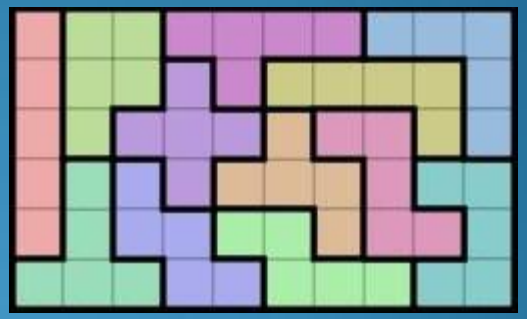
Cutting to the Core: Tangrams and Pentominoes



2012 NCTM Regional Conference Chicago

Annemarie Newhouse

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- What did the little acorn say to the BIG oak tree when she grew up?

Gee, I’m a tree ☺ Geometry!

Pentominoes and Tangrams

- Today we will look at investigating geometry and fractions through tangrams and pentominoes, keeping in mind the NCTM Process and Content Standards and the Common Core State Standards, including Mathematical Practices.

Common Core Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.

I am learning to:

- Make sense of problems
- Make a plan for solving problems
- Try different strategies, even when a problem is hard
- Solve a problem in more than one way
- Check whether a solution makes sense
- Find connections between mathematical ideas

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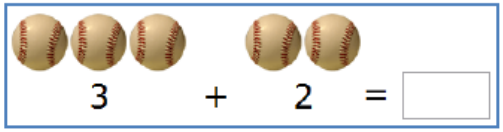
2. Reason abstractly and quantitatively.



I am learning to:

- Represent problem situations with objects, pictures, numbers, words, and symbols
- Explain to others the meanings of objects, pictures, numbers, words, and symbols

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3. Construct viable arguments and critique the reasoning of others.

I am learning to:

- Explain both what to do and why it works
- Make sense of others' mathematical thinking

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4. Model with mathematics.

I am learning to:

- Solve real-world problems using mathematics
- Use models such as graphs, drawings, tables, symbols, and diagrams to solve problems

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Common Core Standards for Mathematical Practice

5. Use appropriate tools strategically.

I am learning to:

- Choose appropriate tools to solve problems
- Use mathematical tools correctly and efficiently
- Estimate an answer before using a tool

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6. Attend to precision.

I am learning to:

- Clearly communicate my thinking to others
  - *Speak*, *Read*, *Write*, and *Listen* mathematically
- Decide whether an estimate or exact answer is needed
- Be accurate when I count, measure, and compute

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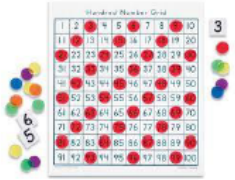
7. Look for and make use of structure.



I am learning to:

- Notice, continue, and create patterns
- Use patterns to solve problems

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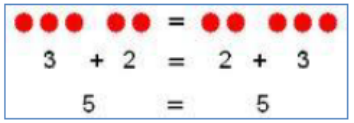
8. Look for and express regularity in repeated reasoning.



I am learning to:

- Use patterns to create and explain rules and shortcuts
- Use rules and properties of number to more easily solve problems
- Reflect on my thinking before, during, and after solving a problem

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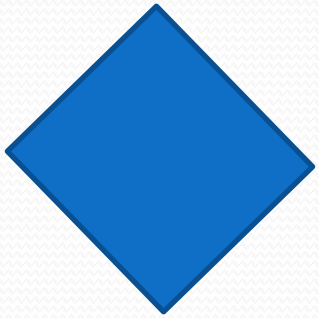
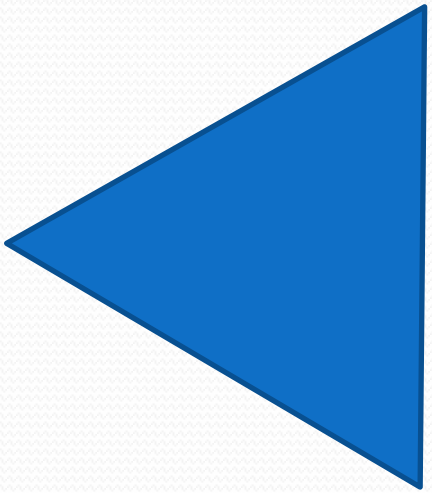
What did the Pirate say when his parrot flew away?

Poly gon…



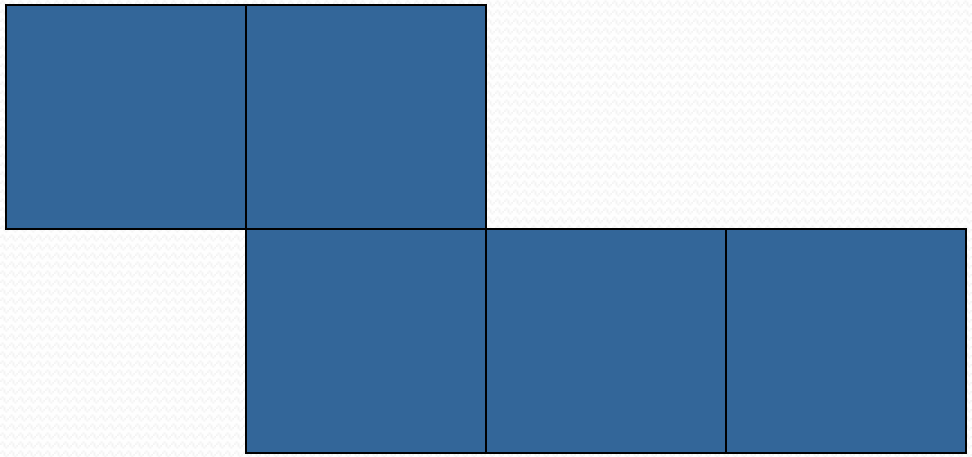


Polyominoes

- made with adjacent squares that share a common edge.
- Monominos using 1 square
- Dominoes using 2 squares
- Triomino using 3 squares
- Tetrominoes using 4 squares
- Pentominoes using 5 squares
- Hexominoes using 6 squares

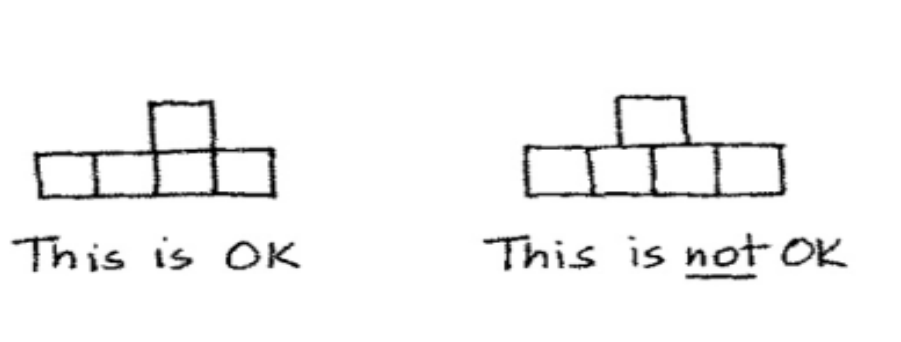
PENTOMINOES

are a type of polygon/ polymino that uses **FIVE** square blocks joined edge to edge.



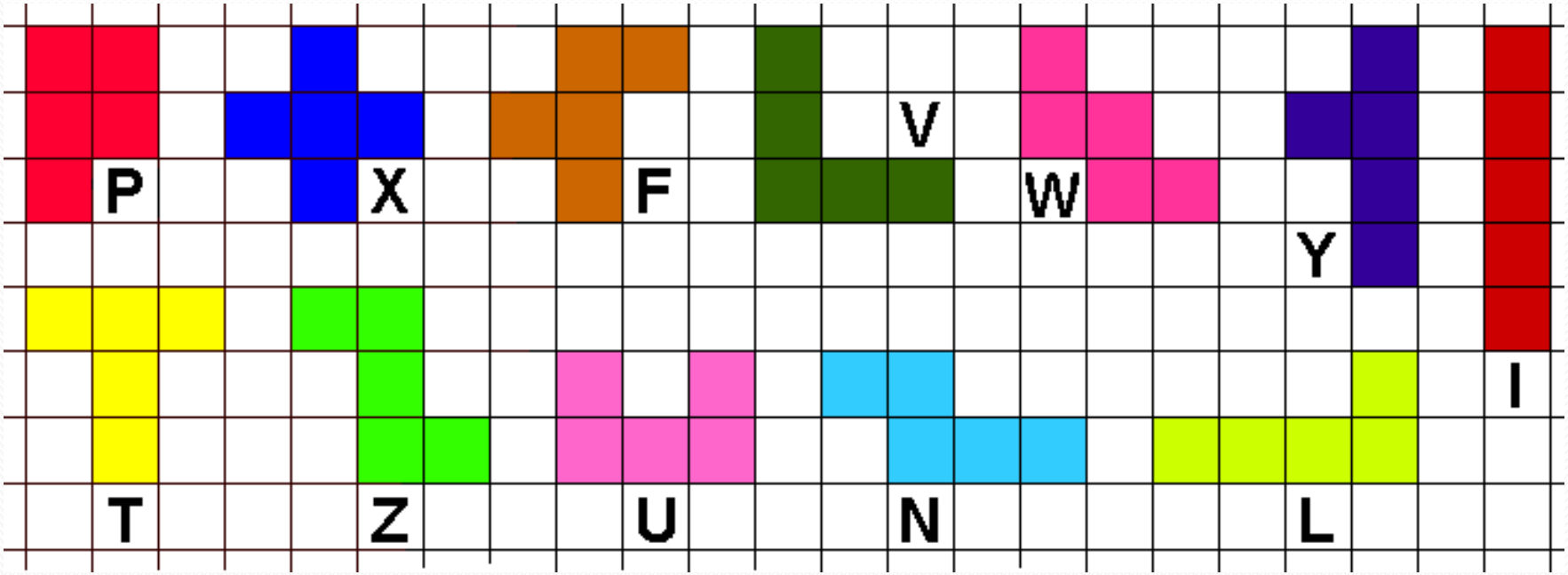
Pentominoe Challenge

- **Create as many different pentominoes possible from five square tiles.**
- **Record your pentominoes on grid paper.**



- **Remember:**
  - **Each square must share one common side with another square**
  - **Each pentomino must be unique – no reflections or rotations!**

PENTOMINO MANIA



- Pentomino shapes can be named by letters

Puzzling Pentominoes – Question 1

Other than P, choose any two pentominoes. Combine them so they connect along a single square's edge. Draw your combined figure on grid paper and calculate its area and perimeter. Remember to use correct units in your answers.

Puzzling Pentominoes

Question 2

Choose two pentominoes you didn't choose in Question 1, again not using P. Combine them so they connect along a single square's edge. Draw your new combined figure on grid paper and calculate its area and perimeter. Remember to use correct units in your answers.

Puzzling Pentominoes

How do the areas and perimeters for the figures in Questions 1 and 2 compare? Can you think of an explanation for your observations? Do you think they would hold for any combination of two pentominoes? Explain your thinking mathematically.

Puzzling Pentominoes

Question 1

Combine pentomino P and another pentomino so they connect along a single square's edge. Draw your combined figure on grid paper and calculate its area and perimeter. Remember to use correct units in your answers.

Puzzling Pentominoes II

Compare your results combining Pentomino ***P*** with another piece to your previous results. Can you think of a mathematical explanation for your observations?

Can you generalize?

Write a rule for the area given the number of pentominoes.

What is the least perimeter of a combined figure of two pentominoes?

What is the greatest perimeter of a combined figure of two pentominoes?

Write a rule to predict the greatest perimeter given the number of pentominoes.

NCTM PSSM Geometry 3-5

- Identify, compare, and analyze attributes of two-and three-dimensional shapes and develop vocabulary to describe the attributes;
- Classify two- and three- dimensional shapes according to their properties and develop definitions of classes of shapes such as triangles and pyramids.
- Investigate, describe, and reason about the results of subdividing, combining, and transforming shapes.
- Explore congruence and similarity;
- Make and test conjectures about geometric properties and relationships and develop logical arguments to justify conclusions.

Apply transformations…3-5

- Predict and describe the results of sliding, flipping, and turning two-dimensional shapes.
- Describe a motion or a series of motions that will show that two shapes are congruent.
- Identify and describe line and rotational symmetry in two-and three-dimensionl shapes and designs.

Use visualization… 3-5

- Build and draw geometric shapes
- Use geometric modeling to solve problems in other areas of mathematics, such as number and measurement.
- Recognize geometric ideas and relationships and apply them to other disciplines and problems that arise in the classroom or in everyday life.

Grade 3 Common Core Standards

Geometry 3.G

Reason with shapes and their attributes.

2. Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole.

Number and Operations—Fractions 3.NF

Develop understanding of fractions as numbers.

1. Understand a fraction 1/*b* as the quantity formed by 1 part when a whole is partitioned into *b* equal parts; understand a fraction *a*/*b* asthe quantity formed by *a* parts of size 1/*b*.

Grade 3 Common Core Standards

Measurement and Data 3.MD

Geometric Measurement: understand concepts of area and relate area to multiplication and division.

5. Recognize area as an attribute of plane figures and understand concepts of area measurement.
   a. A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area.
6. Measure areas by counting unit squares.

Geometric Measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.

8. Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.

Number and Operations—Fractions 3.NF

3. Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.

a. Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.

b. Recognize and generate simple equivalent fractions, e.g., 1/2 =2/4, 4/6 = 2/3). Explain why the fractions are equivalent, e.g., by using a visual fraction model.

d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole.

Grade 4 CCSS-M

Geometry **4.G**

Draw and identify lines and angles, and classify shapes by properties of their lines and angles.

3. Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.

Grade 4 CCSS-M

Number and Operations—Fractions 4.NF

Extend understanding of fraction equivalence and ordering.

- 1. Explain why a fraction a/b is equivalent to a fraction $(n \times a)/(n \times b)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

4th Grade CCSS-M

- **Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.**
- 3. Understand a fraction a/b with $a > 1$ as a sum of fractions $1/b$.
- a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.
- b. Decompose a fraction into a sum of fractions with the same denominator in more than one way…
- .

5th Grade CCSS-M

Classify two-dimensional figures into categories based on their properties.

3. Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. *For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.*
4. Classify two-dimensional figures in a hierarchy based on properties.

Extension to 3-D Geometry

Grade 5

Geometric measurement: understand concepts of volume and relate
volume to multiplication and to addition.

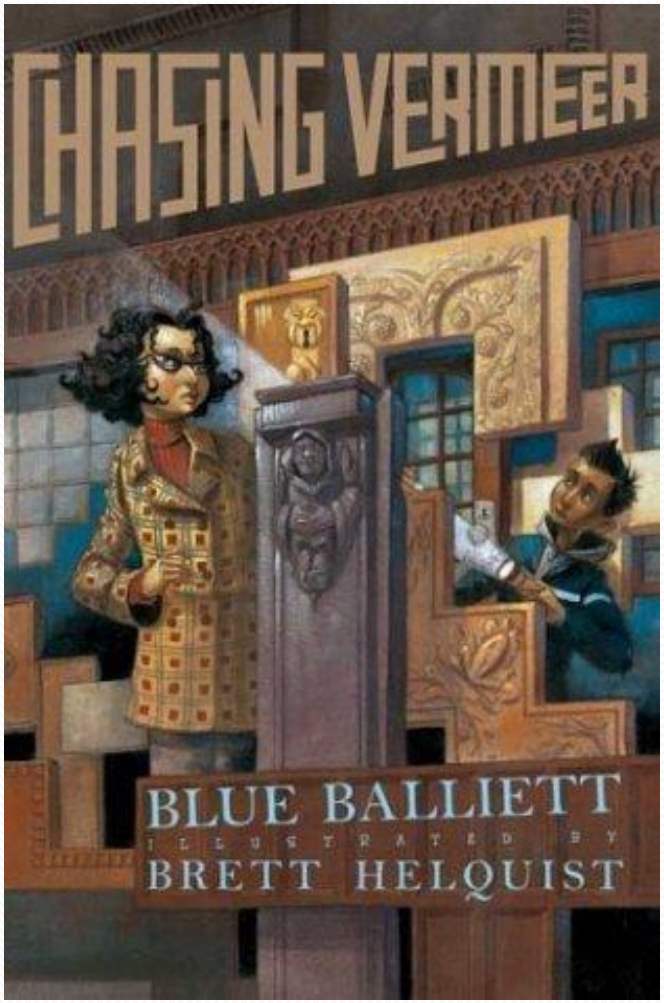
3. Recognize volume as an attribute of solid figures and understand
concepts of volume measurement.

a. A cube with side length 1 unit, called a “unit cube,” is said to have
“one cubic unit” of volume, and can be used to measure volume.

b. A solid figure which can be packed without gaps or overlaps
using *n* unit cubes is said to have a volume of *n* cubic units.

4. Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft., and improvised units.

Chasing Vermeer



By Blue Balliett
Illustrations: Brett Helquist

"Chasing Vermeer is a puzzle, wrapped in a mystery, disguised as an adventure, and delivered as a work of art.“

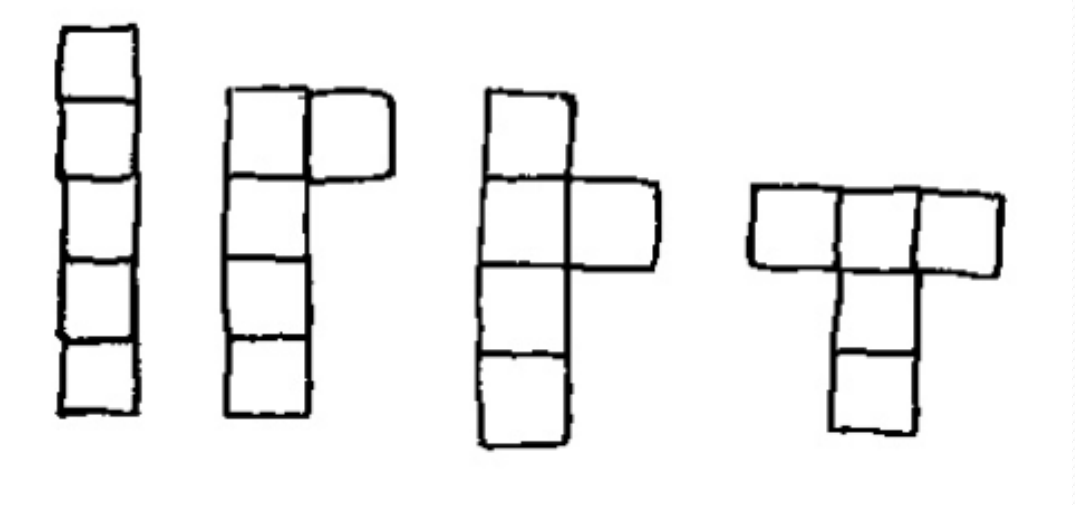
-Nancy Bosch

Illustrations give the reader clues…

Chasing Verneer

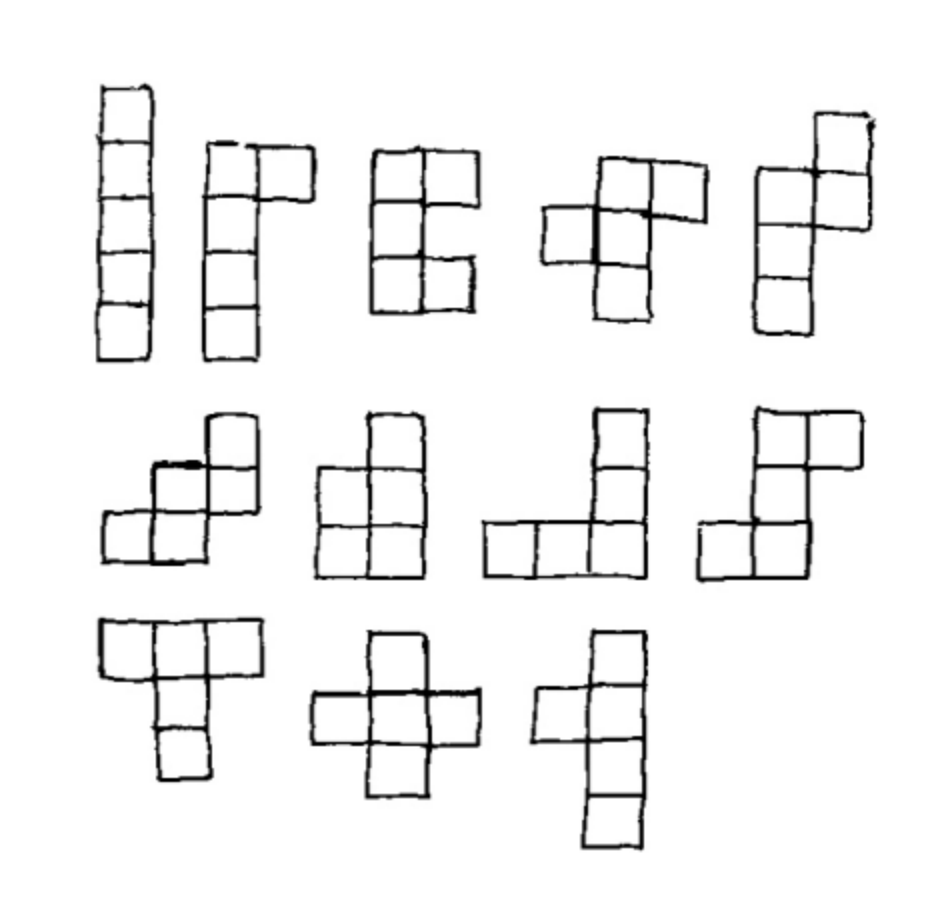
A Lady Writing - Vermeer

Arrange your pentomino pieces into a line, so that only one square needs to moved to change a shape into the one next to it.



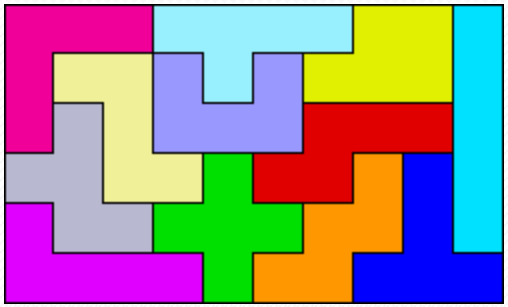
Adapted from a Math Solutions Chasing Vermeer Lesson…

- One example



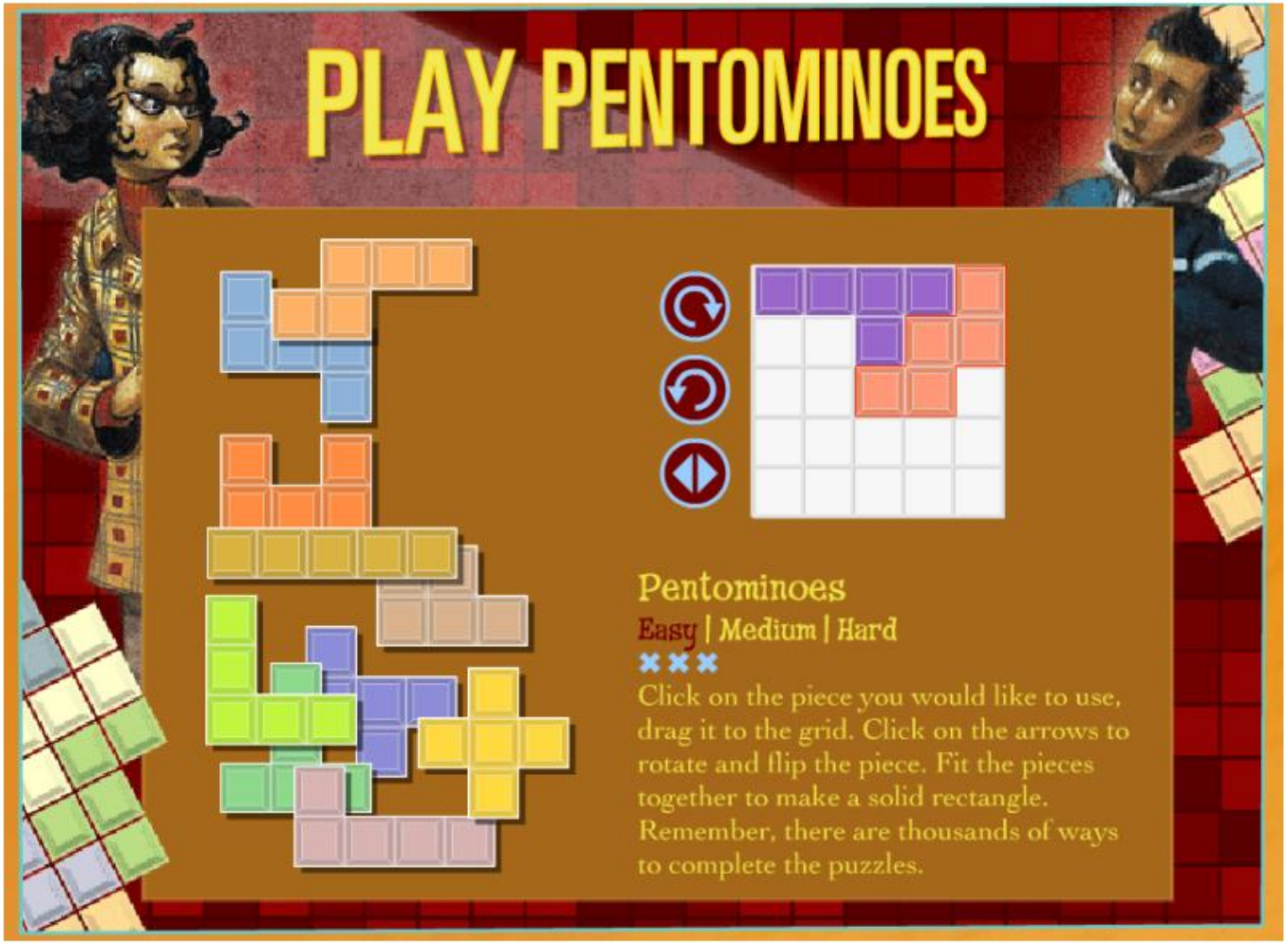
Fun With Pentominoes

- There are many challenging puzzles that can be done with pentominoes.



- There are over 2,339 ways to arrange the petominoes in a 6x10 grid.

http://www.scholastic.com/blueballiett/games/pentominoes_game.htm



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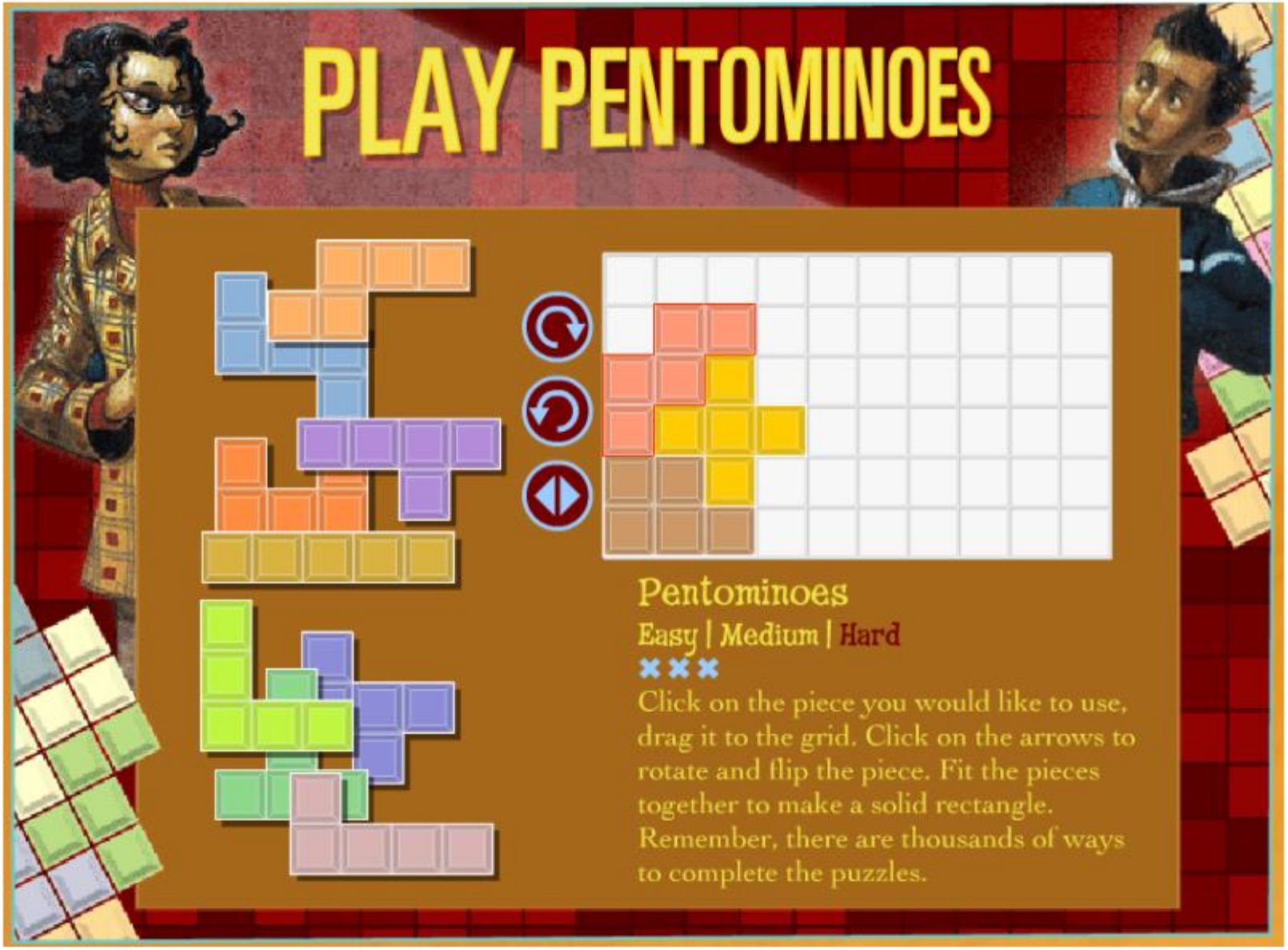


http://www.scholastic.com/blueballiett/games/pentominoes_game.htm

Mathematical Practices

1. **Make sense of problems and persevere in solving them.**
2. **Reason abstractly and quantitatively.**
3. **Construct viable arguments and critique the reasoning of others.**
4. **Model with mathematics.**
5. **Use appropriate tools strategically.**
6. **Attend to precision.**
7. **Look for and make use of structure.**
8. **Look for and express regularity in repeated reasoning.**

Assessment

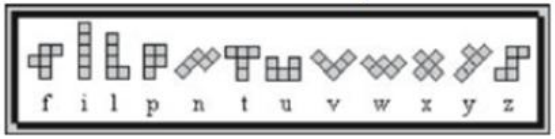
Create an image or design using your pentominoes. Calculate the area and perimeter and explain what area and perimeter mean.

Pentomino Extensions

- Area
- Perimeter
- Symmetry
- Tessellations
- Spatial Visualization
  - Sort by which ones can fold up into an open box
  - Mark an X where the bottom of the box would be

Peter Liljedahl's Pentomino challenge…

A pentomino is a shape that is created by the joining of five squares such that every square touches at least one other square along a full edge. There are 12 such shapes, named for the letters they most closely resemble.

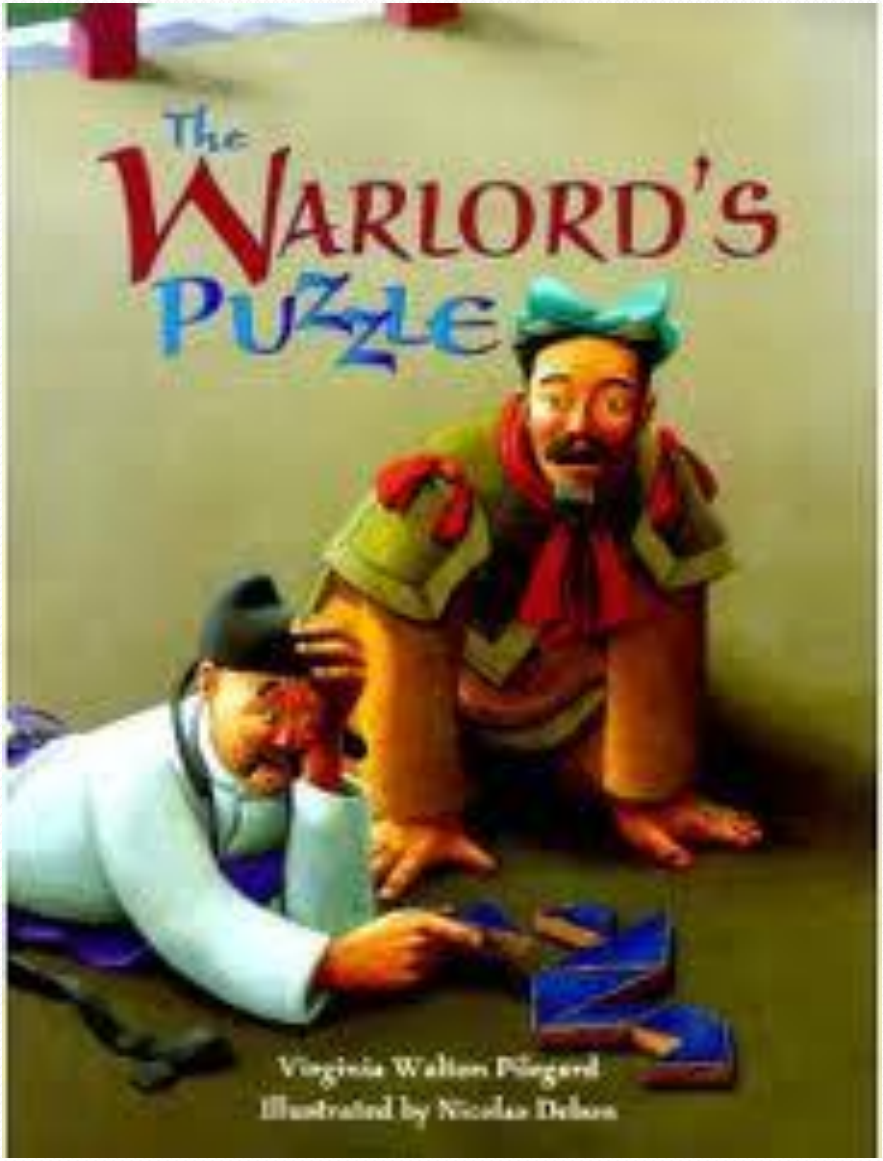


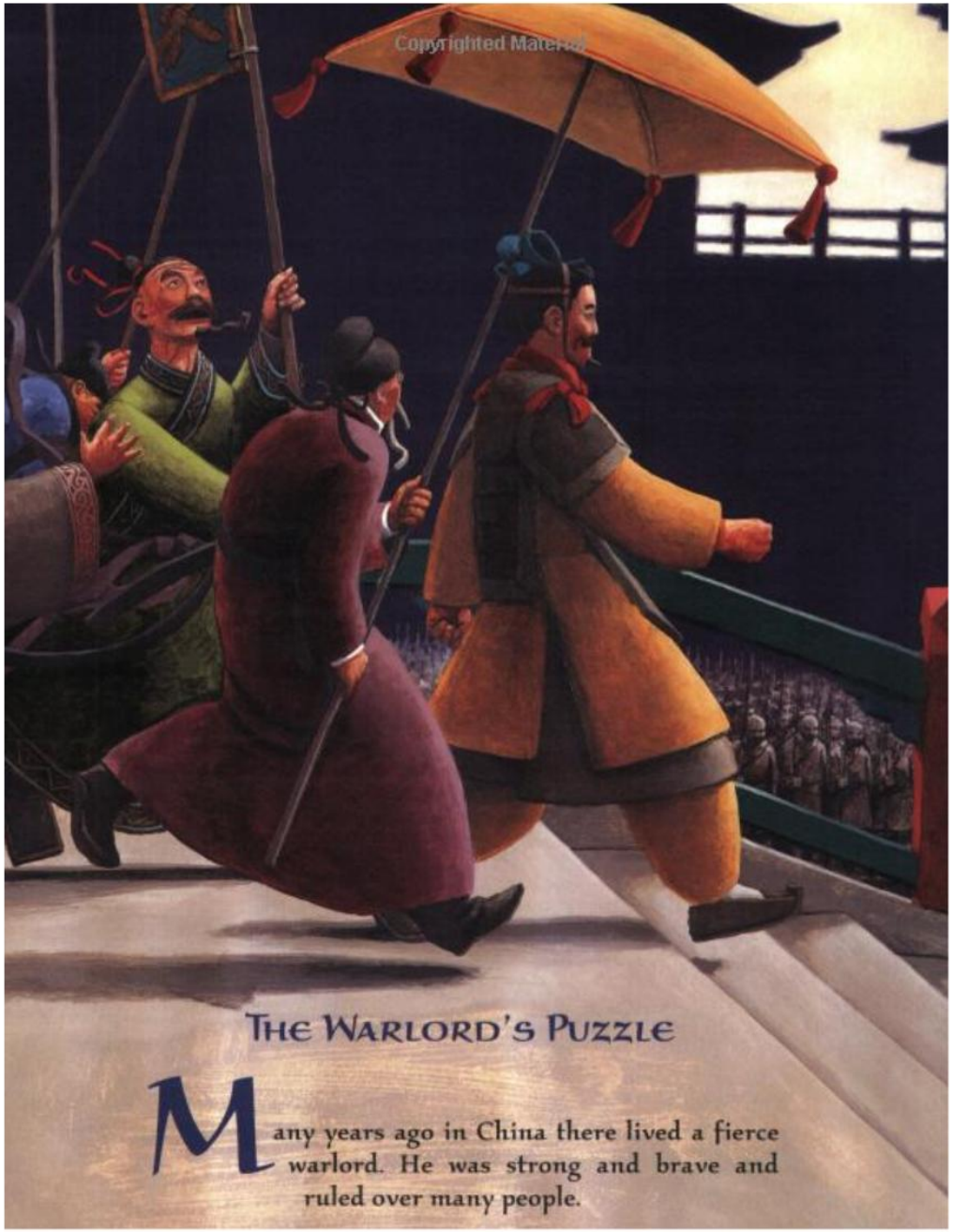
Now consider a 100's chart.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

If a pentomino is placed somewhere on a 100's chart will the sum of the numbers it covers be divisible by 5? If not, what will the remainder be?

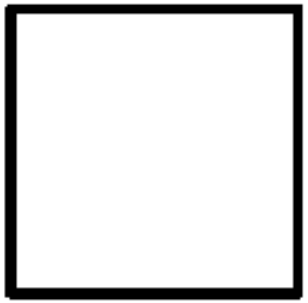
The Warlord's Puzzle



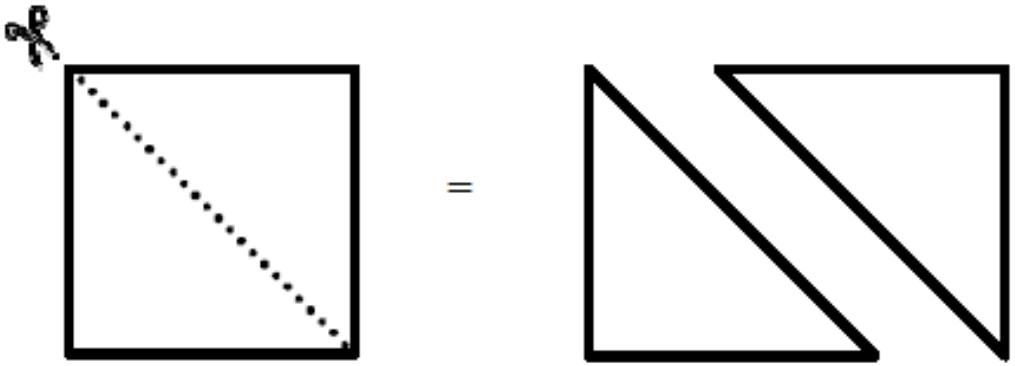

Copyrighted Material
THE WARLORD'S PUZZLE
Many years ago in China there lived a fierce warlord. He was strong and brave and ruled over many people.

Folding and Cutting a Tangram

1. **Begin with a square (4x4, 6x6, etc.)**

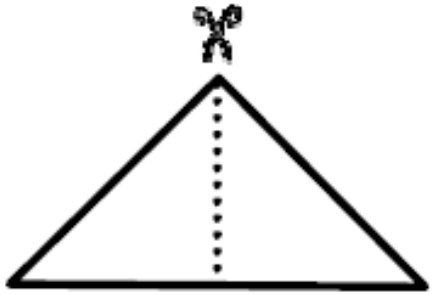


2. **Fold the square along a diagonal matching corners and edges very carefully. Cut along the crease. This forms two large right triangles.**

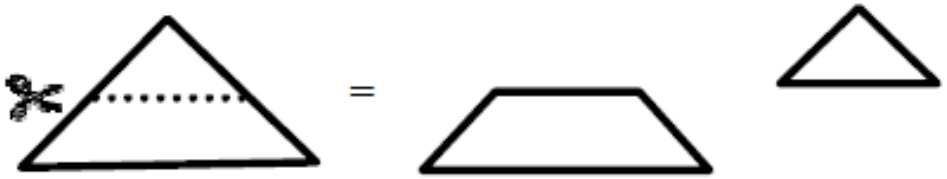


Tangram

3. **Hold one of the right triangles so that the right angle is at the top. Fold in half from left to right matching corners and edges very carefully. Cut along the crease forming two right triangles. Set these aside.**

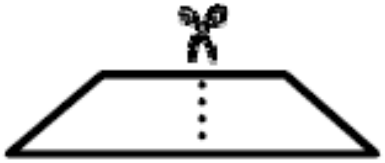


4. **Hold the other large triangle so that the right angle is at the top. Fold this top point down so that it touches the bottom of the triangle at the midpoint of that side. Crease carefully, and cut along the crease. This forms a middle-sized triangle. Set it aside.**

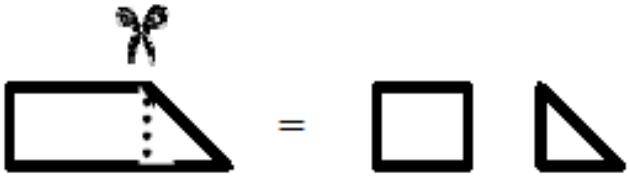


Tangram

5. **The remaining piece is a trapezoid. Fold it in half from left to right matching corners and edges carefully. Cut along the crease. This forms two smaller trapezoids.**

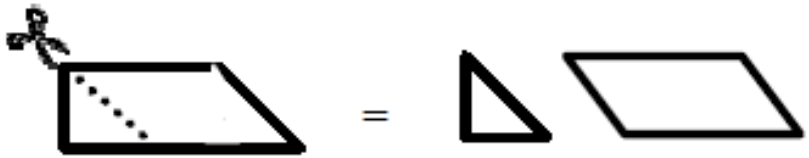


6. **Take one of the smaller trapezoids and fold so that the long point touches the right angle adjacent to it. Crease carefully making sure edges are even. Cut along the crease to from a square and a small triangle.**



Tangram

7. **Hold the second trapezoid so that it resembles a shoe. Fold the "heel" point to the "laces" point. Crease carefully and cut along the crease to form a parallelogram and another small triangle.**



8. **You now have the seven tans that form a tangram.**

Is it possible…

- to make a square using only ONE tangram piece?
  - Two? Three? Four? Five? Six? All seven pieces?

to use all seven tangram pieces and make a:

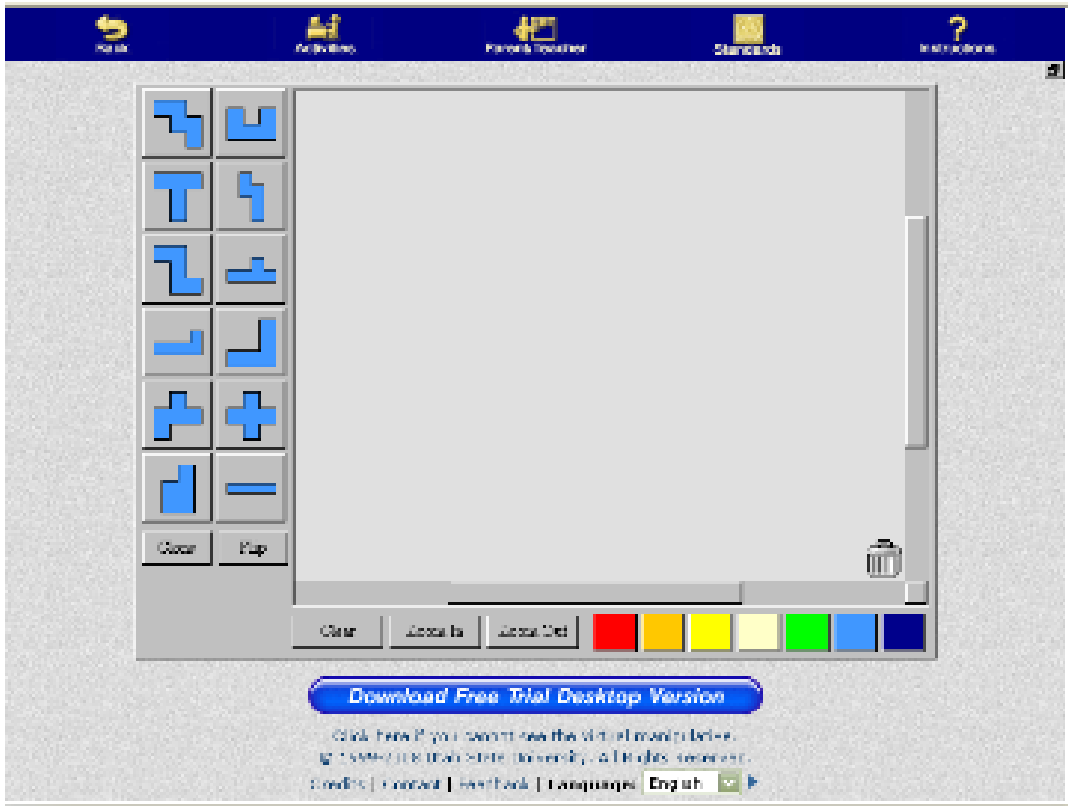
trapezoid?

rectangle that is not a square?

parallelogram that is not a square?

a triangle?

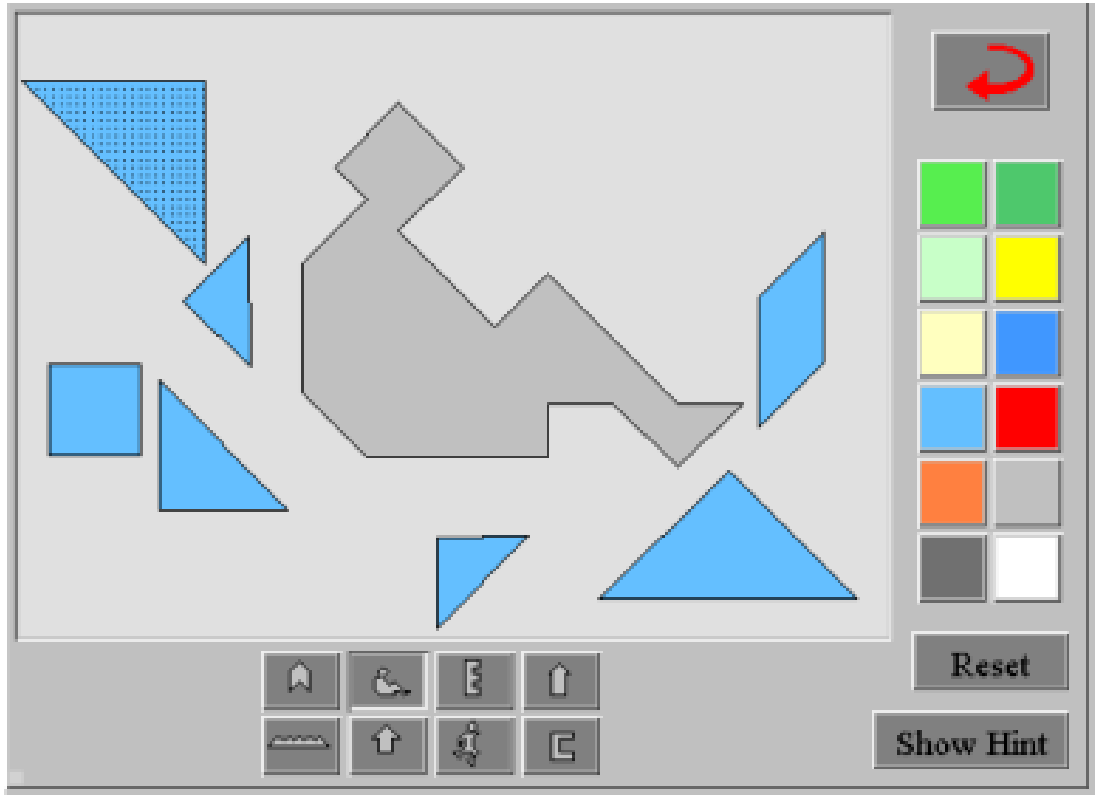
National Library of Virtual Manipulatives - NLVM



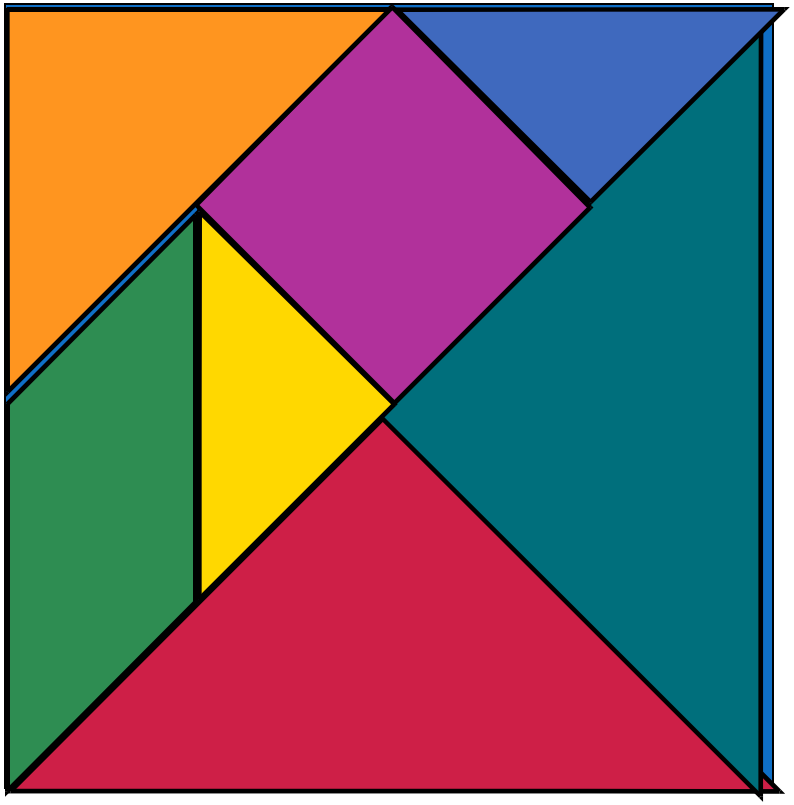
http://nlvm.usu.edu/en/nav/frames_asid_114_g_2_t_3.html?open=activities&from=category_g_2_t_3.html

NCTM Illuminations Applet

- http://www.nctm.org/standards/content.aspx?id=25012



Tangrams

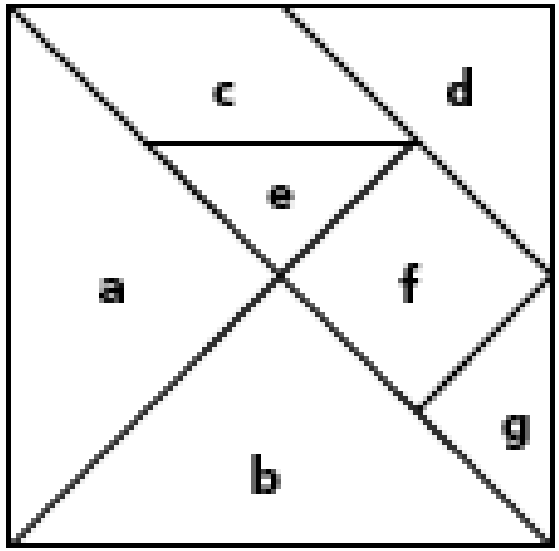


Tangrams as part of a fraction unit…

Lessons in the Unit

Lesson	Title and Description
1	**Design of Fractions**; Students will use plastic square tiles to make a design that fits the constraints of specific fractional parts.
2	**Race to One;** Students play a game with equivalent fractions and adding fractions as they move various game pieces from 0 to 1 whole.
3	**Fraction Buckets;** Students compare fractions using the benchmarks 0, ½, and 1.
4	**Fraction Chain**; Students place fractions on a number line and use the number line model to compare fractions.
5	**Kendall's Candy Company**; Students decompose a fraction into unit fractions in the context of a candy store.
6	**Who am I Puzzles;** Students use various fraction models to solve Fraction Mystery puzzles. They apply their knowledge of comparing fractions in this lesson.
7	**Fractions Tangrams**; Students explore with a set of tangrams and find the fractional value of each tangram piece if all of the tangrams represented one whole region.
8	**Fraction Relay Race**; Students decompose 1 whole into fractional pieces on the number line. This lesson is situated in the context of a relay race where students need to determine how far each person runs so that the sum is 1 whole.

In this tangram puzzle, you can make many different shapes by putting the seven pieces together in different ways.



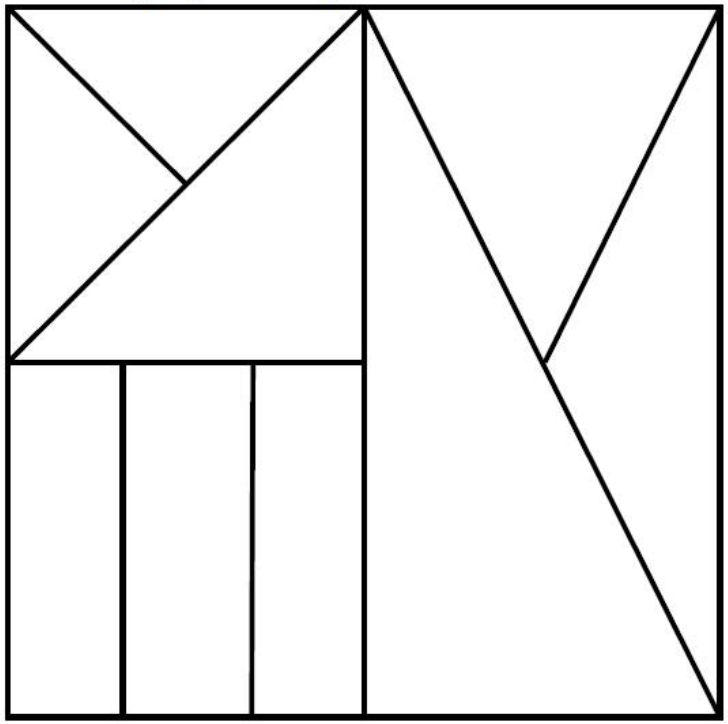
If the area of the entire puzzle is one square unit, find the area of each of the pieces.

What if the area of shape f is one? What is the area of each of the other shapes?

Explain your thinking.

Tangram Task Sheet 2

Record the size of each tangram piece in the picture below.



How did you discover the size of each piece?

_____

_____

_____

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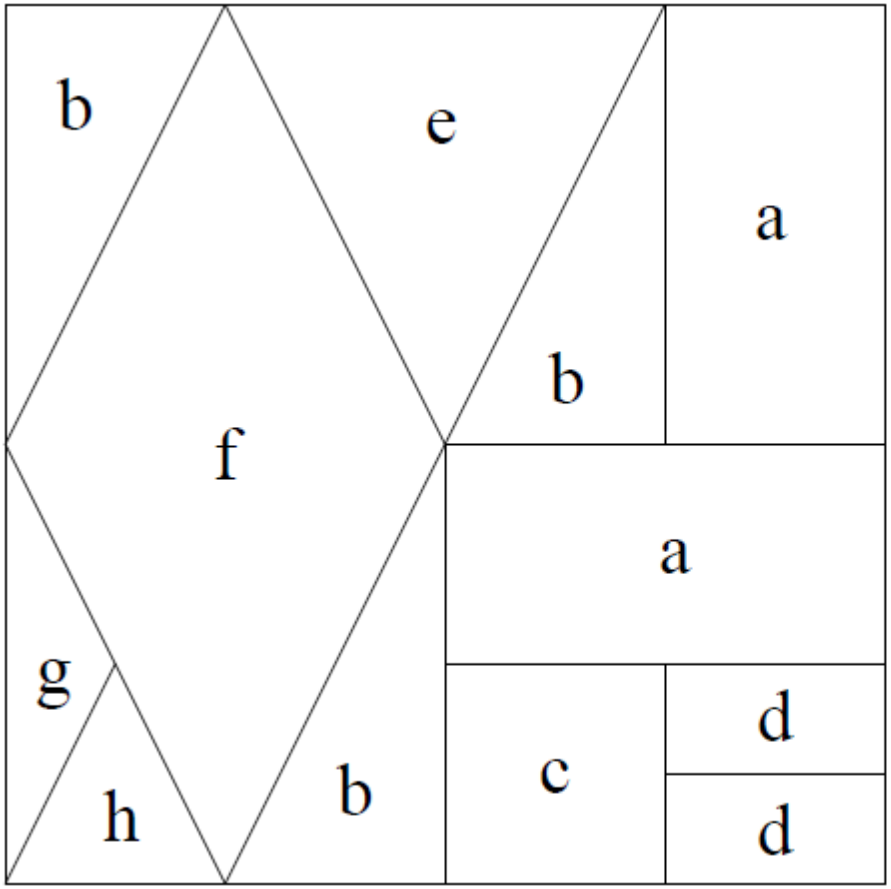
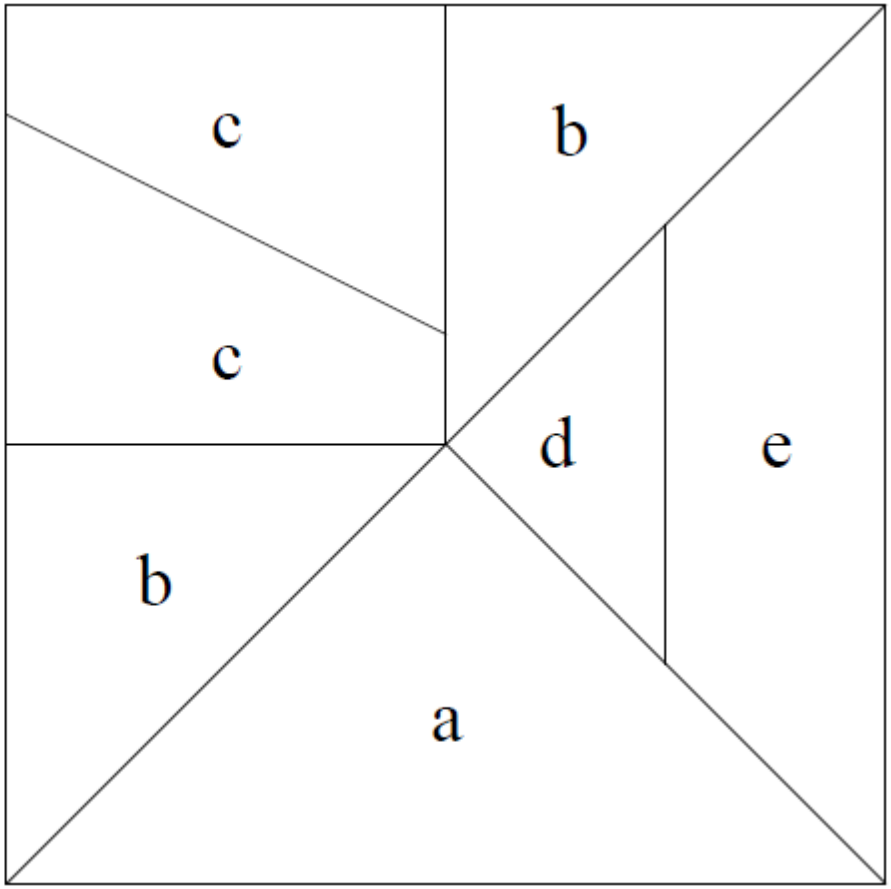
_____

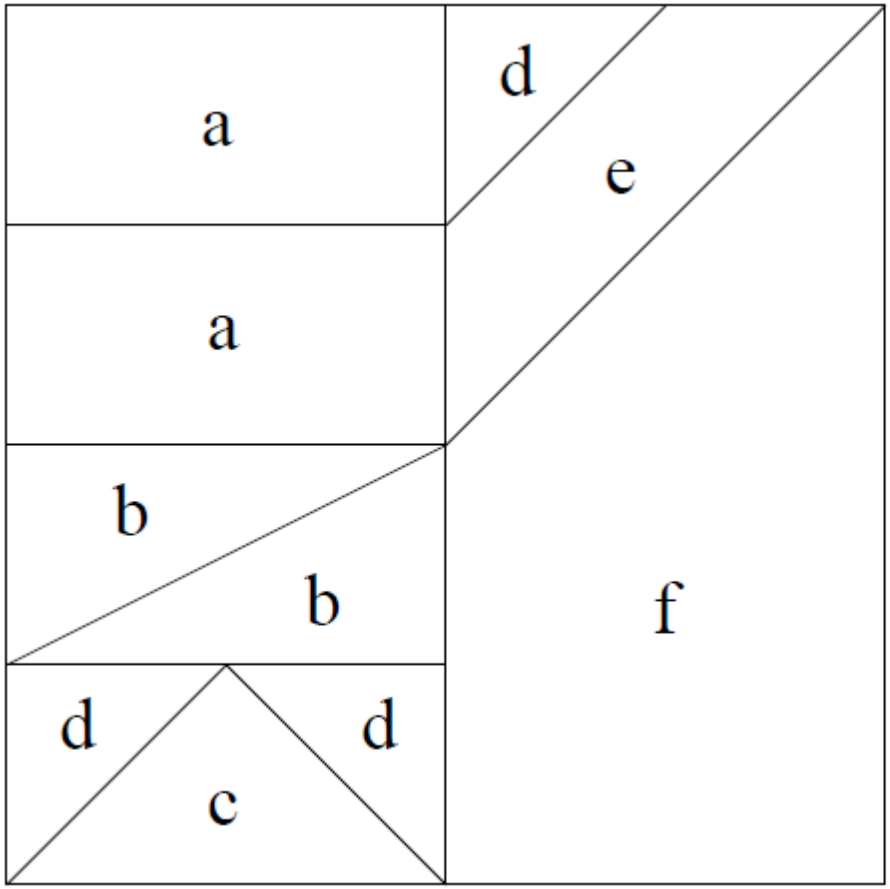
_____



Tangram 3

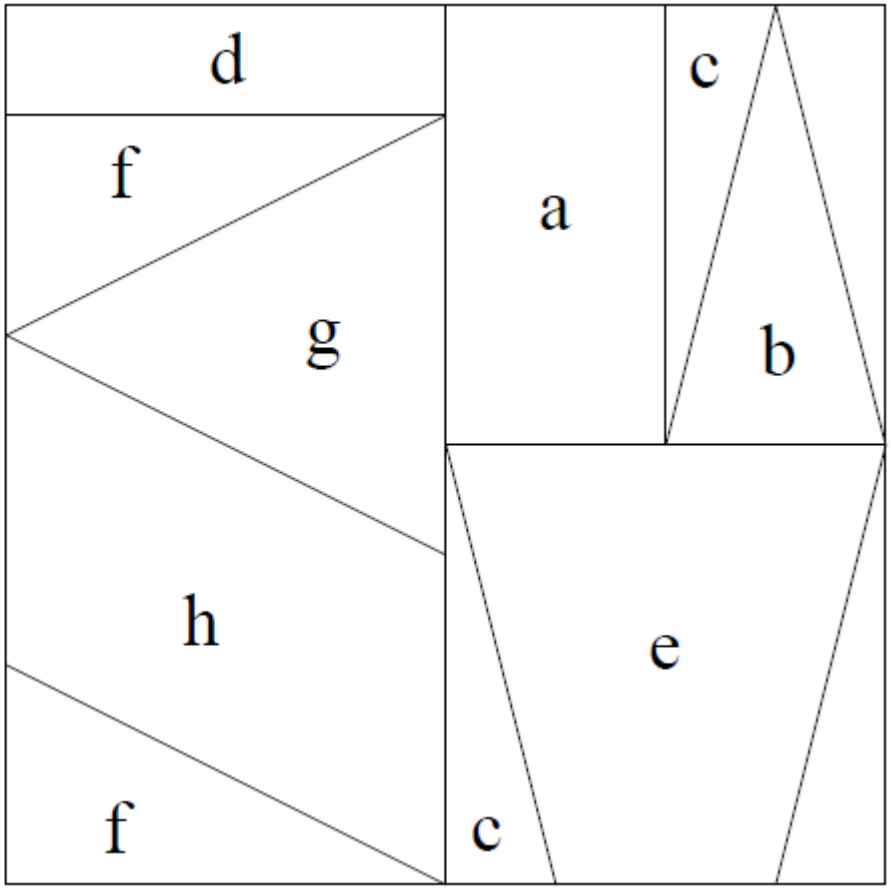
Tangram 4

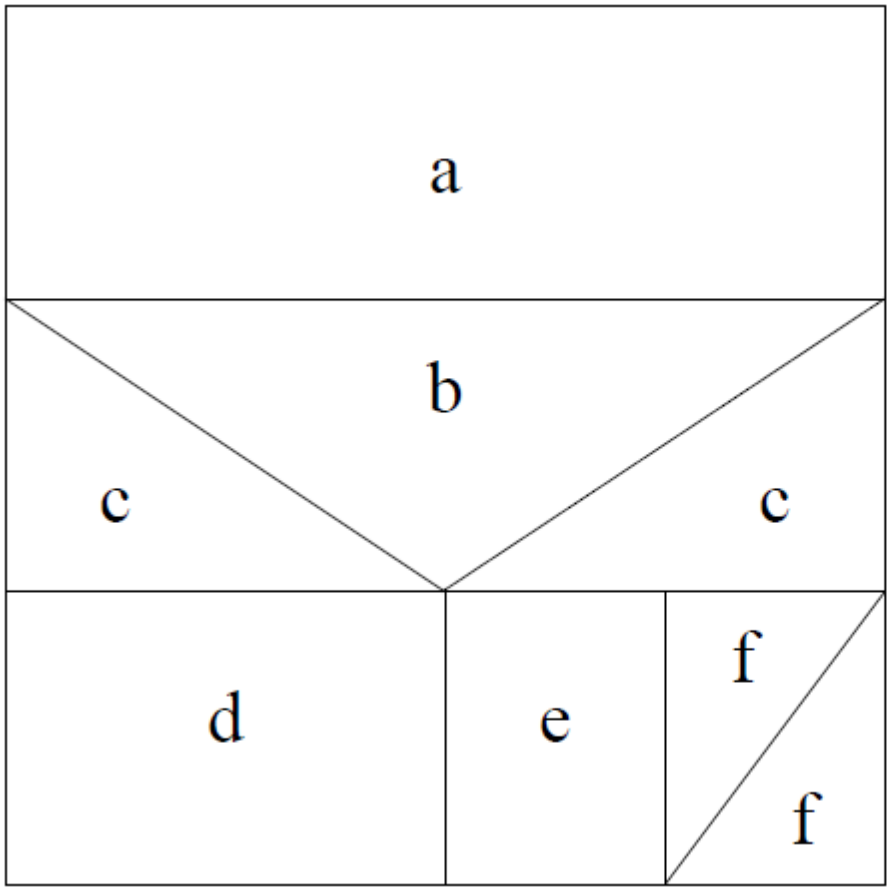




Tangram 5

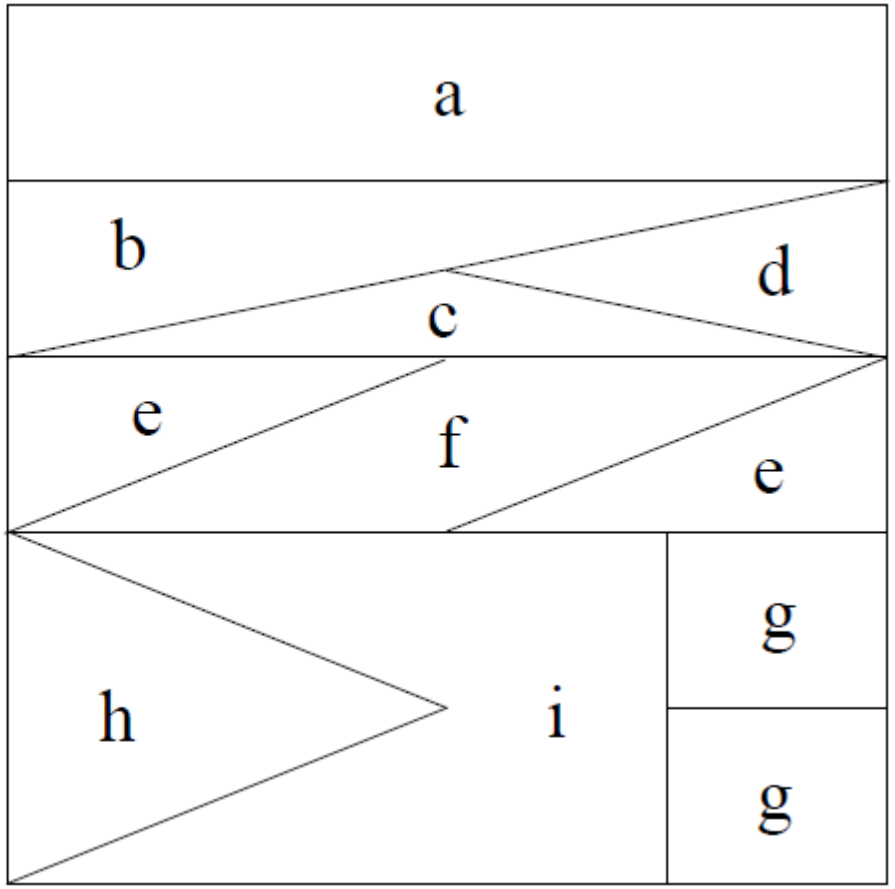
Tangram 6



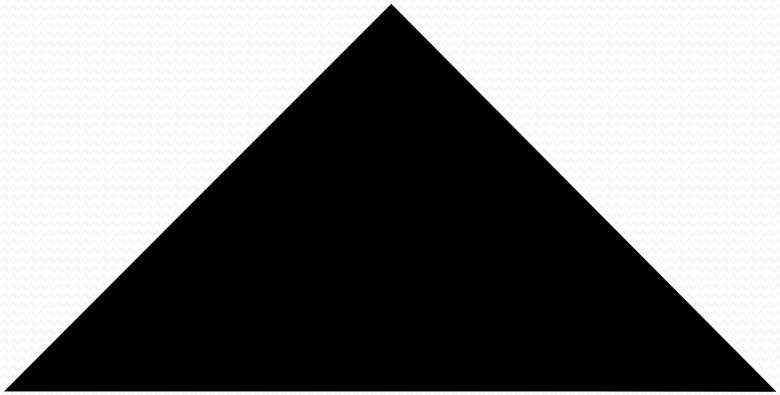


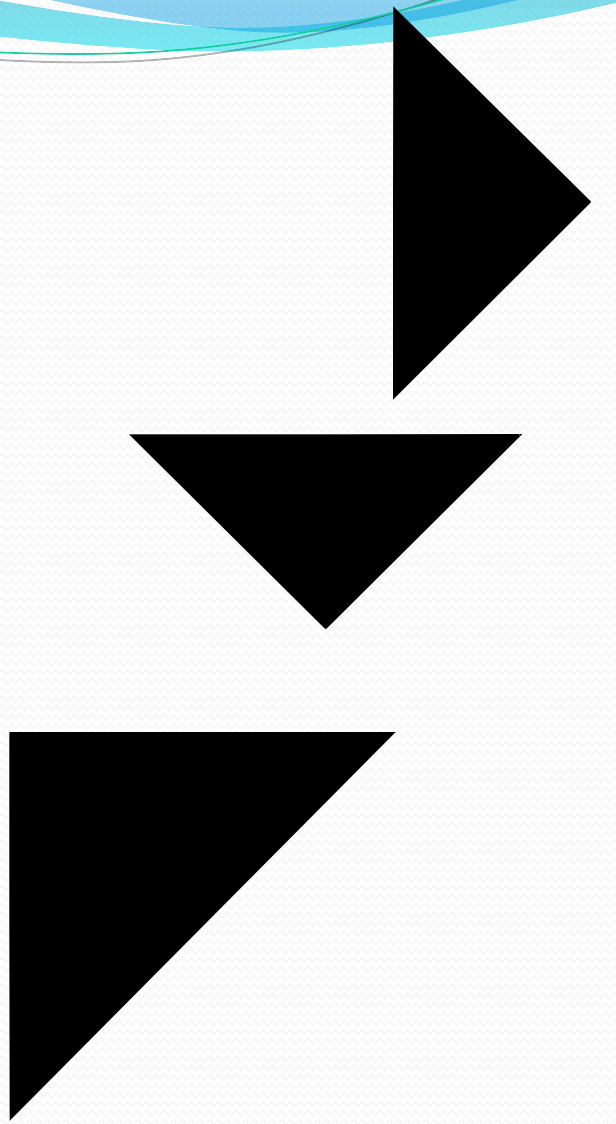
Tangram 7

Tangram 8

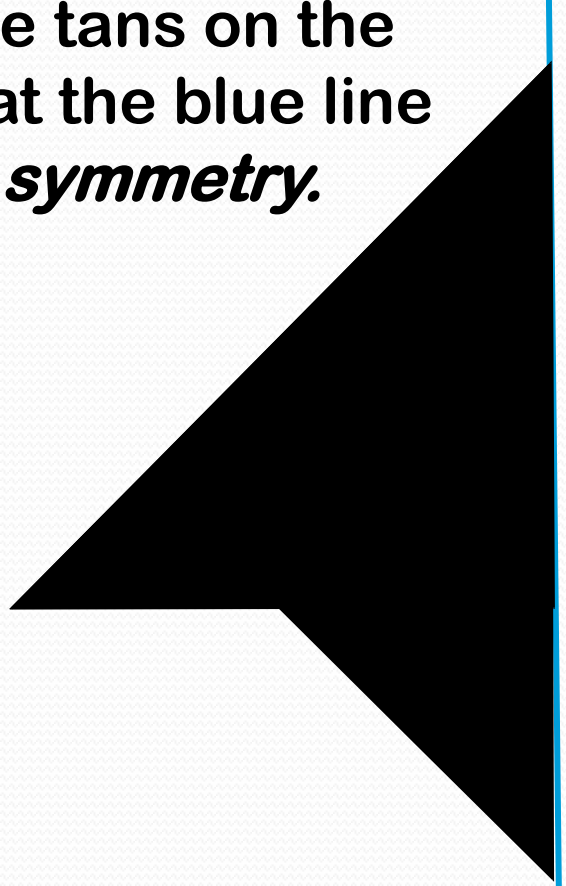


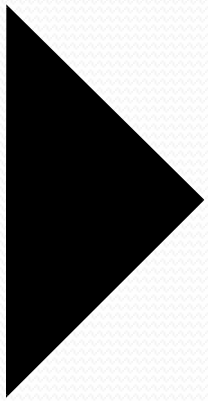
Arrange the tans on the right to form a triangle ***congruent*** to the one on the left.

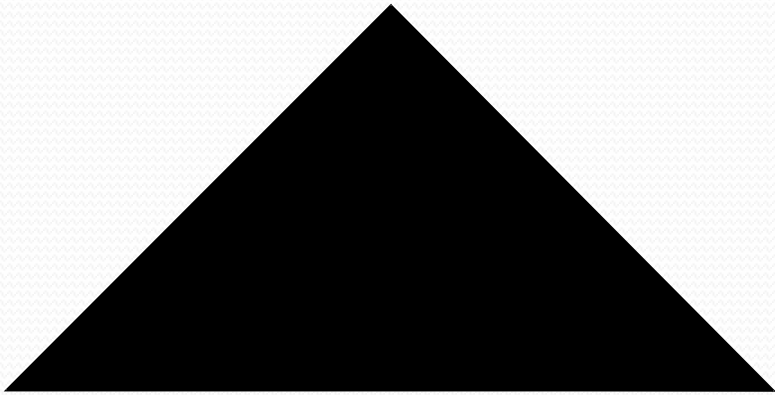




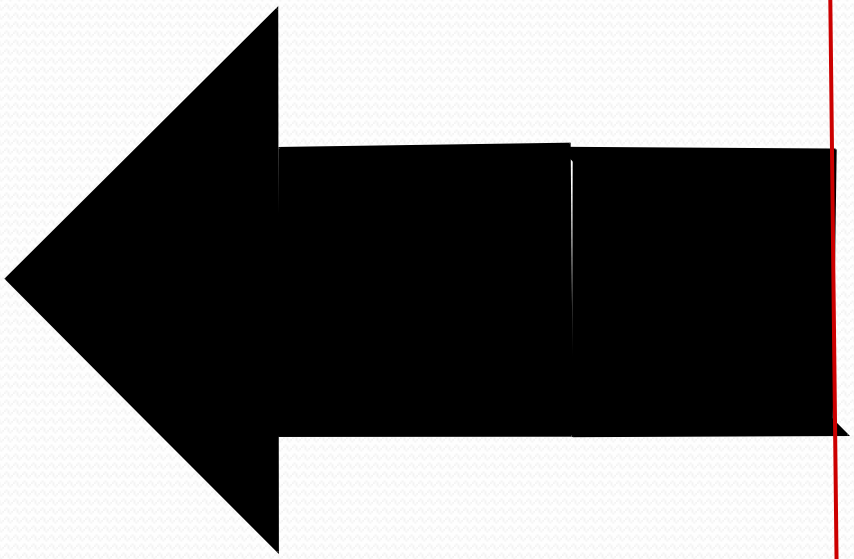
Arrange the tans on the right so that the blue line is a ***line of symmetry.***

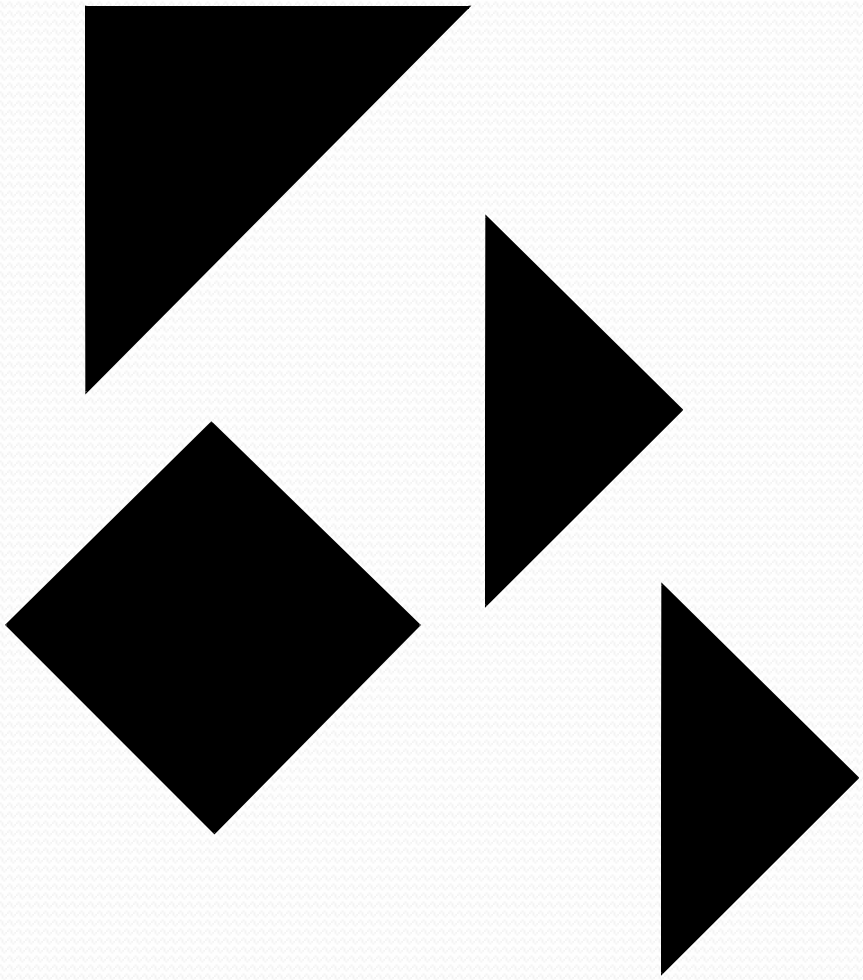




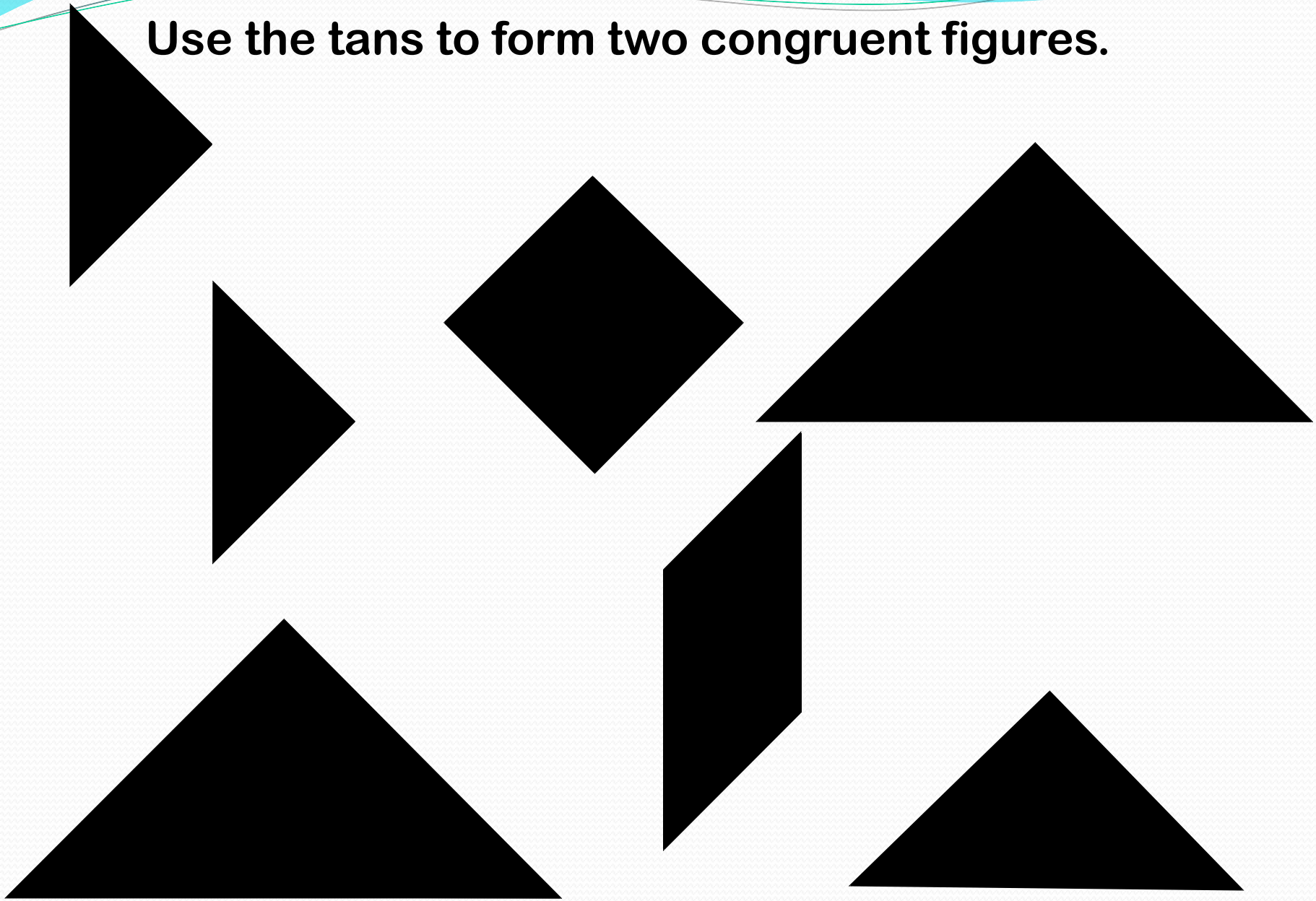


Arrange the tans on this side to show a reflection of the arrow on the left.

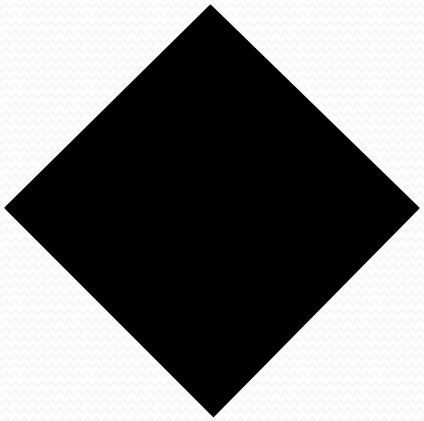


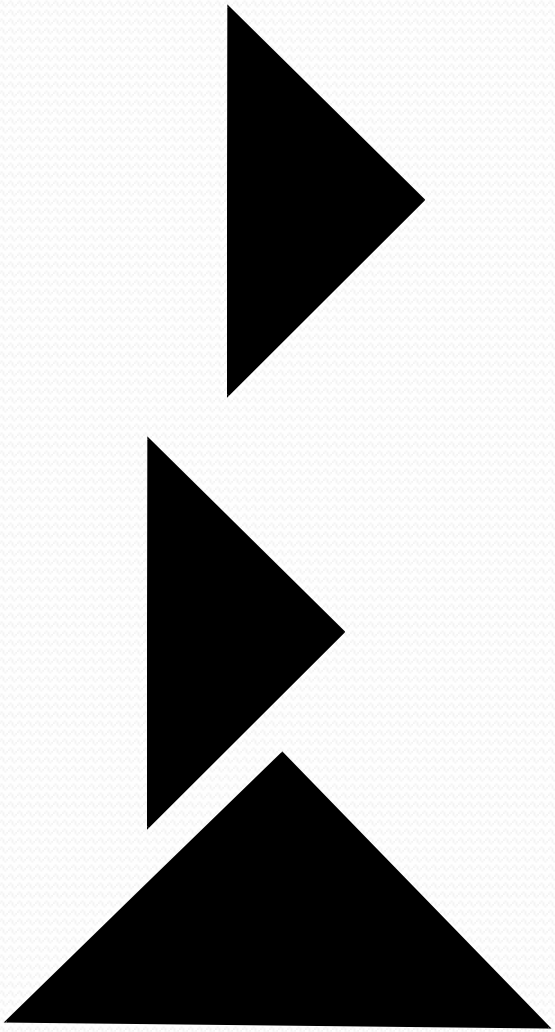


Use the tans to form two congruent figures.



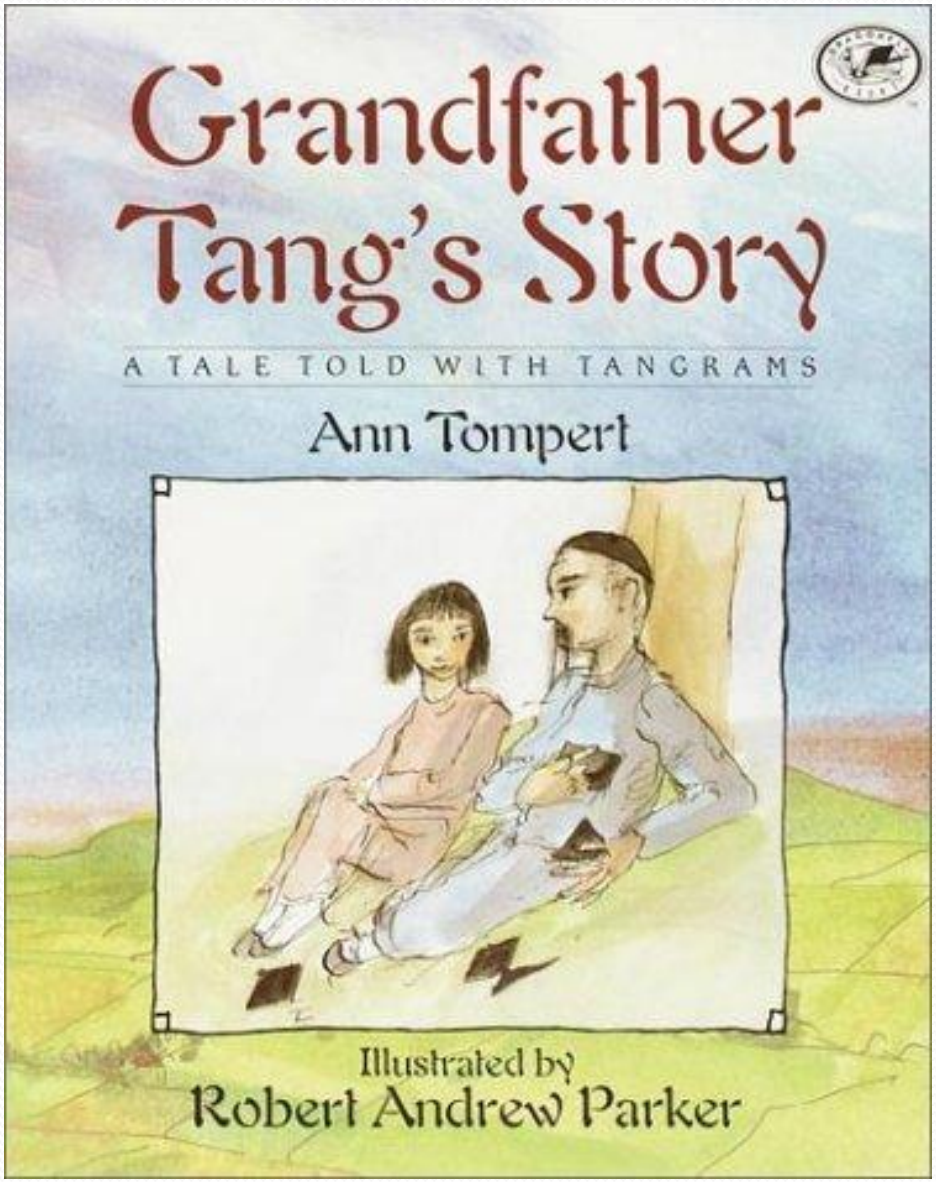
Arrange the tans on the right to form a polygon similar to this one.

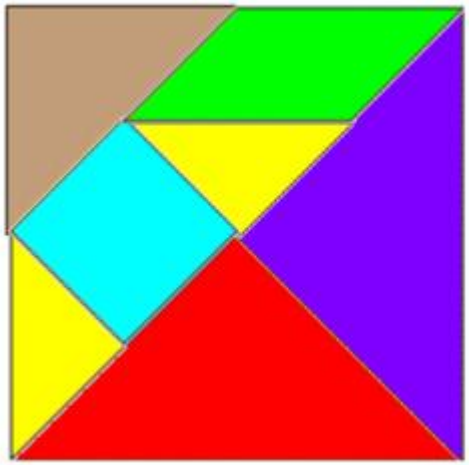




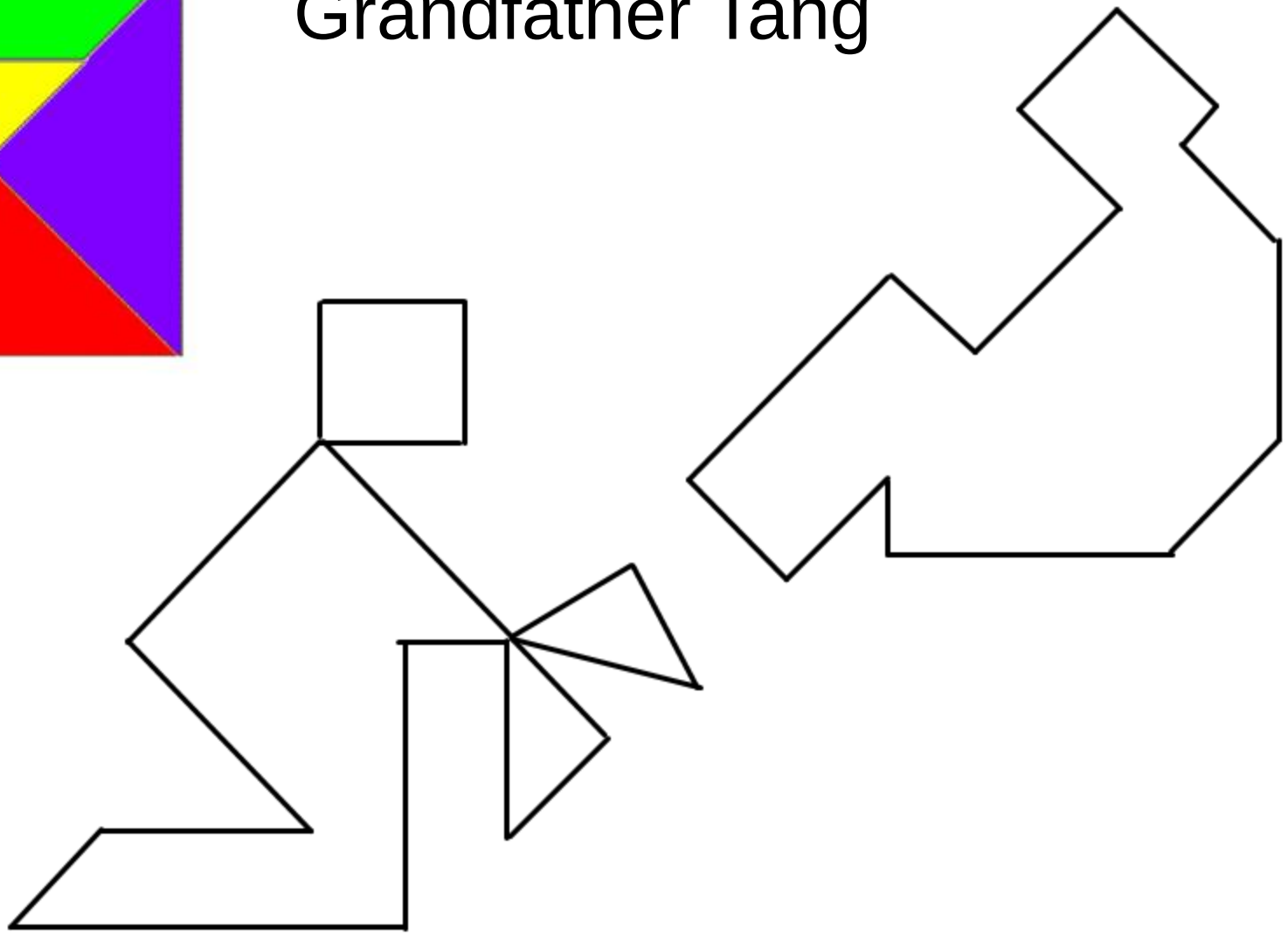
Tangrams

- **Area**
- **Perimeter**
- **Symmetry**
- **Tessellations**
- **Spatial Visualization**
  - **Sort by which ones can fold up into an open box**
  - **Mark an X where the bottom of the box would be**

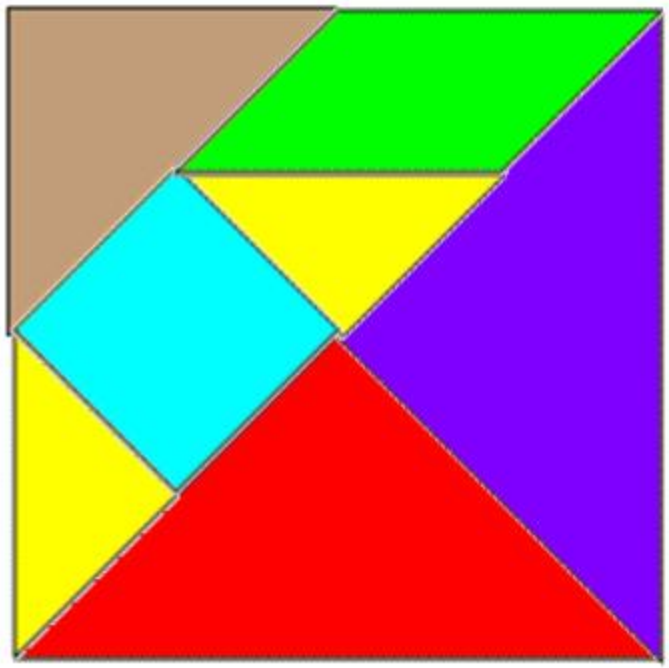

Grandfather Tang's Story
A TALE TOLD WITH TANGRAMS
Ann Tompert
Illustrated by
Robert Andrew Parker

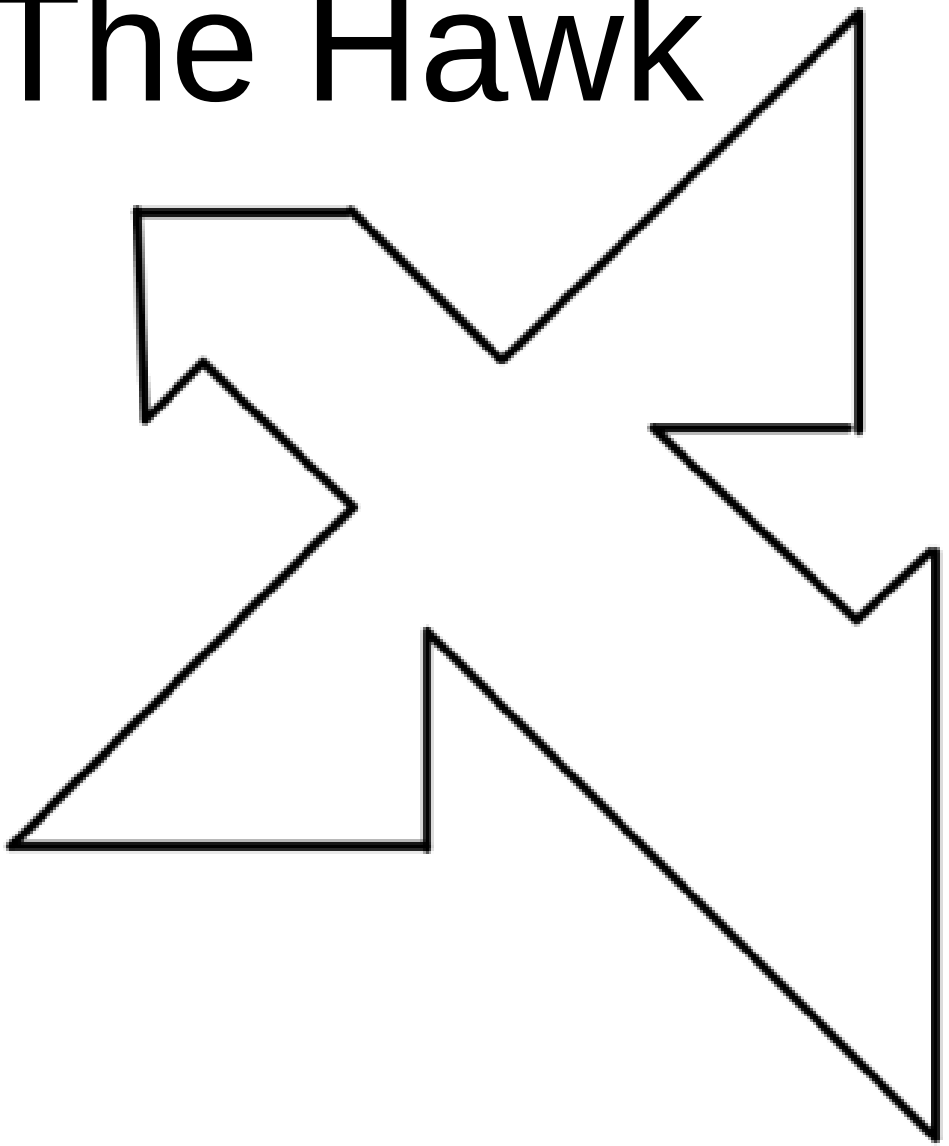


Grandfather Tang



The Hawk





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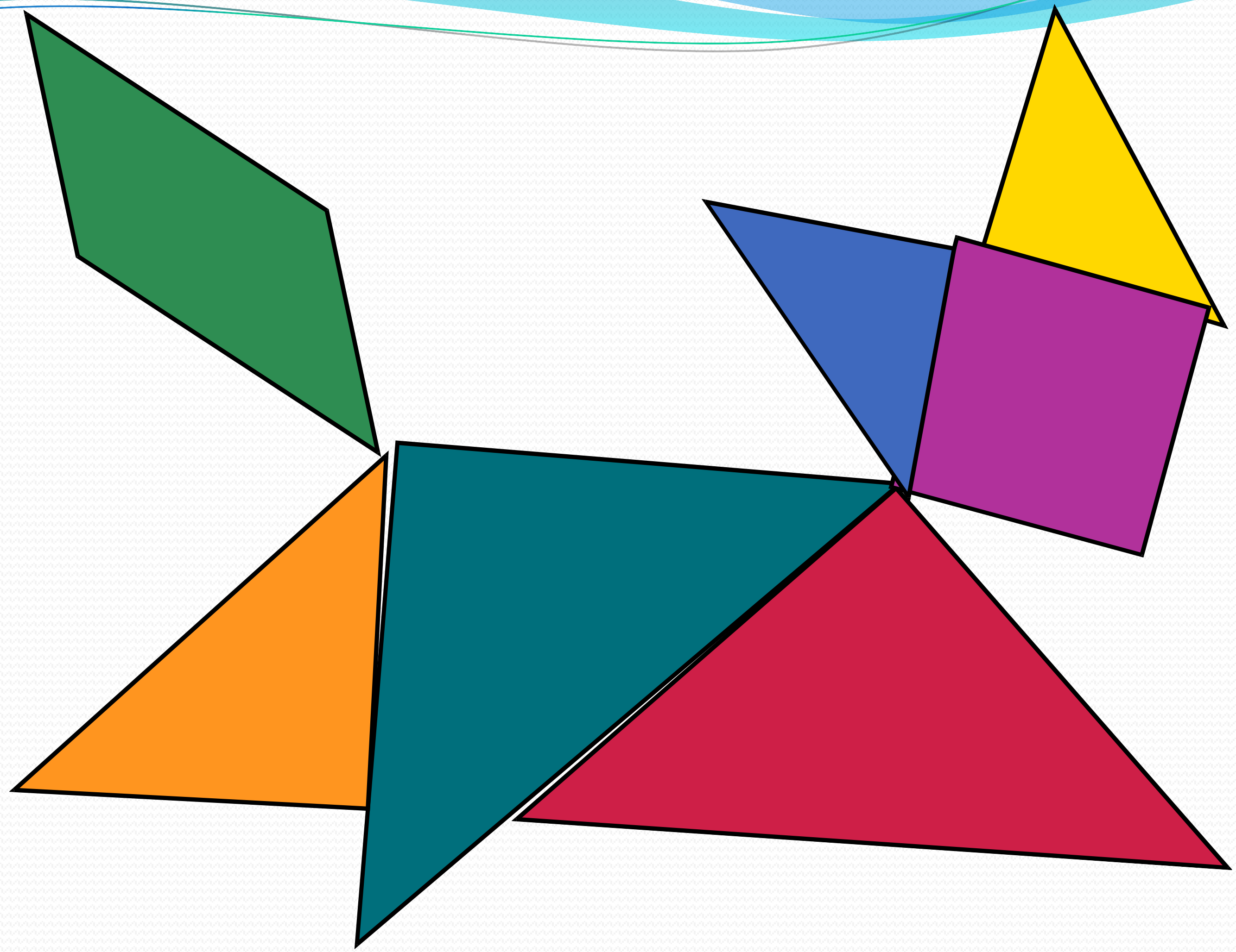
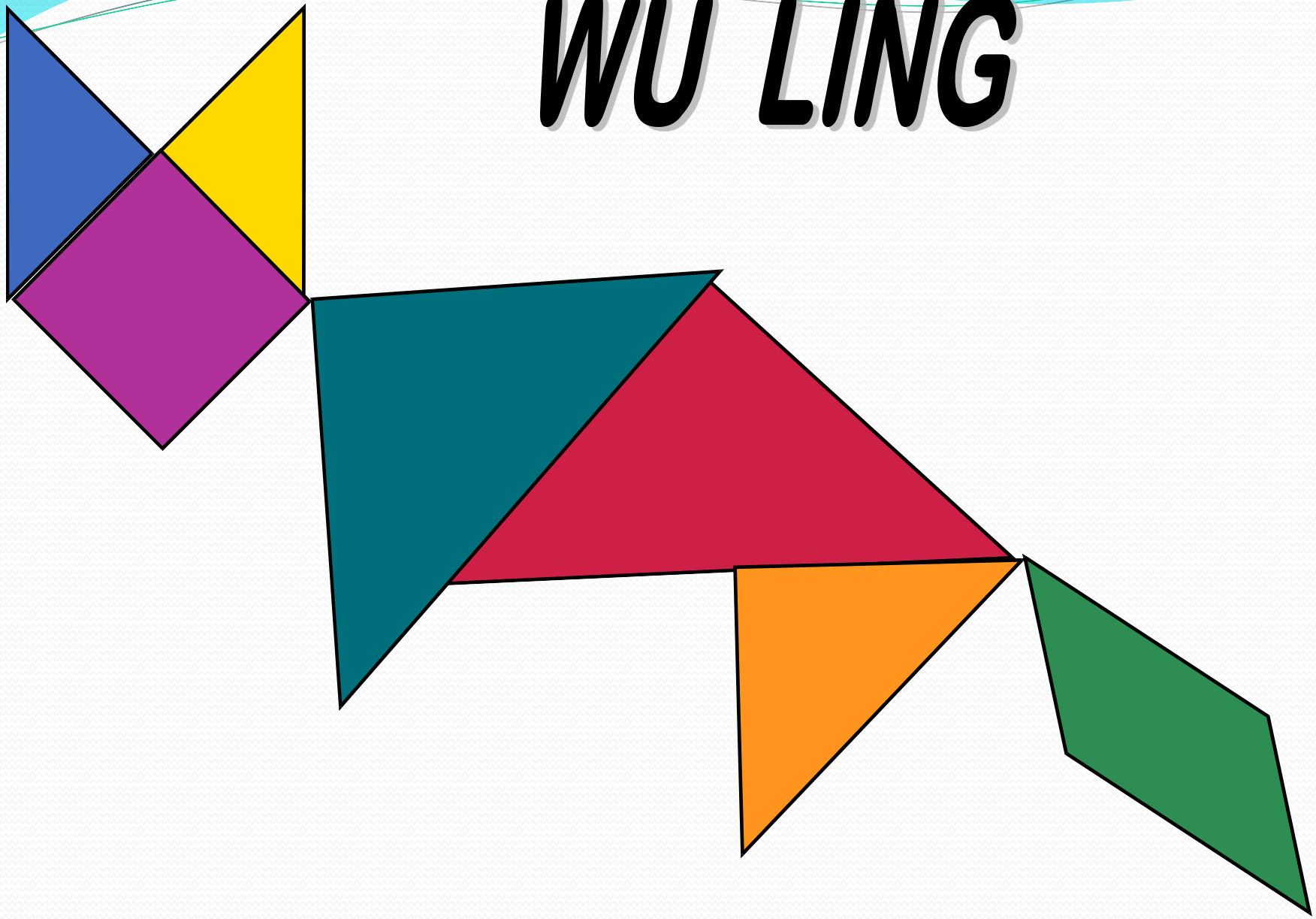
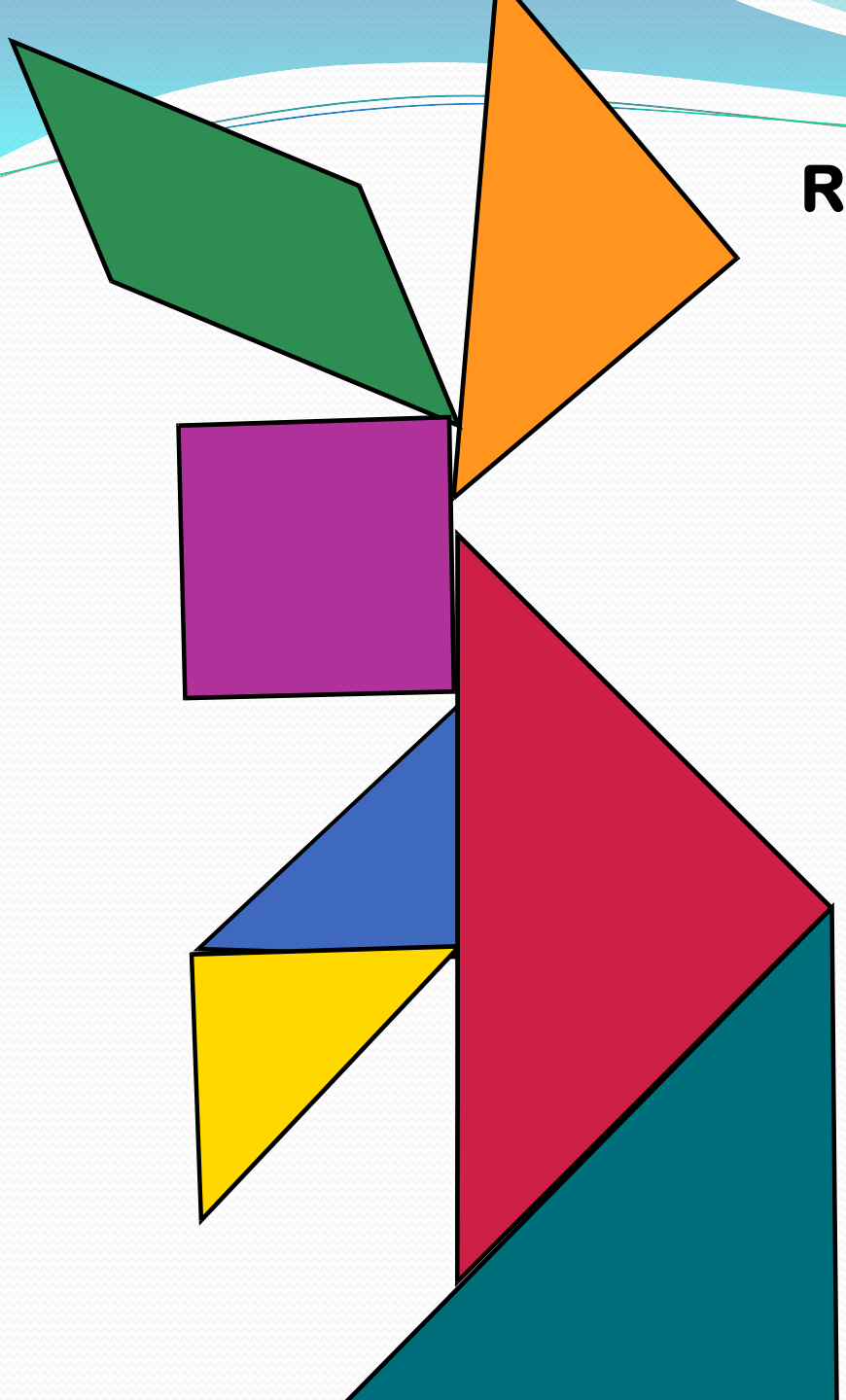


WU LING



RABBIT



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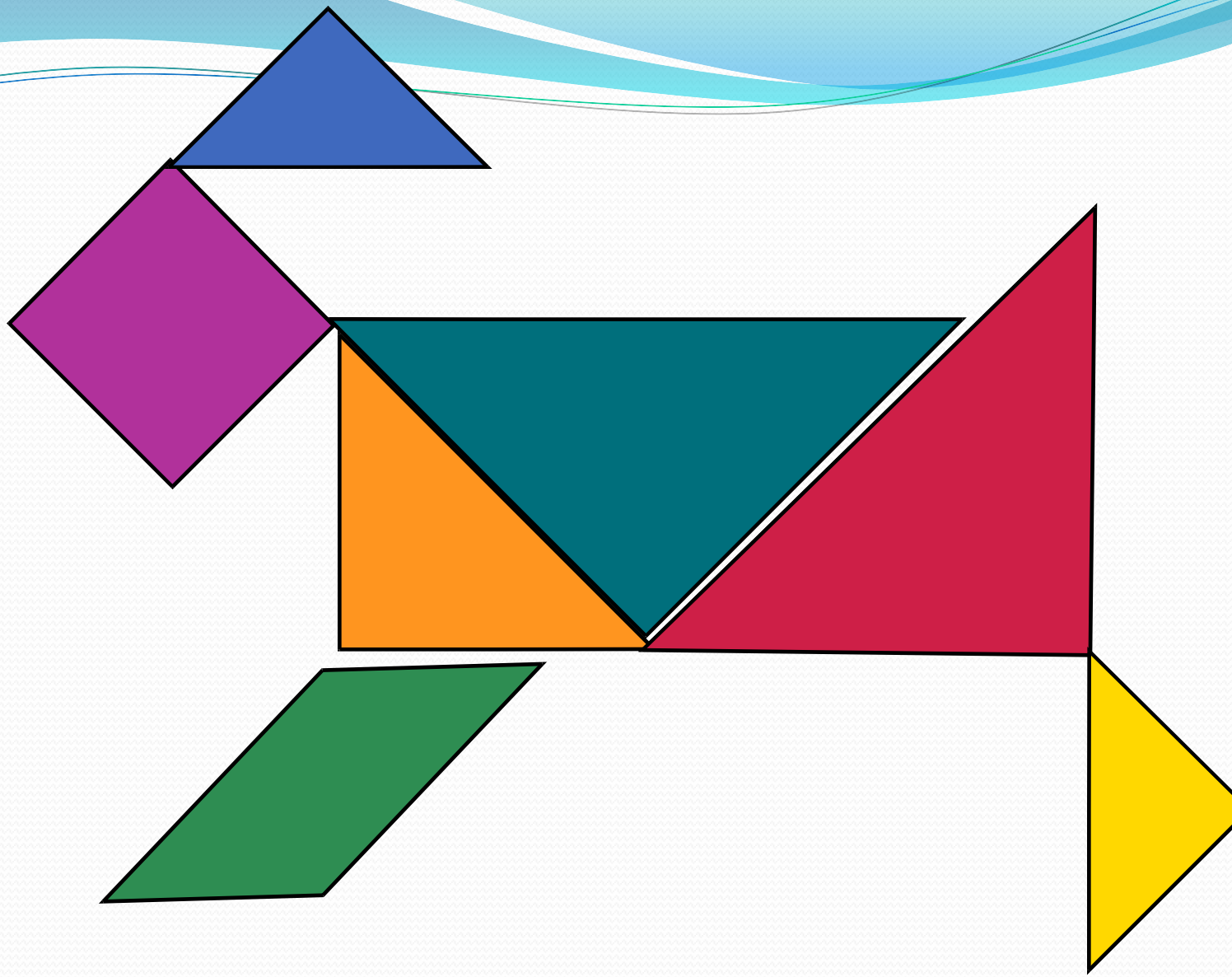
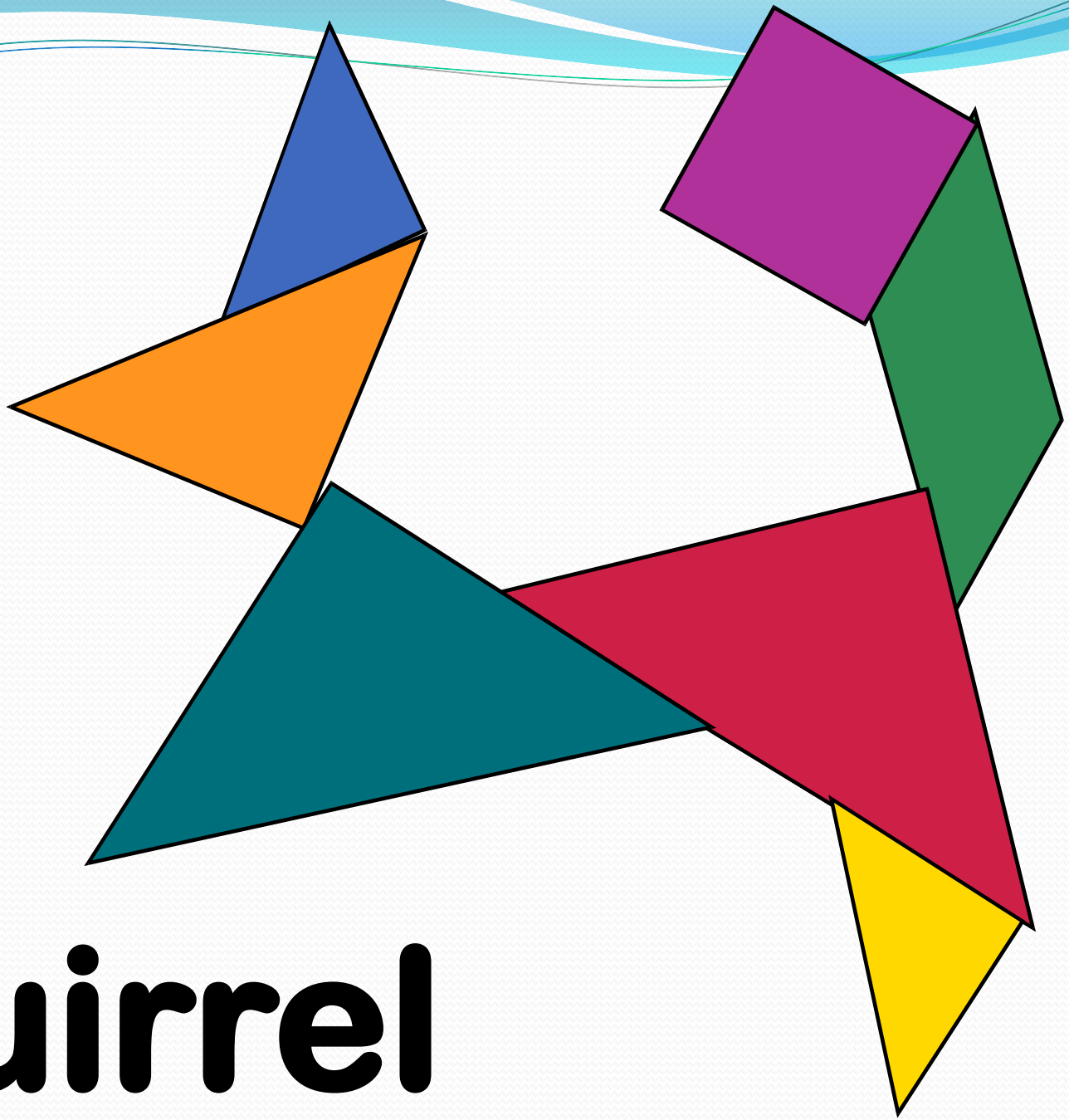
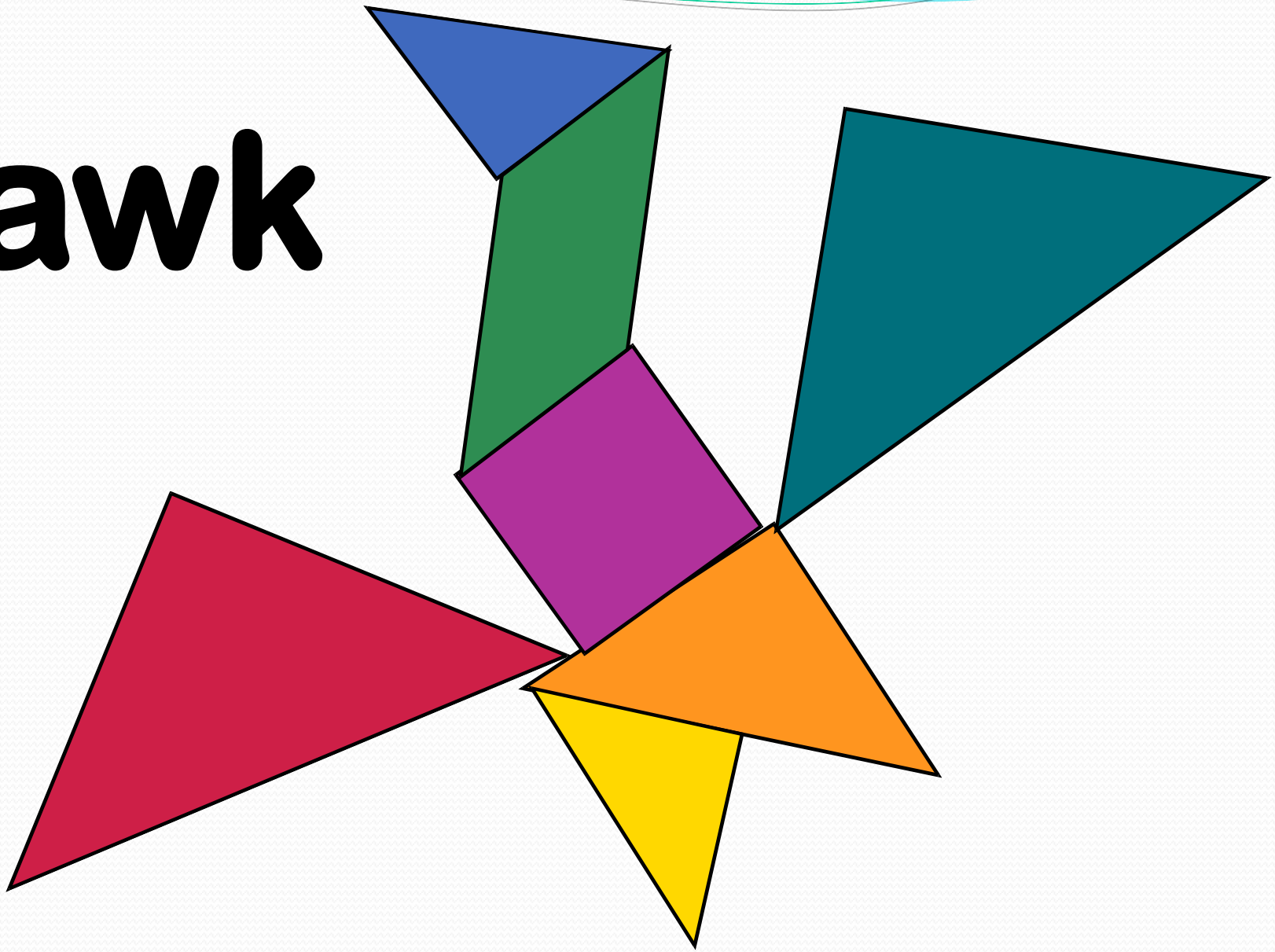


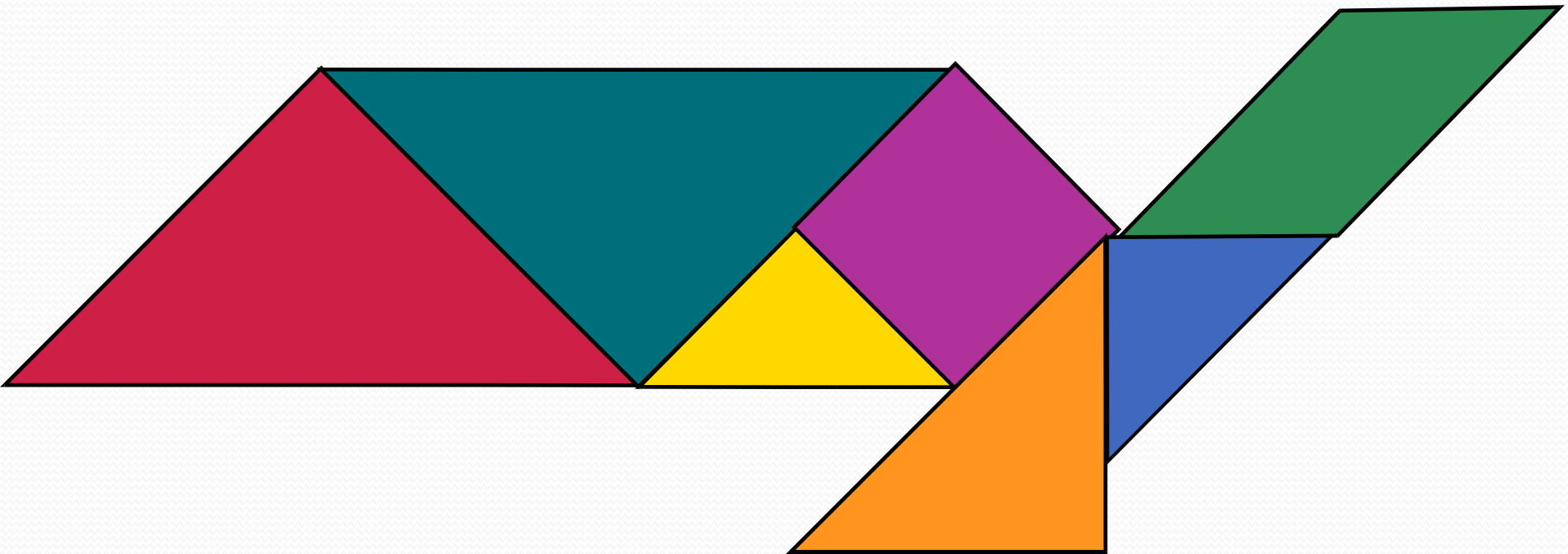


Squirrel

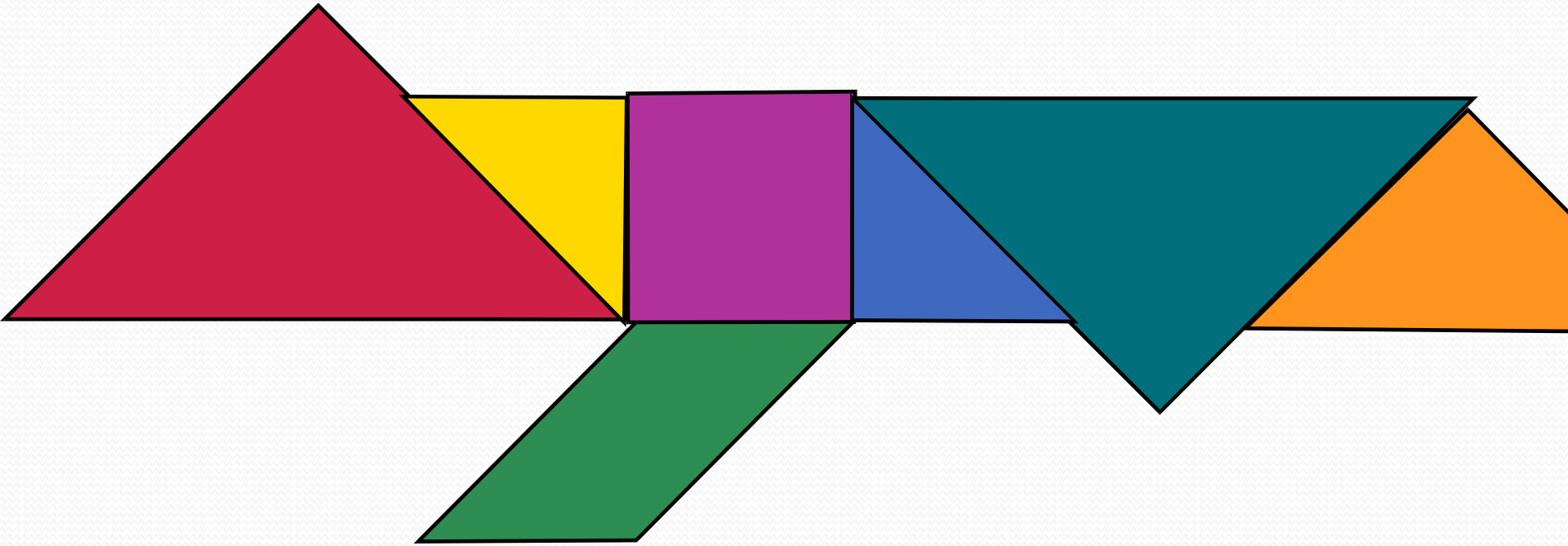
Hawk

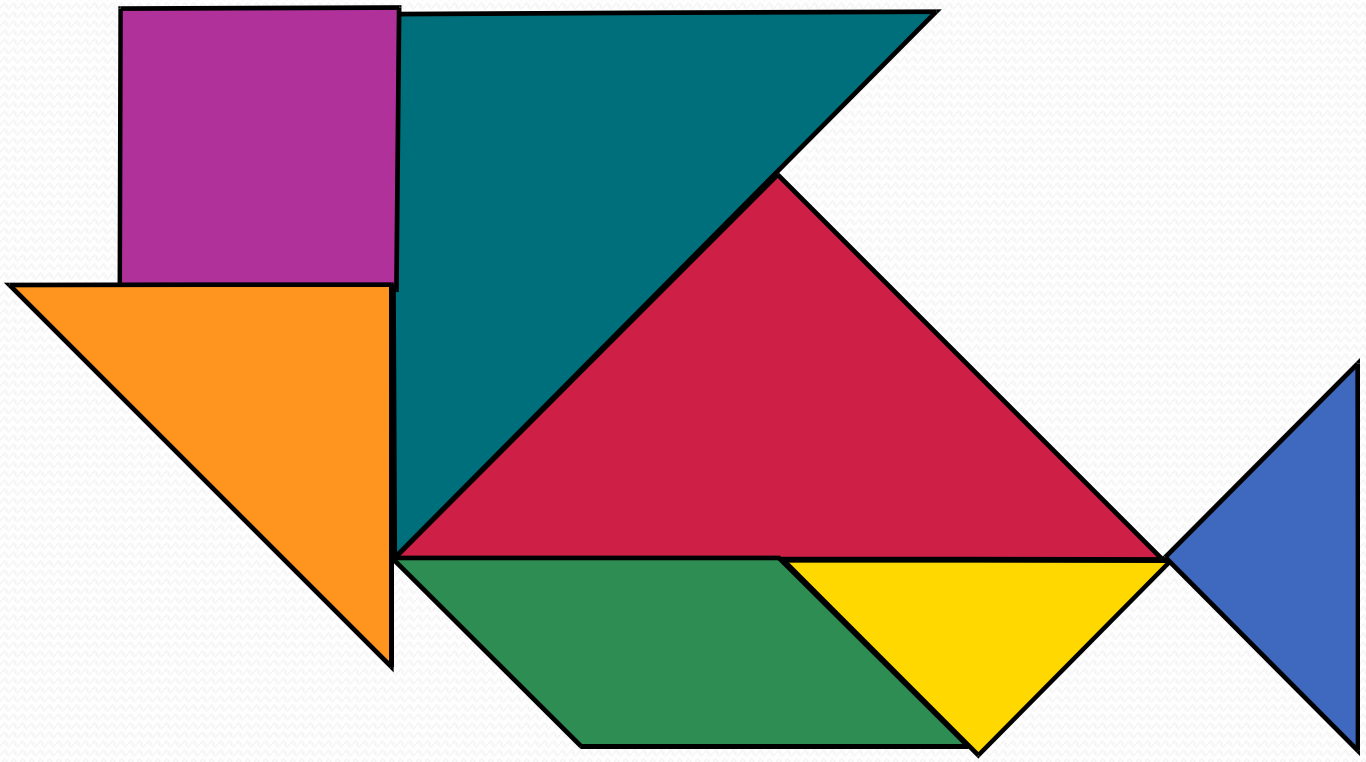


TURTLE



CROCODILE





GOLDFISH

GOOSE

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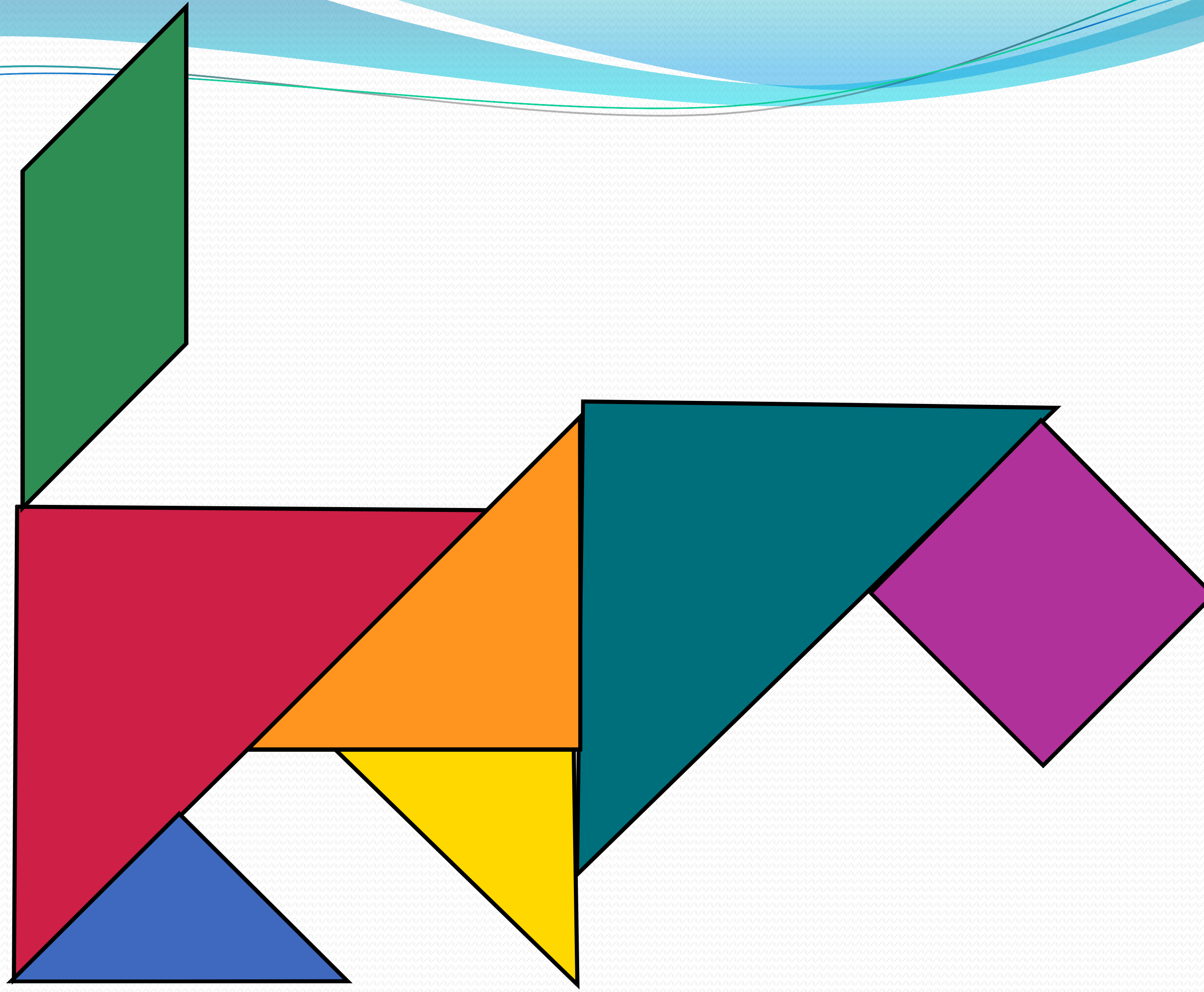
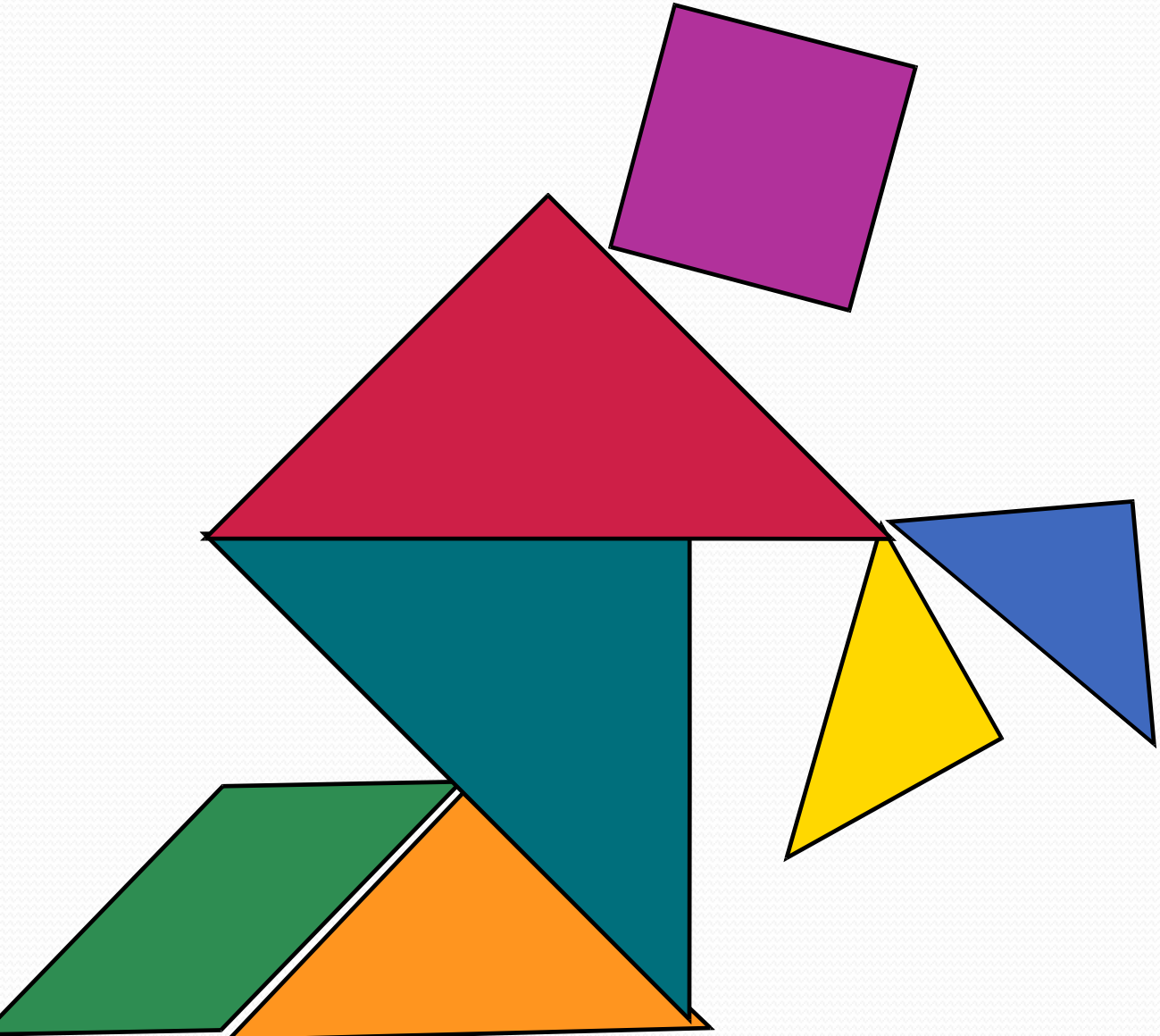
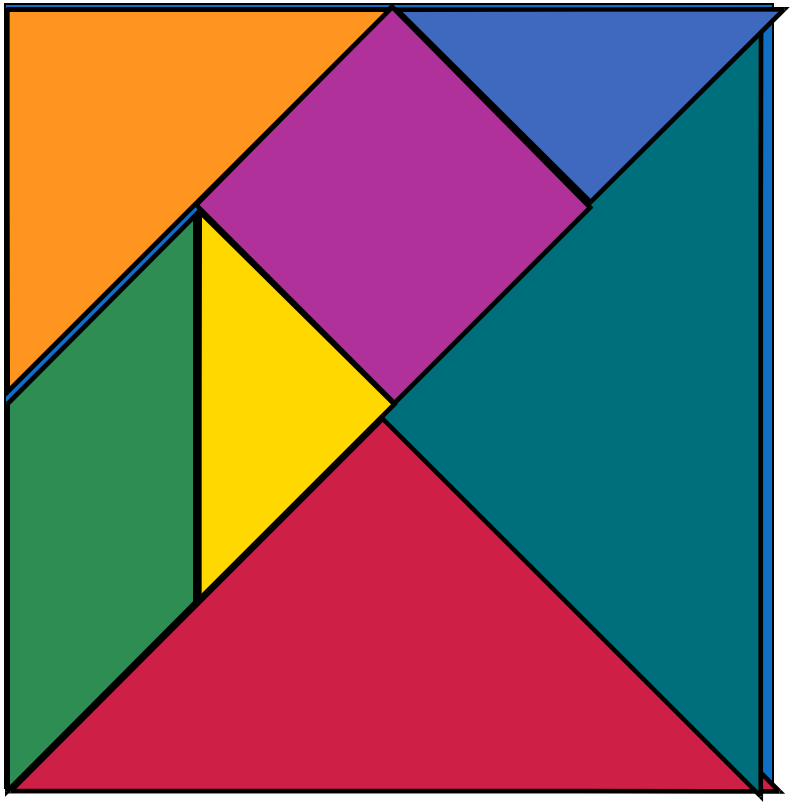




The end



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