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Putting the *Thinking* back into Problem Solving

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5-year, $9,235,407, Targeted Math Science Partnership funded by NSF

Primarily Math

An initiative to strengthen the teaching and learning of mathematics in grades K-3 through the creation of a mathematics specialist certification program

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On a 3x5 card, quickly jot down a story problem suitable for students in grades K-2 that is solved by:

(A) addition; label the card (A)

(B) subtraction; label the card (B)

Brainstorm list of key words

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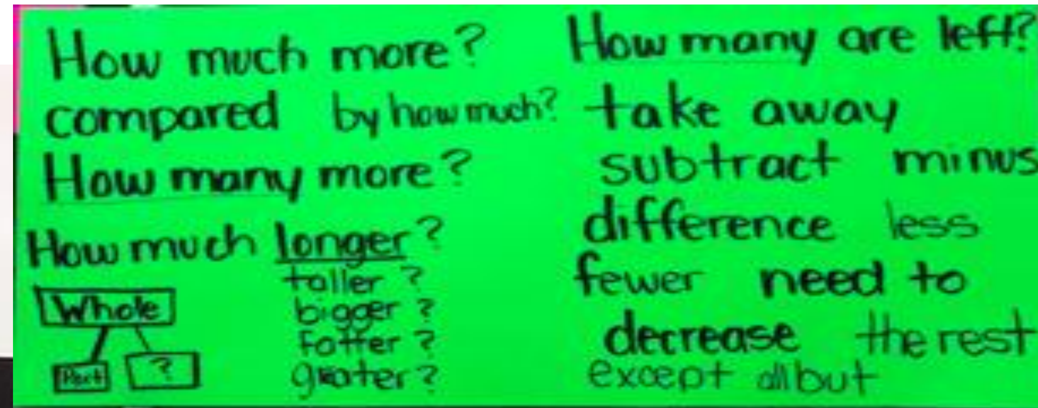
(A) Addition

(B) Subtraction



(A) *Juan has 5 marbles. Anna gives him 7 more. How many marbles does Juan have in all?*

(B) *Nakita made 18 cookies. She gave 12 of them to her teacher. How many cookies does Nakita have left?*

Key words

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(A) *Juan has 5 marbles. Anna gives him 7 more. How many marbles does Juan have in all?*

(B) *Nakita made 18 cookies. She gave 12 of them to her teacher. How many cookies does Nakita have left?*

Key words

Let's make some small changes…

(A) *Juan has 5 marbles. Anna gives him ~~7~~ some more. ~~How many marbles does~~ Juan have in all?*

(B) *Nakita made 18 cookies. She gave 12 of them to her teacher. How many cookies does Nakita have left?*

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(A) *Juan has 5 marbles. Anna gives him ~~7~~ some more. ~~How many marbles does~~ Now Juan ~~have~~ has 12 marbles in all. How many marbles did Anna give to Juan?*

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(A) *Juan has 5 marbles. Anna gives him some more. Now Juan have has 12 marbles in all. How many marbles did Anna give to Juan?*

(B) *Nakita made ~~18~~ some cookies. She gave 12 of them to her teacher. ~~How many cookies does~~ Nakita has 6 cookies left. How many cookies did she make?*

Problems (A) and (B) revised:

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(A_R) *Juan has 5 marbles. Anna gives him some* more. *Now* Juan *has 12 marbles in all. How many marbles did Anna give to Juan?*

Solve

(B_R) *Nakita made some cookies. She gave 12 of them to her teacher.* Nakita *has 6* cookies *left. How many cookies did she make?*

What operations did you use?

Problems (A) and (B) revised:

(A_R) *Juan has 5 marbles. Anna gives him some more. Now Juan has 12 marbles in all. How many marbles did Anna give to Juan?*

So what's the problem with key words?

(B_R) *Nakita made some cookies. She gave 12 of them to her teacher. Nakita has 6 cookies left. How many cookies did she make?*

Key words

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"*What is the fundamental message the kids get when told to look for the key/cue word*?

Don't read the problem. Don't imagine the situation. Ignore the context. Abandon your prior knowledge….You don't have to read; you don't have to think. Just grab the numbers and compute."

-Hyde, Comprehending Math, 2006

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A careful look…

Problem (A) began in the form

number	+	number	=	?

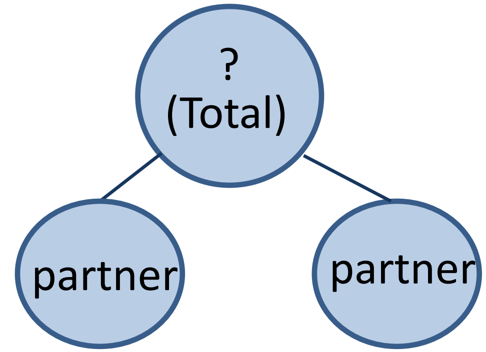
Revised problem (A_R) is in the form

number	+	?	=	number

Additional representations:

number + number = ?

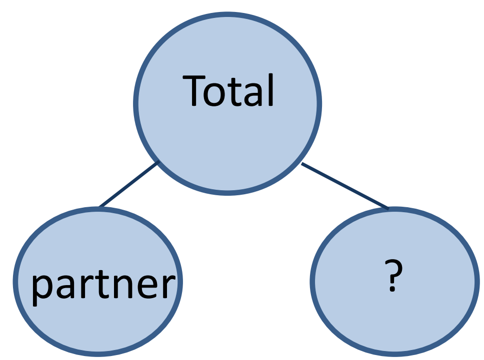
?	
number	number



had + got = now

number + ? = number

number	
number	?



A careful look…

Problem (B) began in the form

number	–	number	=	?

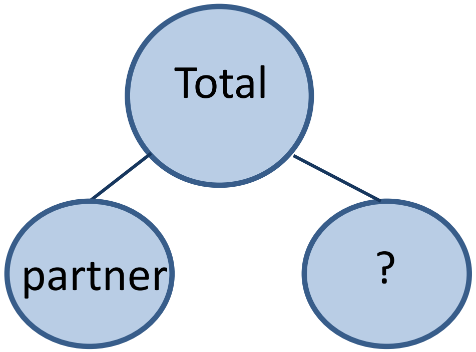
Revised problem (B_R) is in the form

?	–	number	=	number

Additional representations:

number	–	number	=	?

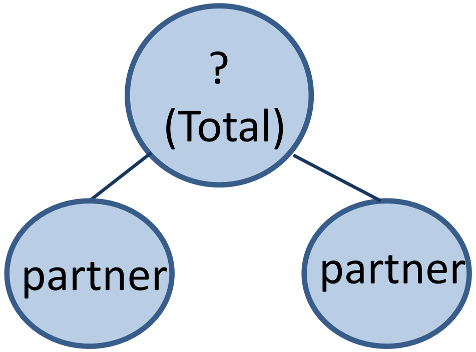
number	
number	?



had – went away = now

?	–	number	=	number

?	
number	number



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*Research indicates that students whose teachers emphasize higher-order thinking skills (in addition to skills), particularly those involving the development of strategies to solve different types of problems, **outperform their peers by about 40% of a grade level***

- Wenglinkiy, 2000.

Now it's your turn...

On separate 3x5 cards, rewrite your own stories (A) and (B) changing the position of the unknown. Label them (A_R) and (B_R) .

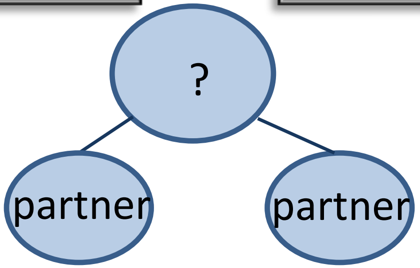
Now it's your turn…

- Identify a *face partner.*
- Exchange your revisions with your face partner*;* the partner reads the revised stories (A_R) and (B_R) and draws two representations for each on the back of the card.

Examples:

? – number = number

?	
number	number



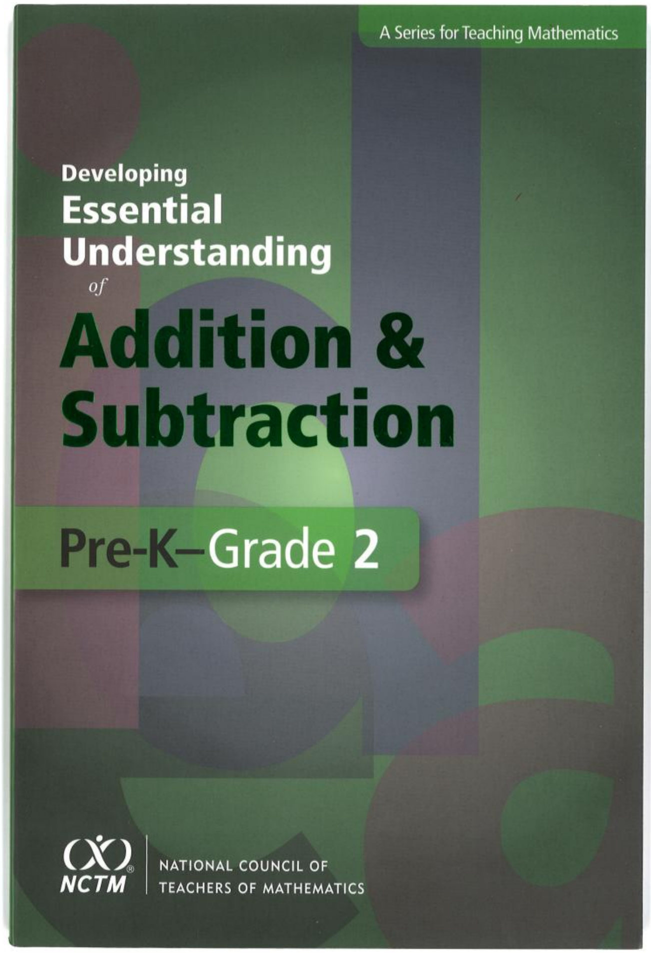
- Give the cards back to your face partner and discuss.
  - Do you agree with each others' representations? (are the unknowns the right places?)
  - Are the representations helpful in understanding the problem? Are some more helpful than others?

- Return to your seats.

NCTM series

Developing Essential Understanding

Pre-K – Grade 2



	Result Unknown	Change Unknown	Start Unknown
Add to	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? 2 + 3 = ?	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? 2 + ? = 5	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? ? + 3 = 5
Take from	Five apples were on the table. I ate two apples. How many apples are on the table now? 5 - 2 = ?	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? 5 - ? = 3	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? ? - 2 = 3
	Total Unknown	**Addend Unknown**	**Both Addends Unknown**
Put Together/ Take Apart	Three red apples and two green apples are on the table. How many apples are on the table? 3 + 2 = ?	Five apples are on the table. Three are red and the rest are green. How many apples are green? 3 + ? = 5, 5 - 3 = ?	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? 5 = 0 + 5, 5 = 5 + 0 5 = 1 + 4, 5 = 4 + 1 5 = 2 + 3, 5 = 3 + 2
	Difference Unknown	**Bigger Unknown**	**Smaller Unknown**
Compare	("How many more?" version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? ("How many fewer?" version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? 2 + ? = 5, 5 - 2 = ?	(Version with "more"): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? 2 + 3 = ?, 3 + 2 = ?	(Version with "more"): Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? 5 - 3 = ?, ? + 3 = 5

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Review the chart categorizing the different types of addition & subtraction problems.

- Share some observations at your table
- What is the difference between these categories:

Put Together/ Take Apart	vs	**Add To** and **Take From**

Note: Some refer to Put Together/Take Apart as **Part-Part-Whole** problems

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Put Together/ Take Apart

Example:

Levi has 12 marbles. Five of the marbles are red and the rest of the marbles are blue. How many blue marbles does Levi have?

	Result Unknown	Change Unknown	Start Unknown
Add to	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? 2 + 3 = ? A	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? 2 + ? = 5 A_R	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? ? + 3 = 5
Take from	Five apples were on the table. I ate two apples. How many apples are on the table now? 5 - 2 = ? B	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? 5 - ? = 3	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? ? - 2 = 3 B_R
	Total Unknown	**Addend Unknown**	**Both Addends Unknown**
Put Together/ Take Apart	Three red apples and two green apples are on the table. How many apples are on the table? 3 + 2 = ?	Five apples are on the table. Three are red and the rest are green. How many apples are green? 3 + ? = 5, 5 - 3 = ?	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? 5 = 0 + 5, 5 = 5 + 0 5 = 1 + 4, 5 = 4 + 1 5 = 2 + 3, 5 = 3 + 2
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Where do Juan and Nakita fit in?

With a ***shoulder partner***

You and your partner should have eight problems between you (4 original, 4 revised)

1. Determine which types of problems you and your partner wrote and where they fit on the chart.

2. Determine which categories are not represented and together write two more problems (C) and (D) to fit an unrepresented category as described:

(C) An Add To, Take From or Put Together/Take Apart problem that fits in the 2nd or 3rd column

(D) A comparison problem which fits in the 2nd or 3rd column

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C

D

2. With your shoulder partner write new problems for these categories.

With your ***shoulder partner***

3. Check your problems with another pair at your table; do all agree in which categories new problems (C) and (D) should be placed?
4. For discussion: Which types of problems are easiest for you to write? Which are more difficult? Why?

Compare CCSS for grades K and 1:

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CCSS Operations and Algebraic Thinking K.OA

Represent and solve problems involving addition and subtraction.

1. *Represent addition and subtraction with objects, fingers, mental images, drawings, sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.*
2. *Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problems.*

CCSS Operations and Algebraic Thinking 1.OA

Represent and solve problems involving addition and subtraction.

1. *Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.*

Compare CCSS for grades K and 1:

<u>CCSS Operations and Algebraic Thinking K.OA</u>

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Compare CCSS for grades 1 and 2:

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CCSS Operations and Algebraic Thinking 1.OA

Represent and solve problems involving addition and subtraction.

1. *Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.*

CCSS Operations and Algebraic Thinking 2.OA

Represent and solve problems involving addition and subtraction.

1. *Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using ~~objects,~~ drawings, and equations with a symbol for the unknown number to represent the problem.*

Compare CCSS for grades 1 and 2:

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<u>CCSS Operations and Algebraic Thinking 1.OA</u>

Represent and solve problems involving addition and subtraction.

1. *Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.*

<u>CCSS Operations and Algebraic Thinking 2.OA</u>

Represent and solve problems involving addition and subtraction.

1. *Use addition and subtraction within 100 to **<u>solve one- and two-step word problems</u>** <u>involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions,</u> e.g., by using drawings, and equations with a symbol for the unknown number to represent the problem.*

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Take from	Five apples were on the table. I ate two apples. How many apples are on the table now? $5 - 2 = ?$	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? $5 - ? = 3$	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? $? - 2 = 3$
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- We have seen a variety of types of problems and why they are important
- We have seen why focusing exclusively on key words can be misleading

This is why it's so important for students to really *think* about the problems they are solving.

So how do we get them to do this?

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Mathematical problem solving…..

“a hallmark of mathematical activity and a major means of developing mathematical knowledge.”

- NCTM, 2000

Traditional Approaches

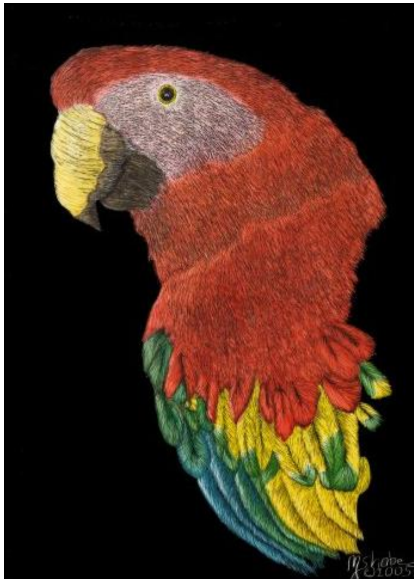
- Rereading problem multiple times and asking guiding questions that remove thinking tasks for students

- "Stepping In"

- "Moving On"

A Change in Our Approach…

Help students avoid mechanical problem-solving approaches. Rather, assist them in working to make sense of the problem situation.



Our goal..

We must change our students from being "problem performers" to "problem solvers!"

Problem Performers	Problem solvers
Do it just like the teacher did. .	Think before you start.
Write your numbers straight.	Draw a picture to understand the problem.
Don't talk when you work. That's cheating.	Explain how you solved it, using math vocabulary.
Find an answer and move on.	Check your work. Do it a different way.

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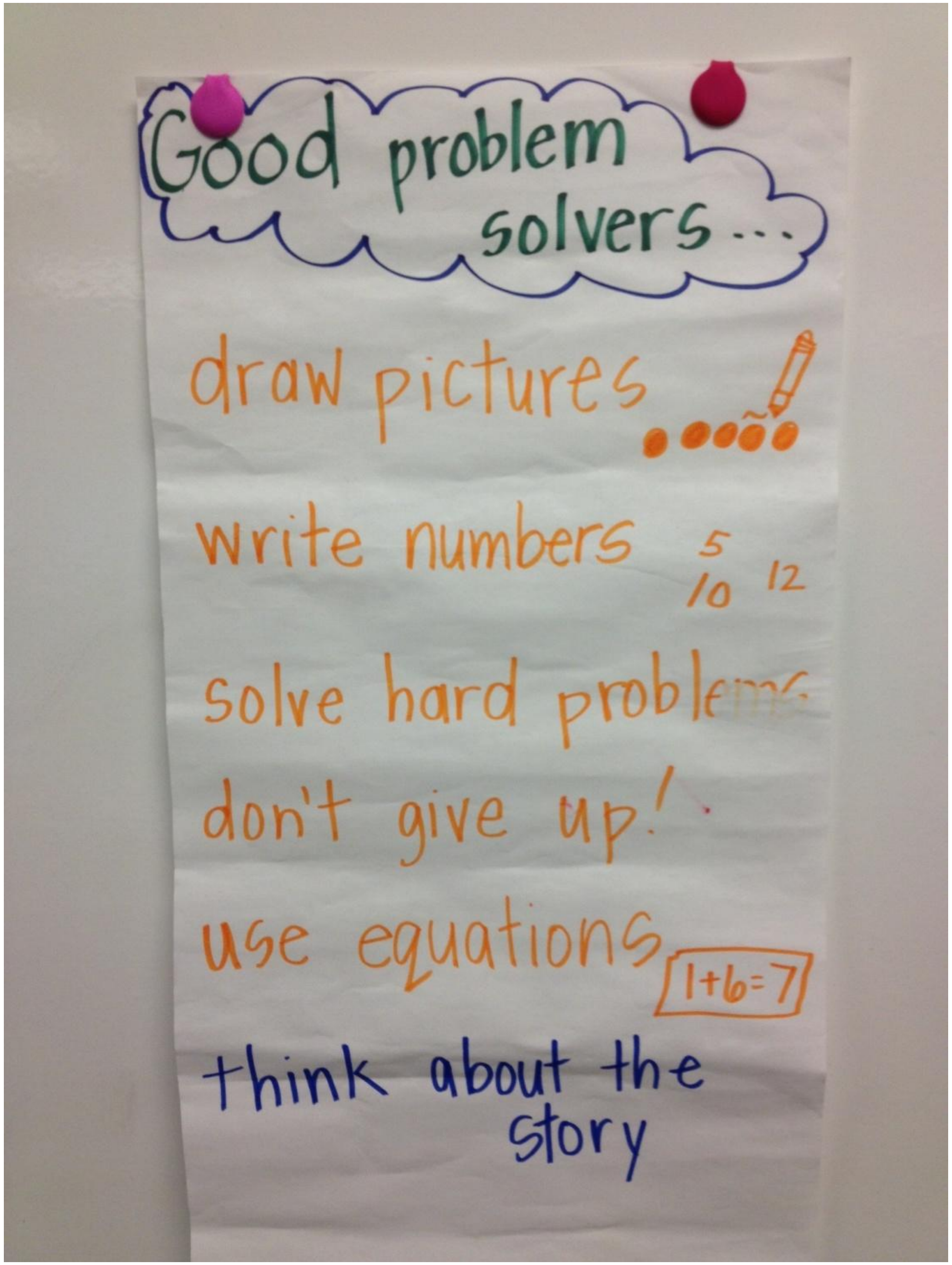

Effective Instruction

Effective mathematics instruction includes:

-carefully selected problems and materials

-opportunities for math discourse

-use of assessment practices to support student thinking

Four Step Model

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- Created by George Polya in 1940's
- Provides a structure for students to follow

Solving problems is a practical art, like swimming, or skiing, or playing the piano: you can learn it only by imitation and practice. . . . if you wish to learn swimming you have to go in the water, and if you wish to become a problem solver you have to solve problems.

Four Steps for Problem Solving

- Understanding the problem
- Devising a plan
- Carrying out the plan
- Look Back

- Understand
- Plan
- Solve
- Look Back

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Understand

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Plan

Solve

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Look Back

Catchy Tune

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Math Problem Solving is FUN!
Math Problem Solving is FUN!

First you take the problem and you Read it, you Read It!
First you take the problem and you Read it, you Read It!

Math Problem Solving is FUN!
Math Problem Solving is FUN!

Then you use the information and you Plan it, you Plan it!
Then you use the information and you Plan it, you Plan it!

Math Problem Solving is FUN!
Math Problem Solving is FUN!

Next you use the plan and you Solve it, you Solve it!
Next you use the plan and you Solve it, you Solve it!

Math Problem Solving is FUN!
Math Problem Solving is FUN!

Last you look back and you Check it, you Check It!
Last you look back and you Check it, you Check It!

Math Problem Solving is FUN!
Math Problem Solving is DONE!

Understand

- *Key idea…No strategy can be used effectively unless one understands the problem.* (Comprehending Math, Arthur Hyde 2006)

- Utilize comprehension strategies taught in literacy

- Promote math talk

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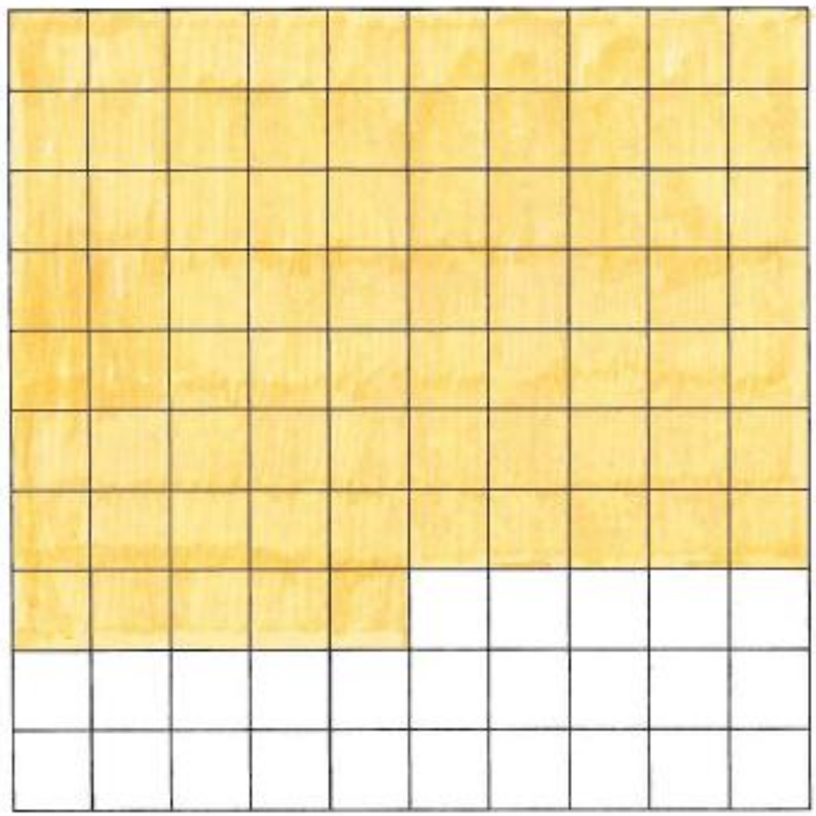
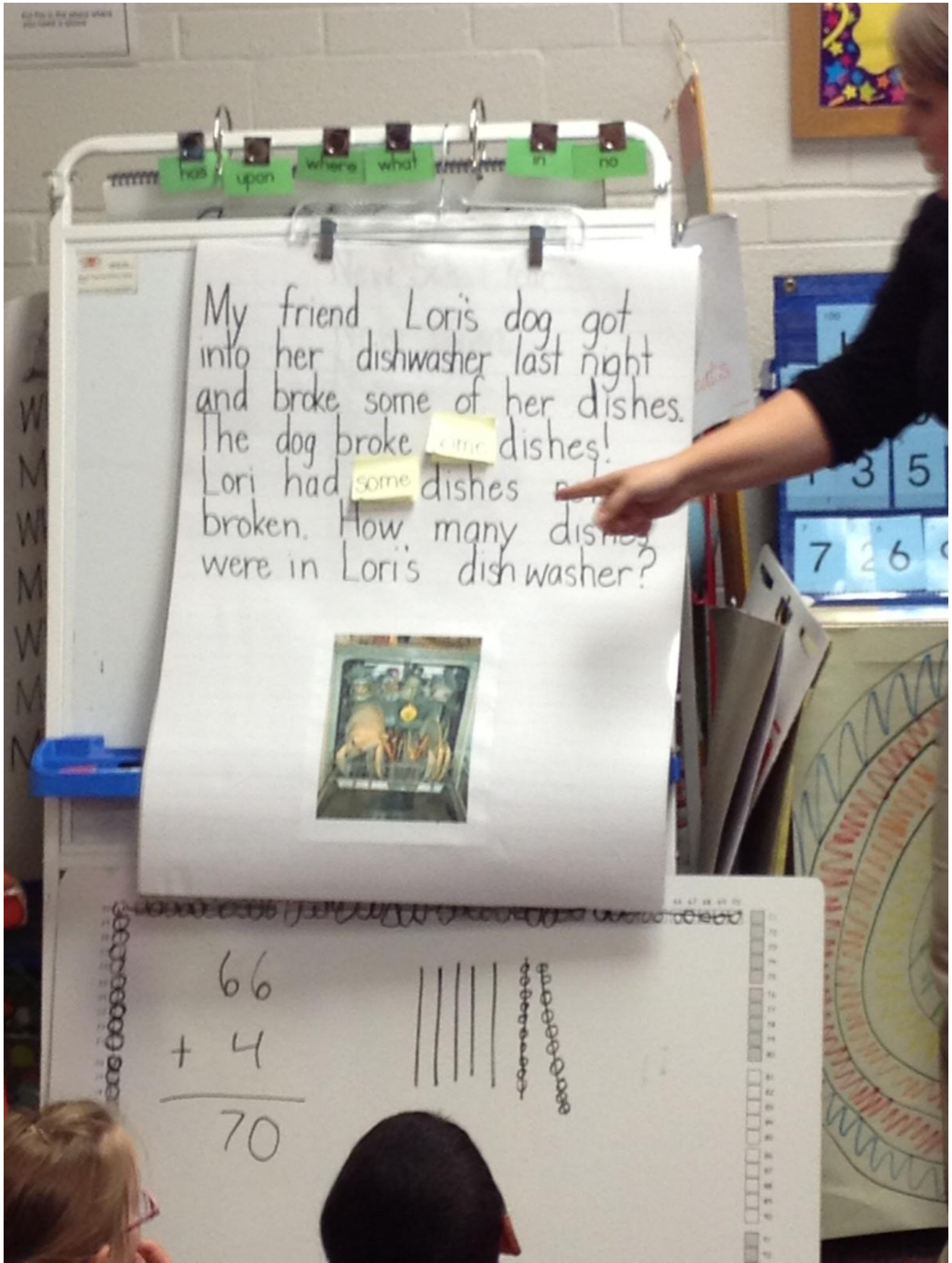
75% of the time spent on a problem





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Problem on Video

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Alina had 17 beads. She used 9 of them to make a bracelet. How many beads does she have left?

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- Video of Jessica goes here

Lucy is a curious dog.

One day Lucy got into the dishwasher.

There were 18 dishes in the dishwasher.

Lucy broke 9 of them.

How many dishes are in the dishwasher now?

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- Video of Susie goes here

SUHUPU

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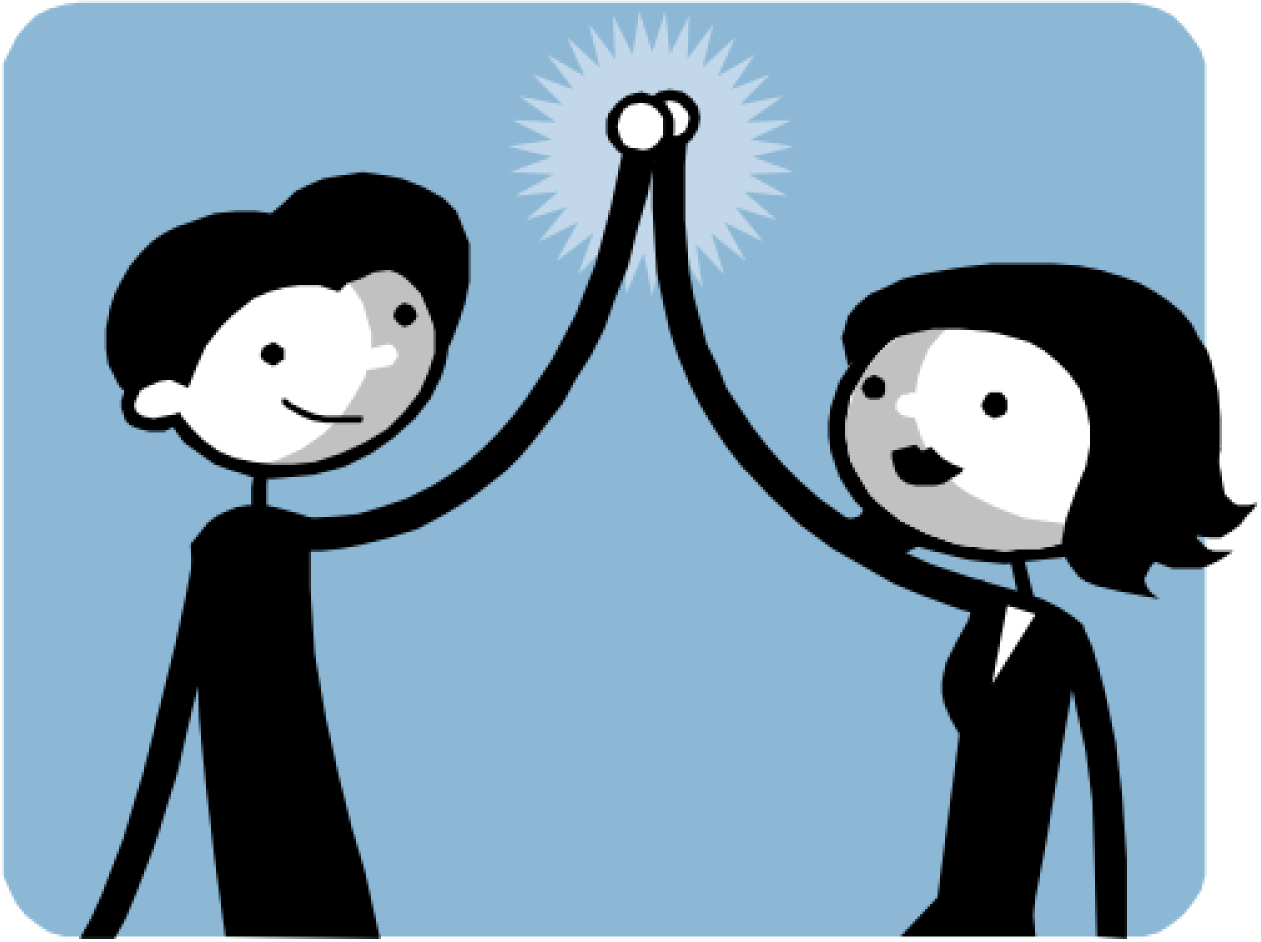
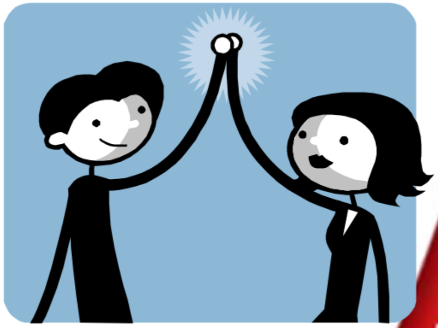


Discussion Questions

- What teacher moves were apparent to help the students understand the situation?

- Discuss the instructional strategies that were used to help students understand the mathematical situation.



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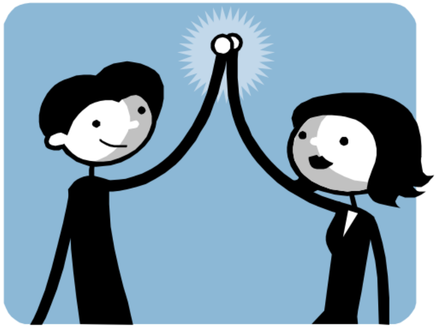
- *How might you utilize this problem in your classroom?*



50 people were on the bus.
10 people got off at the first stop.
30 people got on at the next stop.
How many people are on the bus now?

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A Research Companion to Principles and Standards for School Mathematics

The most powerful problem-solving approach is to understand the situation deeply--that is, to be able to draw it or otherwise represent it to oneself.

Plan

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- Encourage students to “retrieve strategies from their mental toolbox”
- Students determine what representations they can use to help them solve the problem
- Have students verbalize their plans

Solve

- Work on problem
- “Show your thinking along the way!”
- Should I try another strategy?
- Answer is determined

Look Back

- Check the answer
  - Does it look right?
  - Does it make sense?
  - Can you show it another way?
  - Is the answer reasonable?

- Include the correct label

Math Talk

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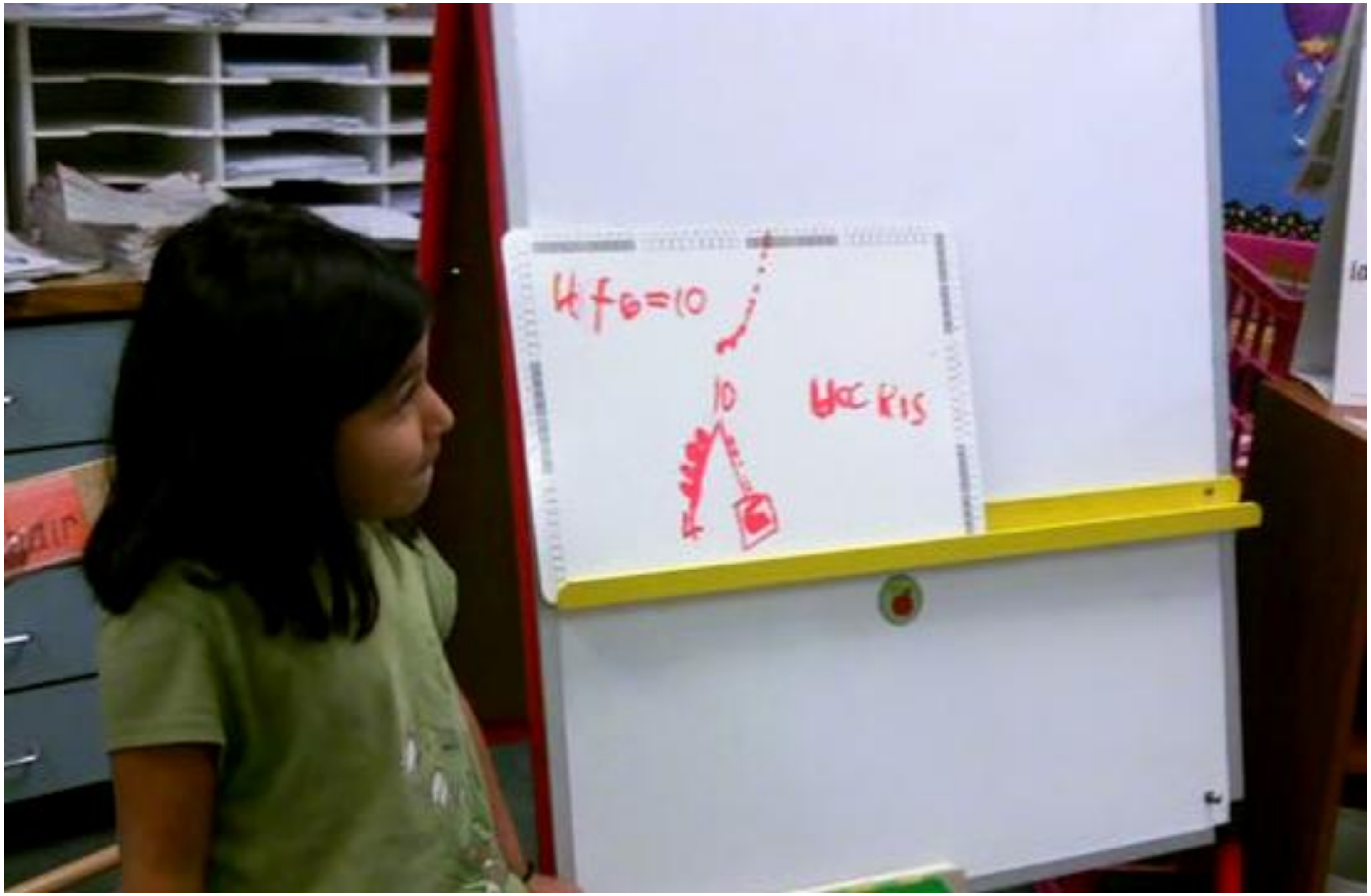

- Teachers make instructional choices that support the opportunity for all students to learn important mathematics

- Teachers find ways to support students as they work through challenging tasks without taking over the process of thinking for them (Don’t eliminate the challenge!!)

Teacher Moves to Elicit Thinking

Teacher Moves to Support a Child's Thinking before a Correct Answer Is Given

Category	Sample Teacher Moves
Ensure that the child understands the problem.	Ask him to explain what he knows about the problem. Rephrase or elaborate the problem. Use a more familiar or personalized context in the problem.
Change the mathematics in the problem to match the child's level of understanding.	Change the problem to use easier numbers. Change the problem to use an easier mathematical structure.
Explore what the child has already done.	Ask him to explain a partial or incorrect strategy. Ask specific questions to explore how what he has already done relates to the quantities and relationships in the problem.
Remind the child to use other strategies.	Ask him to consider using a different tool. Ask him to consider using a different strategy. Remind him of relevant strategies he has used before.

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Teacher Moves to Extend a Child's Thinking after a Correct Answer Is Given

Category	Sample teacher moves
Promote reflection on the strategy the child just completed.	Ask her to explain her strategy. Ask specific questions to clarify how the details of her strategy are connected to the quantities and mathematical relationships in the problem.
Encourage the child to explore multiple strategies and their connections.	Ask her to try any second strategy. Ask her to try a second strategy connected to her initial strategy in deliberate ways (e.g., more efficient counting or abstraction of work with manipulatives). Ask her to compare and contrast strategies.
Connect the child's thinking to symbolic notation.	Ask her to write a number sentence that "goes with" the problem. Ask her to record her strategy.
Generate follow-up problems linked to the problem the child just completed.	Ask her to solve the same or a similar problem with numbers that are more challenging. Ask her to solve the same or a similar problem with numbers that are strategically selected to promote more sophisticated strategies.

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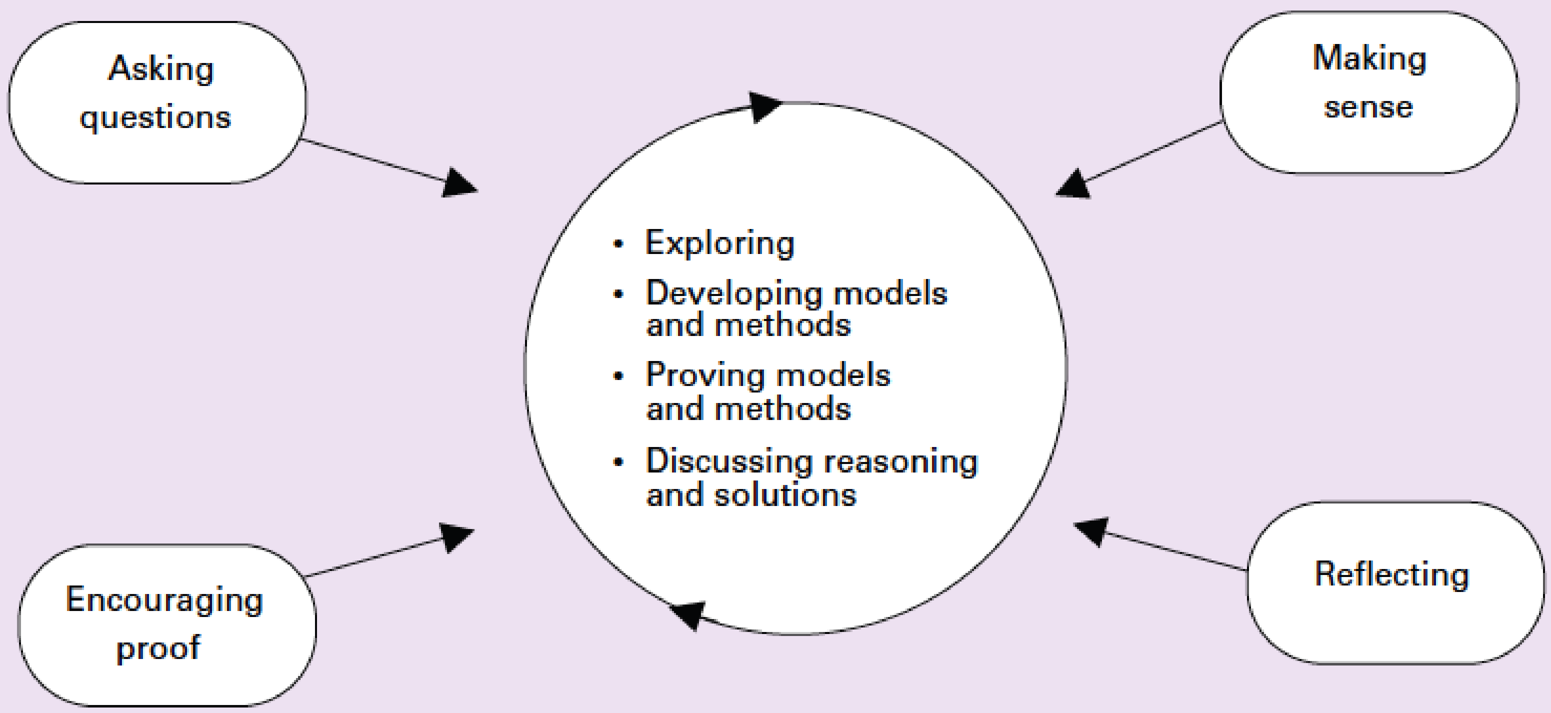
Mathematical problem-solving process



Adapted from Rigelman (2002, p. 184)

Where do you begin?

- Deepen your own professional knowledge on the subject
- MODEL, MODEL, MODEL
- Scaffold learning opportunities
- Give students numerous problem solving opportunities

NCTM states…

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"The challenge at this level (K-5) is to build on children's innate problem-solving inclinations and to preserve and encourage a disposition that values problem solving…By allowing time for thinking, believing that young students can solve problems, listening carefully to their explanations, and structuring an environment that values the work that students do, teachers promote problem solving and help make their strategies explicit."

(Principles and Standards for School Mathematics, 2000)

Talking Chips

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- Which of your current practices related to teaching problem solving were affirmed by today's presentation?
- What topic related to problem solving would you like to continue to explore and why?
- What is the first thing you'd like to try in your classroom when you return?
- How will your instruction for problem solving change from what you experienced today?
- What is your biggest "take away" from today's session?

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Building students' mathematical reasoning and problem-solving skills requires teachers to teach mathematics as the power of thought rather than the power of discrete facts.

McCrel. (2010). *What we know about mathematics teaching and learning.* Bloomington, IN: Solution Tree Press.

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Questions?

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Student’s Misguide to Problem Solving

Rule #1: If at all possible, avoid reading the problem. Reading the problem only consumes time and causes confusion.

Rule #2: Extract the numbers from the problem in order they appear. Be on the watch for numbers in written in words.

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Rule #3: If rule 2 yields three or more numbers, the best bet is adding them together.

Rule #4: If there are only 2 numbers which are approximately the same size, then subtraction should give the best results.

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Rule #5: If there are only two numbers and one is much smaller than the other, then divide if it goes evenly—otherwise multiply.

Rule #6: If the problem seems like it calls for a formula, pick a formula that has enough letters to use all the numbers given in a problem.

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Rule #7: If rules #1-6 don't seem to work, make one desperate attempt. Take the set of numbers found by rule #2 and perform about two pages of random operations using these numbers. You should circle about five or six answers on each page just in case one of them happens to be the answer. You might get some partial credit for trying hard.