

Renewing Our Focus on *Principles and Standards*: A Vision for Mathematics Education

**Dane Camp, NCTM Board of Directors
Linda Gojak, President, NCTM**



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An Agenda for Action



- Problem solving be the focus of school mathematics
- Basic skills encompass more than computational facility;
- Take full advantage of the power of calculators and computers;
- The success of mathematics programs and student learning be evaluated by a wider range of measures than conventional testing;
- More mathematics study be required for all students and a flexible curriculum with a greater range of options be designed to accommodate students' diverse needs;
- Mathematics teachers demand of themselves and their colleagues a high level of professionalism;
- Public support for mathematics instruction be raised to a level commensurate with the importance of mathematical understanding to individuals and society.



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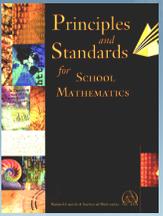
Standards...a brief history



1989 NCTM
Curriculum and Evaluation Standards

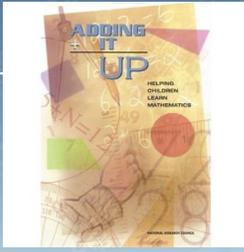


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2000 NCTM *Principles and Standards for School Mathematics*
www.nctm.org

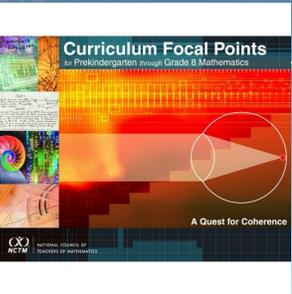
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2001 National
Research Council
Adding it Up
(www.nap.edu)



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2006 NCTM
Curriculum Focal Points



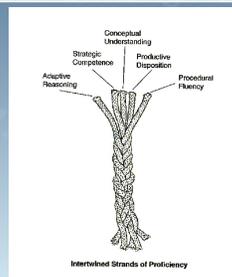
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NCTM Process Standards

- *Problem Solving* is an integral part of mathematics learning.
- *Reasoning and Proof* are ways of expressing justification.
- *Communication* is an essential part of mathematics education.
- *Connections* are critical in mathematics, both across mathematical topics and to contexts outside mathematics.
- *Representations* of mathematical ideas are fundamental to enhancing mathematical understanding.

What Does it Mean to be Mathematically Proficient?



- **Adaptive Reasoning**
- Strategic Competence
- Conceptual Understanding
- Productive Disposition
- Procedural Fluency

CCSS Standards for Mathematical Practice

The Common Core proposes a set of Standards for Mathematical Practice that all teachers should develop in their students.

The Mathematical Practice Standards are strongly informed by the NCTM Process Standards in *Principles and Standards for School Mathematics*. These standards form the basis for standards documents nationally and internationally.

CCSS Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

NCTM Process Standards and the CCSS Mathematical Practice Standards

NCTM Process Standards	CCSS Mathematical Practices
Problem Solving	Make sense of problems and persevere in solving them. Use appropriate tools strategically
Reasoning and Proof	Reason abstractly and quantitatively. Critique the reasoning of others. Look for and express regularity in repeated reasoning
Communication	Construct viable arguments Attend to precision.
Connections	Look for and make use of structure
Representations	Model with mathematics.

Stretch Your Mind

Which is greater? Justify your reasoning using more than one way.

$$\frac{96}{200} \quad \text{or} \quad \frac{71}{150}$$



Share your reasoning with your neighbor(s). What justifies your methods?

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Promoting Student Reasoning



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From IMAP Video Collection: *Children's Mathematical Thinking Clips*. Pearson Higher Education

Reflection

- Talk with 1-2 partners about what you recorded.
- What does a teacher need to do to create the sort of learning environment seen in the video?



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Hexagon Train Task

Each figure in the pattern below is made up of hexagons that measure 1 cm on each side



Figure 1
Perimeter = 6 cm



Figure 2
Perimeter = 10 cm



Figure 3
Perimeter = 14 cm



Figure 4
Perimeter = 18 cm

If the pattern of adding 1 hexagon to each figure is continued, what will be the perimeter of the 25th figure in the pattern? Justify your answer.

Eliciting Student Reasoning



From JAGUAR, an NSF-funded project led by Sean Larsen, Megan Staples, Jill Newton & Eva Thanheiser

Understanding Student Methods

If the pattern of adding one hexagon to each figure is continued, what will be the perimeter of the 25th figure in the pattern? Explain your thinking and show your work.

$$\begin{array}{r}
 18 \text{ cm} \\
 \times 2 \\
 \hline
 36 \text{ cm} \\
 + 36 \\
 \hline
 72
 \end{array}$$

$$\begin{array}{r}
 72 \\
 + 36 \\
 \hline
 108 \\
 + 6 \\
 \hline
 114
 \end{array}$$

I noticed a pattern; the pattern was that every time you add on a hexagon so I decided to add 4 to 18 and got 22 then I multiplied 22 x 5 because 5 x 5 = 25 and 22 is the perimeter of five hexagons together x 5 would equal the perimeter of 25 hexagons. So I got 110 cm as the perimeter for 25 hexagons.

Reflection

- Talk with 1-2 partners about what you recorded.
- What does a teacher need to do to create the sort of learning environment seen in the video?



NCTM Process Standards

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- *Representations* of mathematical ideas are fundamental to enhancing mathematical understanding.

Please mark each circle when you find a bullet that is represented in the video clip. In the next column list specific evidence that represents that bullet.

Adapted from:

http://staffdev.henrico.k12.va.us/mathsummit/media/math_summit_lesson_analysis.doc

Process Standards from NCTM		Evidence from the video
<p>1 - Problem Solving: Instructional programs should enable all students to...</p> <ul style="list-style-type: none"> • Build new mathematical knowledge through open-ended questions and more-extended exploration; • Allow students to recognize and choose a variety of appropriate strategies to solve problems; • Think & Discuss and Share & Summarize to allow students to reflect on their own and other strategies for solving problems. 	<p style="text-align: center;">○ ○ ○</p>	
<p>2 - Reasoning & Proof: Instructional programs should enable all students to...</p> <ul style="list-style-type: none"> • Recognize and create conjectures based on patterns they observe; • Develop and evaluate mathematical arguments; • Investigate math conjectures and prove that in all cases they are true or that one counterexample shows that it is not true; • Explain and justify their solutions. 	<p style="text-align: center;">○ ○ ○</p>	
<p>3 - Communication: Instructional programs should enable all students to...</p> <ul style="list-style-type: none"> • Organize and consolidate their mathematical thinking in written and verbal communication; • Communicate their mathematical thinking clearly to peers, teachers, and others; • Analyze and evaluate the mathematical thinking of others; • Use the language of mathematics to express mathematical ideas precisely. 	<p style="text-align: center;">○ ○ ○</p>	
<p>4 - Connections: Instructional programs should enable all students to...</p> <ul style="list-style-type: none"> • Understand that mathematical ideas are interconnected and that they build and support each other; • Recognize and apply connections to other contents; • Solve real world problems with mathematical connections. 	<p style="text-align: center;">○ ○ ○</p>	
<p>5 - Representation: Instructional programs should enable all students to...</p> <ul style="list-style-type: none"> • Emphasize a variety of mathematical representations including written descriptions, diagrams, equations, graphs, pictures, and tables; • Select, apply, and translate among mathematical representations to solve problems; • Model real-life situations. 	<p style="text-align: center;">○ ○ ○</p>	