Fold Some Geometry

Presented By

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Paper Folding and Origami

Origami has numerous Mathematical Applications

- To explore patterns
- To interpret and follow directions
- To develop mathematical vocabulary
- To develop geometric concepts
- To develop fractional concepts
- To investigate three-dimensional objects and develop spatial sense
- To investigate angle measurements and relationships
- To develop problem solving and critical thinking

Suggestions when doing paper folding or Origami:

- Fold on a firm surface.
- Precision … fold neatly and accurately.
- Crease each fold at least three times (using fingernail or pencil).
- Be patient and practice.

The Magic Circle

Materials: one 7 inch paper circle (a thin paper plate works wonderfully), pencil

1. What is the shape of the plate?

2. What is the distance around the outside of the circle called?

3. Fold your circle in half and crease it well. Open the circle. What is the crease you made called?

4. Fold your circle in half again matching up the end points of the diameter. What is this new crease called?

5. Do the line segments (diameters) intersect?

6. Is there something special about the way these line segments intersect?

7. Place a small dot at the point where the creases connect. What is this point called?

8. Using your pencil, trace one line segment from the center to the edge of the circle. What is the traced line segment called?

9. Fold in one of the outer, curved edges of the circle until it just touches the dot in the middle (the center point). Crease it well.

10. Open the fold and look at the crease you just made. Is it a diameter? Is it a radius? Why or why not? What is this line segment called?

11. Look at the curved part of the circle between the points where this line touches the outside of the circle. This is called an arc. Can you find other arcs on your circle?

12. Refold the circle on the chord. On the opposite side of your circle, fold another chord so that the outside of the circle touches the center point and it shares an endpoint with the previous chord. You will now have a figure that looks like an ice cream cone. Crease it well.

13. Fold the top of your ice cream cone down until the curved part just touches the center of the circle. The corners should make perfect points, crease well. What shape do you now have?

14. Do you notice anything special about this triangle? Look at all of the angles; they are the same measure. The sides are all the same measure as well. What is this type of triangle called?

15. With the flaps still folded but facing up, fold the new triangle in half by matching up two of the vertices. Crease well. The new crease splits the original triangle in half, this new crease is called the height or altitude. Can you figure out anything about this new triangle?

16. Open the right triangle up to the equilateral triangle.

17. Take the top corner of the big triangle and fold it down so that the top vertex touches the opposite side of the triangle at the point where the previous crease was formed. You will now see three smaller triangles. What type of triangles are these?

18. Turn the paper over so that you do not see the three separate triangles. What is this shape called?

19. Turn it back over so that you now see all of the creases. Fold one of the outer triangles in so that it lies directly on top of the center triangle. Turn it back over and describe the shape you now see.

20. Turn your shape back over and fold the last outer triangle over onto the center once again. You should now have a smaller equilateral triangle.

21. Open up all three of the small triangles. Bring the three loose points together so that you now have a pyramid. At this point you can discuss faces, edges, vertices, bases and points and the fact that this is a triangular pyramid and not a squared pyramid like those built in Egypt.

22. Open your pyramid back up to the large equilateral triangle.

23. Fold over one of the vertices so that it just touches the dot in the middle. What shape have you re-created?

24. Fold another vertex of the triangle in so that it just touches the dot in the middle. Now what shape do you have?

25. Now fold in the last vertex of the large triangle. What shape is it now?

26. Take two of the small triangles that you just formed and raise them up (with the main hexagon still on the desk) and place one inside the flap of the other. You may have to try more than one. Choose the one that makes the best fit. Slide the last corner under/inside the others. You have now created a truncated tetrahedron.

Extensions: For any of the shapes during this activity you can have students calculate the surface area, volume, perimeter and/or area. Students can also prove the steps based on the geometric relationships.

Folding a Triangular Hexahedron or a Cube

Materials for Cube: 6 sheets of origami paper (2 each of 3 different colors), preferably 7×7
Materials for Triangular Hexahedron: 3 sheets of origami paper (different colors), preferably 7×7

Steps 1-22 are directions to fold a single component of the figure. You will need to fold a total of 3 of these for assembling the triangular hexahedron in steps 23-27 and 6 of these for assembling the cube in steps 28-31.

1. With the white side of the paper up, fold the paper in half using a book fold. A book fold means that the square will be folded into two congruent rectangles. Make sure to have a crisp crease.

2. Open the paper back up so you again have the white side facing you with the ½ fold line running horizontally.

3. Fold each rectangle in half so that the new folds run parallel with the book fold made in step (1). This is done by bringing the bottom edge of the paper and aligning it with the ½ fold line from step (1).

4. Open the paper back up. What do you see? (You will now have 4 small congruent rectangles with all three fold lines running horizontally and parallel to one another.)

5. Take the bottom right corner and fold a small right isosceles triangle so that the length of one leg of the triangle equals the width of one small rectangle. One leg of the triangle will sit directly on the ¼ fold line. What is the shape of the entire paper? Leave this right isosceles triangle folded over. (pentagon)

6. Repeat step 5 for the opposite corner, the top left corner.

7. By leaving the triangles folded over, what shape do you have? (irregular hexagon)

8. For the bottom right isosceles triangle, place your finger on the 45 degree angle located on the ¼ fold line. Bring the hypotenuse of the triangle up so that it sits on the ¼ fold line and fold a new isosceles triangle. You now have an obtuse isosceles triangle created by bisecting the 45 degree angle located on the ¼ fold line. Leave this obtuse isosceles triangle folded over.

9. Once again, repeat step 8 for the upper left corner. What shape do you now have?

10. Refold along the bottom ¼ fold line as well as the top ¼ fold line. (You are essentially tucking away the isosceles triangle you just made.)

11. You now have a rectangle with the ½ fold line slightly visible in the center still running horizontally.

12. Take the bottom left corner and fold it upwards to create a right isosceles triangle so that the length of one leg of the triangle equals the width of the rectangle. One leg of the triangle will sit flush with the top edge of the rectangle. What shape do you have now? Leave this right isosceles triangle folded over. (right trapezoid)

13. Repeat step 12 for the opposite corner, the top right corner.

14. At this point you will no longer see any white (or very little) and you will now have a parallelogram created by two congruent right isosceles triangles.

15. Tuck the 90 degree angle of the triangles into the pockets underneath each.

16. Smooth out your parallelogram.

17. The parallelogram will have a smooth side (no folds) and a folded side (containing fold lines).

18. Have the smooth side of the parallelogram facing up with the longer side running horizontally.

19. Take the top left acute angle of the parallelogram and fold an isosceles right triangle so that the acute angle sits on top of the top right obtuse angle of the parallelogram.

20. Leave this isosceles right triangle folded over. The crease in step 19 is very important and must be as crisp as possible.

21. Repeat steps 19 and 20 for the opposite corner. The bottom right acute angle will sit on top of the bottom left obtuse angle.

22. You now have a square. On one side all folds are tucked tight and on the other you have two flaps which are congruent right isosceles triangles.

Assembly Directions for the Triangular Hexahedron:

23. Fold a total of 3 individual components. You should have three different colors. Check to make sure that all three pieces are folded identically.

24. An isosceles right triangle flap of Color A will tuck into a pocket of the square on Color B. The pockets are the small 45 degree angle pockets on the face of the square. Color A and Color B should take on a perpendicular orientation with one another when inserted.

25. Next fit a flap of Color C into the pocket of Color A.

26. Crease along the diagonal of the square units. Continue to insert the flaps into the corresponding square pockets.

27. Make sure to keep all flaps to the outside of the figure until they are tucked into the corresponding pocket.

Extensions: Discuss the faces (shapes, number) and how this leads to the name of the solid. This solid is also called a “double tetrahedron” or a “triangular bipyramid”.

Assembly Directions for the Cube:

28. Fold a total of 6 individual components. You should have three different colors (2 of each color). Check to make sure that all six pieces are folded identically.

29. The isosceles right triangle flaps of color A will tuck into the pockets of the square of color B. The pockets are the small 45 degree angles pockets on the face of the square.

30. Match up the square face and triangle flaps as follows;

Square Face	Triangle Flaps
Color A	Color B
Color B	Color C
Color C	Color A

31. Keep all flaps to the outside of the figure until they are tucked into the corresponding pocket. The goal is to make sure that every flap is tucked away in a pocket on the square face. Each square will have a similar two-color pattern.

Notes for the Cube:
There are numerous publications and internet sites devoted to Origami, the art of paper folding. The cube described in this activity is a variation of the Sonobe Cube and is categorized as a modular origami design. In modular origami, several sheets of origami paper are used and each sheet of paper is an individual component of the final product. The individual components are usually identical and assembled to create the final product.

Folding a Paper Bag

Materials: one brown paper bag (length > width, bottom is sealed)

1. Start by folding the bag in half lengthwise and unfold. You now have a horizontal fold line (view the bag in landscape orientation with the raw edge to the right).

2. Take the top left vertex point of the bag and bring it down so that the vertex point is on the crease (fold line) from step (1) in such a way that you have created a triangle with the top left and bottom left vertex points of the bag serving as two of the vertex points in the triangle.

3. Can you classify this triangle? Provide a convincing argument for your classification.

4. Flip the bag over so the top visible edge of the bag is longer than the bottom visible edge of the bag.

5. Fold so the left folded edge of the bag sits flush with the top of the bag.

6. Unfold the bag completely and flip the bag back to its original position and repeat step (2) using the bottom left vertex of the bag.

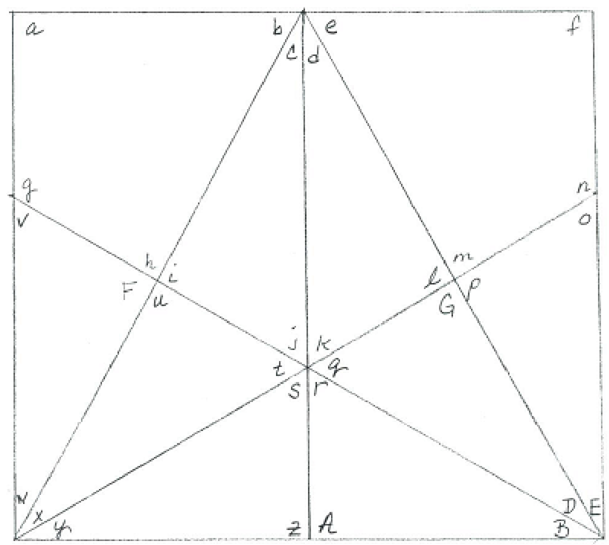
7. Flip the bag over so the bottom visible edge of the bag is longer than the top visible edge of the bag.

8. Fold so the left folded edge of the bag sits flush with the bottom of the bag.

9. Unfold the bag completely and flip the bag back to its original position.

10. Identify the large equilateral triangle. Locate the vertex of the large equilateral triangle that sits on the half crease fold line. Fold a line perpendicular to the half crease fold line at this vertex point. Unfold.

11. Classify the types of triangles you see. How many are there? Use what you know about right triangles, equilateral triangles, parallel lines, transversals, and angle pairs to identify every angle in the figure below (the figure is similar to what you have with your bag).



12. Open the bag and simultaneously pinch the original side edges of the bag together. You now have a geometrical solid. What do you notice about the faces of this solid? How many faces are there? What shape are they? Are they congruent? Provide a convincing argument detailing your discoveries.

13. How many edges are there? How many faces? How many vertices?

14. A polyhedron is a solid whose faces are all polygons. This polyhedron is known as a tetrahedron; a polyhedron (suffix *hedron*) with four faces (prefix *tetra*). To be even more specific, this is known as a regular tetrahedron.

Folding a Dollar Bill

Materials: Dollar bill (or paper cut to the size of a dollar bill)

1. Classify the polygon represented by the dollar bill. Provide a convincing argument.

2. Start by folding the dollar bill in half lengthwise and unfold. You now have a horizontal fold line (view the dollar bill in landscape orientation).

3. Take the right vertex point of the dollar bill and bring it down so that the vertex point is on the crease (fold line) from step (1) in such a way that you have created a triangle with the top right and bottom right vertex points of the dollar bill serving as two of the vertex points in the triangle.

4. Can you classify this triangle? Provide a convincing argument for your classification.

5. Fold along the shortest edge of your triangle so that the hypotenuse of your triangle sits flush with the top of your dollar bill.

6. Can you classify this new triangle? Provide a convincing argument for your classification.

7. Flip the dollar bill over. What polygon do you now have? Classify the polygon and provide a convincing argument for your classification.

8. Flip the dollar bill over again so that the top of the dollar bill is longer than the bottom.

9. Take the right side of your polygon and fold it over the internal edge of the triangle so that the right side sits flush with the bottom of the dollar bill.

10. Take the left side of the polygon and fold it along the internal edge of the triangle. Here you have your first ‘type’ of triangle sitting on top of our second ‘type’ of triangle. How do the areas of these two triangles relate? Provide a convincing argument for your statement.

11. Tuck the smaller triangle (flap) into the larger triangle pocket. You have successfully folded your dollar bill into a triangle.

12. Unfold your dollar bill and place it in front of you in a horizontal (landscape) position. How many triangles do you see? How many are equilateral and how many are right triangles?

13. When folding any rectangle, do you think you will get the same number of equilateral triangles? Why or why not?

14. What other rectangular dimensions would provide the exact same number of equilateral and right triangles as was found in step (12)? Explain why.

15. With your dollar bill unfolded, use what you know about your right triangles, equilateral triangles, parallel lines, transversals, and angle pairs to identify every angle in the figure below (the figure is similar to your dollar bill).