Presentation #216, **Stern**, Methods for Teaching Subtraction from Around the World
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About Teaching Algorithms

Why teach algorithms when calculators are ubiquitous?
There are good reasons to do so.

Teaching is helpful. Imposing is harmful.
Second grade is the point at which many children begin to dislike, feel "bad at" and give up on math. **Learning** occurs when actions make sense to children. Imposing an algorithm can cause frustration that does not lead to accomplishment.

The effective use of manipulative materials
The materials must not become the method of subtraction. They should serve as stepping stones to written and mental methods, something that students can visualize as they move toward the abstraction of using just the numbers.

Numeracy and flexibility remain extremely important
It is vital that children not lose their flexibility with numbers once algorithms are introduced. A child who, asked to subtract 59 from 61, does this:

$$\begin{array}{r} {}^{5}\cancel{6}\ {}^{1}1 \\ \underline{-\ 5\ \ 9} \\ 2 \end{array}$$

is a child in trouble, or one who has been taught that her/his brain is not part of math class and s/he should "do what the teacher demands" and not think.

Naming subtraction methods as a means to encourage flexibility
Encourage children to employ a method that is efficient in a given situation.[1] Here are some of the types of questions and the names chosen by some classes:

"Only 1 place"

$$\begin{array}{r} 83 \\ \underline{-\ 53} \end{array} \qquad \begin{array}{r} 538 \\ \underline{-238} \end{array}$$

"So easy"

$$\begin{array}{r} 358 \\ \underline{-\ 243} \end{array}$$

"So close / nearby"

$$\begin{array}{r} 61 \\ \underline{-\ 59} \end{array}$$

"Slide up or down"

$$\begin{array}{rcr} 92 + 3 = & & 95 \\ \underline{-\ 37} + 3 = & & \underline{-\ 40} \end{array}$$

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[1] This idea stems from Anton Klein, Meindert Beishuizen and Adri Treffers, " The Empty Number Line in Dutch Second Grades: *Realistic* Versus *Gradual* Program Design," Journal for Research in Mathematics Education 29:4 (1998): 443-464.

The Algorithms

Not every school in every country teaches the same way. These are some samples of methods used at some schools.

The **United States, Brazil and Guatemala**: three styles of an algorithm.

The one most **U.S.A. students** are taught:

$$\begin{array}{r} {}^{4}\cancel{\mathbf{5}}\;\;{}^{11}\cancel{\mathbf{2}}\;\;{}^{1}\mathbf{4} \\ \underline{-\;\mathbf{2}\;\;\;\mathbf{8}\;\;\;\mathbf{7}} \\ 2\;\;\;3\;\;\;7 \end{array}$$

subtract 7 from 14 → 7 in the ones place
subtract 8 (tens) from 11 (tens) → 3 in the tens place
subtract 2 (hundreds) from 4 (hundreds) → 7 in the hundreds place

Brazil and **Ghana** use the same method but expect the students to be able to do it without crossing out and writing little numbers – they expect the students to keep track in their heads.

In **Guatemala** and **Haiti** the same is true as for Brazil except that sometimes, when first learning it, with 2-digit numbers one is allowed to write the reduced tens place to the side, like this:

$$\begin{array}{r} 6\;3 \\ \underline{-\;2\;9} \end{array}$$

5 (this 5 is a reminder that there are 5 tens left after a ten was used to subtract the 9 ones)

France: two styles for an algorithm.

In this algorithm, when ten (or 100, 1000, etc.) is added to the minuend it is also added to the subtrahend. (Technically this method is the same as the **Constant Difference** method described below.)

$$\begin{array}{r} 5\;2\;4 \\ \underline{-2\;8\;7} \end{array}$$

$$\begin{array}{r} \mathbf{5}\;{}^{1}\mathbf{2}\;{}^{1}\mathbf{4} \\ \underline{-{}^{1}\mathbf{2}\;{}^{1}\mathbf{8}\;\mathbf{7}} \\ 2\;\;3\;\;7 \end{array}$$

(The little "1s" are circled.)
The 10 that is added to make 14 is subtracted along with the 8 tens in 2<u>**8**</u>7.
The hundred that is added to provide 12 tens is subtracted along with the 2 hundreds.

Notice that this method requires thinking of the little "1s" in two different ways!

Some French schools teach this method with a different writing style.

$$\begin{array}{r} 5\;2\;4 \\ \underline{-2\;8\;7} \end{array} \qquad\qquad \begin{array}{r} \mathbf{5}\;{}^{1}\mathbf{2}\;{}^{1}\mathbf{4} \\ \underline{-\;{}^{3}\cancel{\mathbf{2}}\;{}^{9}\cancel{\mathbf{8}}\;\mathbf{7}} \\ 2\;\;3\;\;7 \end{array}$$

The **constant difference** or **equal addition or equal subtraction** method uses a property that is extremely important for children to understand: that if the same quantity is added to (or subtracted from) both the minuend and subtrahend, their difference, or distance apart on the number line stays the same.

$$\begin{array}{rcrcr} 524 + 3 & = & 527 + 10 & = & 537 \\ \underline{-287} + 3 & = & \underline{-290} + 10 & = & \underline{-300} \\ & & & & 237 \end{array}$$

Facts to ten method

$^{4}\mathbf{\not{5}}\ ^{10+1}\mathbf{\not{2}}\ ^{10+}\mathbf{4}$	
$\underline{-\ \mathbf{2\,8}\quad\ \mathbf{7}}$	subtract 7 from 10 → 3 + 4 → 7 in the ones place
2,(2+1),(3+4)	subtract 8 from 10 → 2 +1→3 in the tens place
2 3 7	subtract 2 from 4 → 2 in the hundreds place.

I've written the 7 under the 10 and the 8 under the 10 to make the method clear. Always subtract from 10 when a digit in the minuend is smaller than the corresponding one in the subtrahend. Those for whom this is difficult can write it like this:

$$\begin{array}{r} {\scriptstyle 4\ \ 10\ \ 10} \\ \mathbf{\not{5}\ \not{2}^{1}\ 4} \\ \underline{-\ \mathbf{2\ 8\ 7}} \end{array}$$

Algorithms for the students who need a greater challenge

The **all-subtraction** method employs negative numbers.

$$\begin{array}{r} 524 \\ \underline{-287} \end{array}$$

$3\,^{-}6\,^{-}3 = 300 - 60 - 3 = 240 - 3 = 237$

The **Chinese** algorithm. It could be called the (almost) **all addition** method.

$$\begin{array}{r} 524 \\ \underline{-287} \end{array}$$

Subtract the subtrahend from 999 to get	712
Add the minuend	$+\underline{524}$
	1236
Drop the high-order place 1 and add 1	237

Some questions to pose to students to whom you show this method:

How does this algorithm work?
How can subtraction be performed on an abacus?
Is this algorithm easier than trying to remove an amount on an abacus?