls on Track signed up to run a relay race with Coach Erica. Sayla ran $\frac{2}{3}$ of a mile, Cassidy ran $\frac{5}{6}$ of a mile, and Mia ran $\frac{1}{4}$ of a mile. As a team, how far did they run? In the space below, solve the problem and create a model that explains the solution.

<p><u>K</u></p> <ul style="list-style-type: none"> Cassidy ran $\frac{5}{6}$ mi Sayla ran $\frac{2}{3}$ mi Mia ran $\frac{1}{4}$ mi 	<p><u>Key</u></p> <p>C = Cassidy M = Mia S = Sayla</p>												
<p><u>W</u></p> <p>As a team how far did the run? or All together how far did the group run.</p>	<table border="1"> <tr> <td>S =</td> <td>$\frac{1}{3} \times \frac{4}{4} = \frac{4}{12}$</td> <td>$\frac{2}{3} \times \frac{4}{4} = \frac{8}{12}$</td> <td>$\frac{3}{3} \times \frac{4}{4} = \frac{12}{12}$</td> </tr> <tr> <td>C =</td> <td>$\frac{1}{6} \times \frac{2}{2} = \frac{2}{12}$</td> <td>$\frac{2}{6} \times \frac{2}{2} = \frac{4}{12}$</td> <td>$\frac{3}{6} \times \frac{2}{2} = \frac{6}{12}$</td> </tr> <tr> <td>M =</td> <td>$\frac{1}{4} \times \frac{3}{3} = \frac{3}{12}$</td> <td>$\frac{2}{4} \times \frac{3}{3} = \frac{6}{12}$</td> <td>$\frac{3}{4} \times \frac{3}{3} = \frac{9}{12}$</td> </tr> </table>	S =	$\frac{1}{3} \times \frac{4}{4} = \frac{4}{12}$	$\frac{2}{3} \times \frac{4}{4} = \frac{8}{12}$	$\frac{3}{3} \times \frac{4}{4} = \frac{12}{12}$	C =	$\frac{1}{6} \times \frac{2}{2} = \frac{2}{12}$	$\frac{2}{6} \times \frac{2}{2} = \frac{4}{12}$	$\frac{3}{6} \times \frac{2}{2} = \frac{6}{12}$	M =	$\frac{1}{4} \times \frac{3}{3} = \frac{3}{12}$	$\frac{2}{4} \times \frac{3}{3} = \frac{6}{12}$	$\frac{3}{4} \times \frac{3}{3} = \frac{9}{12}$
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C =	$\frac{1}{6} \times \frac{2}{2} = \frac{2}{12}$	$\frac{2}{6} \times \frac{2}{2} = \frac{4}{12}$	$\frac{3}{6} \times \frac{2}{2} = \frac{6}{12}$										
M =	$\frac{1}{4} \times \frac{3}{3} = \frac{3}{12}$	$\frac{2}{4} \times \frac{3}{3} = \frac{6}{12}$	$\frac{3}{4} \times \frac{3}{3} = \frac{9}{12}$										
<p><u>I</u></p> <p>Make all fractions have a like denominators, then add.</p>	$\left(\frac{10}{12} + \frac{8}{12} \right) + \frac{3}{12} = \frac{21}{12}$ $\frac{21}{12} \div \frac{3}{3} = \frac{7}{4} = \boxed{1\frac{3}{4}}$												

Three Areas of Focus

Organization	Accuracy	Representation

Common Core State Standards for Mathematical Practices: Model with Mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Design of Task 1

Provided students with a table with locations and distances from a given point. Students asked to place the locations on a number line

But...

The data we got back wasn't what we expected!

- Lots of data on the content being taught
- Little to no data on their ability to create a model



to the Drawing Board

What do we do when we model with mathematics?

- Create
- Compare
- Evaluate

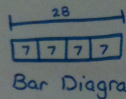
Next, we developed learning targets

- ◎ I can create representations that model a math concept and my thinking
- ◎ I can compare representations
- ◎ I can evaluate representations for how well they model a problem situation (and solution)

Different Ways to Represent Math

- A representation is a visual model of a math concept, procedure, or way of thinking.
 - idea behind or guiding the math
 - specific steps taken, organized method for solving problems
 - the way you understand, figure out, or try to solve the problem in your own mind
- The same math concept can be represented in different ways.

example) Multiplication



or $7 \times 4 = 28$
Equation

- Some representations show math concepts better than others.

Representations we use in class:

1. Number lines
2. Bar diagrams
3. Place value blocks
4. Equations
5. Draw a picture

$B = 2$ wheels
 $T = 3$ wheels

Very organized and clear model!

Tricycles: 3 6 9 12
1 2 3 4

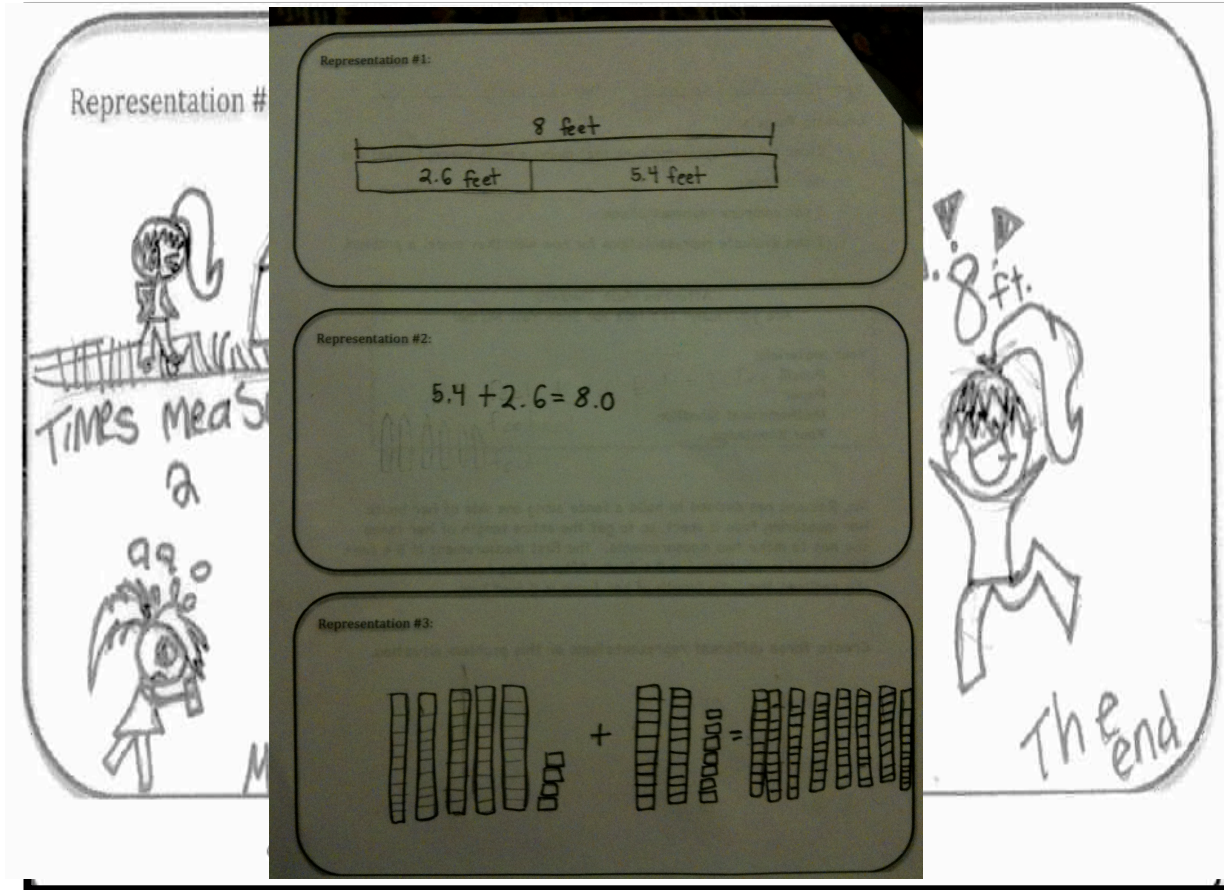
Bikes: 2 4 6 8 10 12
1 2 3 4 5 6

Six wheels + Six wheels =

12

6 6

Teaching Time



© Students asked to create more than 1 representation

© Provided Solution to the Calculation

© New Reflection Questions

Creation of Task #2

Rubric Development

Create

- Accurate
- Organized
- Understood by Others

Translate

- Efficient
- Connections between multiple representations

Evaluate

- Relationship between the representation and the mathematical situation

Facilitating Student Reflections

- There must be time for reflection for students
- Students should be held accountable for identifying where they will apply what they learned from a critique protocol in their own work
- Students need opportunities to apply their learning as soon as possible

Questions?