




From Kindergarten to College Understanding Algebra at Every Grade



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Algebraic Thinking

“Algebraic thinking begins with the very young, expands and deepens and matures through the years, and continues to serve adults long after the end of formal schools”

National Council of Teachers of Mathematics
Navigating through Algebra



Smarter Balanced Assessment Consortium: Item #43052

Tony is buying a used car. He will choose between two cars. The table below shows information about each car.

Car	Cost	MPG	Est. Immediate Repairs
A	\$3200	18	\$700
B	\$4700	24	\$300

Tony wants to compare the total costs of buying and using these cars.

- Tony estimates he will drive at least 200 miles per month.
- The average cost of gasoline per gallon in his area is \$3.70.
- Tony plans on owning the car for 4 years.

Calculate and explain which car will cost Tony the least to buy and use.



Item #43052

Common Core Standards:

- A.CED.1 – Create equations and inequalities in one variable and use them to solve problems.
- A.REI.3 – Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.



Finding the Solution

- What are we trying to find out?
 - Total cost of each car for 4 years of ownership
- What information is given?
 - Car cost
 - Immediate repairs cost
 - Gas mileage
 - Cost of gas
 - Number of miles driven per month
 - Number of years of ownership

How do you calculate the total cost?

8.F.4: Construct a function to model a linear relationship between two quantities.

6.EE.7: Solve real-world and mathematical problems by writing and solving equations of the form $px=q$.

5.OA.2: Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them.

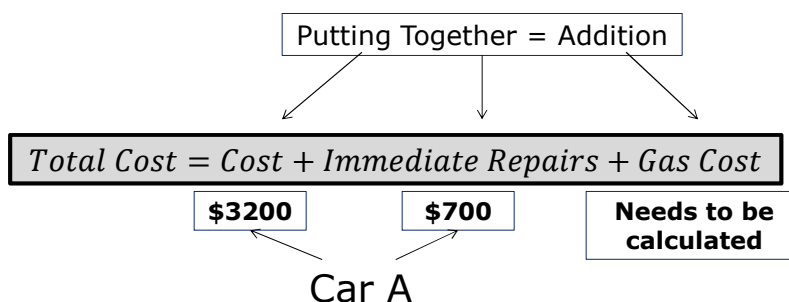
Answer: The sum of the cost, immediate repairs and gas cost

1.OA.2: Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20.

3.OA.3: Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities by using drawings and equations with a symbol for the unknown number to represent the problem.

1.OA.2: Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20.

Tony wants to compare the total costs of buying and using these cars.



1.OA.2: Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20.

- Sandra had 8 pennies, 4 nickels, and 3 dimes. How many coins does Sandra have altogether?
 - $8+4+3=??$
- Sandra had 8 pennies, 4 nickels. George gave her some dimes. Now Sandra now has 15 coins. How many dimes did George give her?
 - $8+4+?=15$
- Sandra had some pennies. George gave her 4 nickels and 3 dimes. Now Sandra has 15 coins. How many pennies did Sandra have to begin with?
 - $?+4+3=15$

1.OA.2: Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20.

Tony wants to compare the total costs of buying and using these cars.

$$\text{Total Cost} = \text{Cost} + \text{Immediate Repairs} + \text{Gas Cost}$$

\$3200

\$700

Needs to be calculated

Car A

3.OA.3: Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities.

$$\text{Gas Cost Per Mile} = \frac{\text{Gas Cost Per Gallon}}{\text{Miles Per Gallon}}$$

If gas costs \$3.70 per gallon and my car gets 18 miles per gallon. How much am I spending on gas per mile?

Measurement division

Total Number of objects
(Dividend)
\$3.70 per gallon

Number of Objects in Each Group
(Divisor)
18 miles per gallon

Number of Groups
(Quotient)
\$.206 per mile

3.OA.3: Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities.

Janet loves jelly beans. Her mom has a bag of 100 jelly beans and gives Janet 20 jelly beans each day. How many days will the jelly beans last?

Measurement division

Total Number of objects
(Dividend)

100 jelly beans

Number of Objects in Each Group
(Divisor)

20 jelly beans per day

Number of Groups
(Quotient)

5 days



3.OA.3: Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities.

$$\text{Gas Cost Per Mile} = \frac{\text{Gas Cost Per Gallon}}{\text{Miles Per Gallon}}$$

If gas costs \$3.70 per gallon and my car gets 18 miles per gallon. How much am I spending on gas per mile?

$$\text{Gas Cost Per Mile} = \frac{\$3.70}{18} \approx \$0.206 \text{ per mile}$$



5.OA.2: Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them.

$$\text{Monthly Gas Cost} = \text{Gas Cost Per Mile} \times \text{Miles Driven per Month}$$

Answer:

Question:

If it costs \$.206 per mile to drive my car, and I drive 200 miles per month, how much will it cost to drive my car for a month?

$$\text{Monthly Gas Cost} = \$.206 \times 200 = \$41.20 \text{ per month}$$



5.OA.2: Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them.

Tammy eats 3 cookies after school each day, how many cookies will Tammy eat in one week?

The total number of cookies eaten is equivalent to the product of the number of cookies eaten each day and the number of days.

$$\text{Total Cookies} = \text{Number of Cookies per day} \times \text{Days of Eating}$$



5.OA.2: Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them.

$$\text{Monthly Gas Cost} = \text{Gas Cost Per Mile} \times \text{Miles Driven per Month}$$

$$\text{Monthly Gas Cost} = \$0.206 \times 200 = \$41.20 \text{ per month}$$

6.EE.7: Solve real-world and mathematical problems by writing and solving equations of the form $px=q$.

$$\text{Gas Cost} = \text{Monthly Gas Cost} \times \text{Number of Months}$$

Direct Proportion →

This amount remains constant at \$3.70

→ This amount varies depending on the number of months.

The total gas cost is the product of the monthly gas cost and the number of months you own the car

The foundations of understanding functional relationships

$$\text{Car A Gas Cost} = \$41.20 \times 48$$

6.EE.7: Solve real-world and mathematical problems by writing and solving equations of the form $px=q$.

Tammy eats 3 cookies after school each day, how many cookies will Tammy eat in one week? Two weeks? One month?

$$C = N \times D$$

Direct Proportion →

This amount remains constant at 3 cookies

This amount varies depending on the number of days.

$$\begin{aligned} \text{Total Cookies in One Week} &= 3 \times 5 = 15 \\ \text{Total Cookies in Two Weeks} &= 3 \times 10 = 30 \\ \text{Total Cookies in One Month} &= 3 \times 20 = 60 \end{aligned}$$

6.EE.7: Solve real-world and mathematical problems by writing and solving equations of the form $px=q$.

$$\text{Gas Cost} = \text{Monthly Gas Cost} \times \text{Number of Months}$$

$$\text{Car A Gas Cost} = \$41.20 \times 48$$

8.F.4: Construct a function to model a linear relationship between two quantities

$$\text{Total Cost} = (\text{Cost} + \text{Immediate Repairs}) + \text{Gas Cost}$$

Fixed cost when you purchase the car no matter how much driving is done.

b

Variable cost depending on the gas mileage of the car.

mx

Interpret the rate of change and initial value of a linear function in terms of the situation it models

8.F.4: Construct a function to model a linear relationship between two quantities

The Relax and Listen Music club offers new members a special introductory plan. You pay \$15.00/month but only \$6.00 for each MP3 album you download. If D represents the number of albums you purchase each month, write a linear equation to represent your monthly cost.

$$M = 6D + 15$$

mx

b

D = # albums purchased per month
 M = Monthly cost

A.REI.3: Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

$$\text{Total Cost} = (\text{Cost} + \text{Immediate Repairs}) + \text{Gas Cost}$$

Fixed cost when you purchase the car no matter how much driving is done.

b

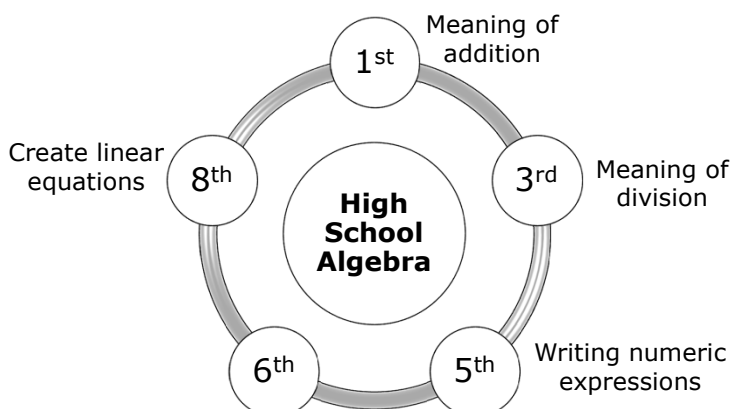
Variable cost depending on the gas mileage of the car.

mx

$$\text{Total Cost} = (\$3200 + \$700) + \$1977.60 = \$5877.60$$



It All Works Together





Contact Info

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