



USING MANIPULATIVES AND SINGAPORE MATH® STRATEGIES IN INTERMEDIATE SCHOOL CLASSROOMS

Tricia Salerno
SMARTTraining, LLC
www.SingaporeMathTraining.com

Singapore Math® is a registered trade mark of Singapore Math Inc.

© SMARTTraining, LLC 2011

Three stages of learning



Concrete



Pictorial

$$7 + 3 =$$

Abstract

Jerome Bruner



© SMARTTraining, LLC 2011

All Instruction Begins Informally

What can happen children learn the algorithm too early?



© SMARTTraining, LLC 2011

$$2 \frac{3}{4} \div \frac{1}{2} =$$

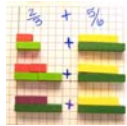
Please:

1. Solve
2. Draw a picture
3. Write a word problem

© SMARTTraining, LLC 2010

Learning About Fractions Using Manipulatives

- Fraction Pieces – NOT JUST CIRCLES
- Cuisenaire rods
- Patty paper
- Paper strips



© SMARTTraining, LLC 2011

PAPER FOLDING

Your patty paper is one whole.

1. Fold it to create two equal parts.
2. Open your paper, make a line on the crease, and label one of the two equal parts $\frac{1}{2}$. Shade that $\frac{1}{2}$.
3. Fold it on the crease again, then fold it in the opposite direction so that the ends meet.
4. How many pieces are there now? What have we learned?
5. Repeat.



(c) SMARTTraining, LLC 2010

PLACE VALUE PLACE VALUE PLACE VALUE

- Conceptual understanding of place value is a key to ongoing success in mathematics!!!

- How many ones are in 300?
- How many tens are in 300?
- How many hundreds are in 300?
- 247 is 22 tens plus ____ tens plus ____ ones

$$300 - 28 = ?$$

© SMARTTraining, LLC 2011

Using Base 10 Blocks and Number Disks



© SMARTTraining, LLC 2010

Word Problems in Singapore Math®

An Introduction to Model Drawing

Singapore Math® is a registered trade mark of Singapore Math Inc.

© SMARTTraining, LLC 2010

How To Use The Part Whole Bar Model and The Comparison Model Methods In **Addition** and **Subtraction**

© SMARTTraining, LLC 2010

Part Whole Discrete Model

Mary has 8 cups. Robby has 4 cups. How many cups do they have all together?

After using manipulatives, this is the next step: creating a pictorial model to represent the problem.

© SMARTTraining, LLC 2010

Part Whole Continuous Model

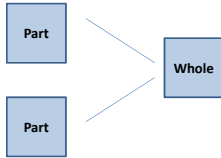
We instruct the students to **VISUALIZE** the numbers used in this problem as a whole made up of two parts (proportionality is important).

$8 + 4 = 12$. They have 12 cups altogether.

© SMARTTraining, LLC 2010

Part - Whole

This was informally taught with number bonds in the primary grades.



© SMARTTraining, LLC 2010

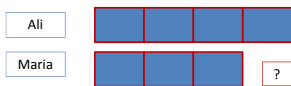
Comparison Model

Ali has 4 pens. Maria has 3 pens. How many more pens than Maria does Ali have?



© SMARTTraining, LLC 2010

Comparison Model



Ali has 4 pens. Maria has 3 pens. How many more pens than Maria does Ali have?

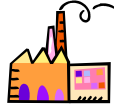
© SMARTTraining, LLC 2010

Part Whole

Let's Try It:

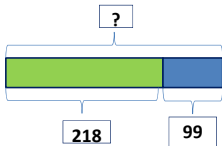
218 women and 99 men worked in a factory.
How many people worked in the factory?

Let's use paper strips to model this problem.



© SMARTTraining, LLC 2010

Part Whole



This bar represents the whole (all the workers) and is divided into two parts: women (218) men (99). From this model you can find the whole by adding the two parts.

218 women and 99 men and worked in a factory.
How many people worked in the factory?

$$218 + 99 = 317$$

317 people worked in the factory.

© SMARTTraining, LLC 2010

Comparison

Let's Try It:

99 men worked in a factory. 119 more women than men work in the factory. How many people work in the factory?



© SMARTTraining, LLC 2010

Comparison

Women	<div style="background-color: #4a7ebb; color: white; padding: 5px; display: inline-block;">?</div>	}	<p>Women $99 + 119 = 218$</p> <p>Men & women $99 + 218 = 317$</p> <p>317 people work in the factory.</p>
Men	<div style="background-color: #4a7ebb; color: white; padding: 5px; display: inline-block;">99</div> \longleftrightarrow 119		

99 men worked in a factory. 119 more women than men work in the factory. How many people work in the factory?

© SMARTTraining, LLC 2010

How To Use The Part Whole Bar Model And The Comparison Model Methods In **Multiplication** and **Division**


© SMARTTraining, LLC 2010

Part Whole-Multiplication

Let's Try It:

7 children shared equally in the cost of a gift for their teacher. Each child paid \$19. What was the cost of the gift?

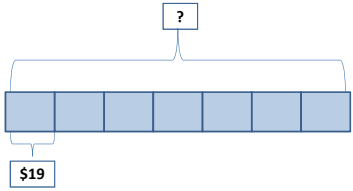
Let's use graph paper to help draw a model for this problem.



© SMARTTraining, LLC 2010

Part Whole-Multiplication

7 children shared equally in the cost of a gift for their teacher. Each child paid \$19. What was the cost of the gift?




$7 \times 19 = 133$ The gift cost \$133.

© SMARTTraining, LLC 2010

Part Whole-Division (Partitive-Sharing)

Let's Try It:

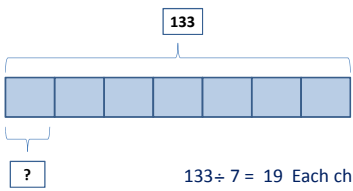
7 children shared equally in the cost of a gift for their teacher. The gift cost \$133. How much did each child pay toward the gift?



© SMARTTraining, LLC 2010

Part Whole-Division

7 children shared equally in the cost of a gift for their teacher. The gift cost \$133. How much did each child pay toward the gift?



We know the whole and the number of parts so we divide the whole by the number of parts.

$133 \div 7 = 19$ Each child paid \$19.

© SMARTTraining, LLC 2010

Comparison-Multiplication

Let's Try It:

A baker baked 6 cakes. He baked 4 times as many cupcakes as he did cakes. How many cupcakes did he bake?

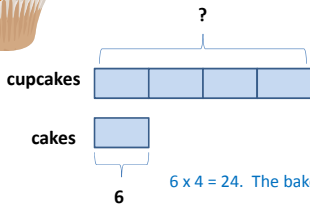


Let's use Cuisenaire rods to model this problem.

© SMARTTraining, LLC 2010



Comparison-Multiplication



Two bars are drawn with unit lengths of 4 and 1 to represent cupcakes and cakes. Given the number of cakes you can find the number of cupcakes.

A baker baked 6 cakes. He baked 4 times as many cupcakes as he did cakes. How many cupcakes did he bake?

© SMARTTraining, LLC 2010


Comparison-Division

Let's Try It:

A baker baked 24 cupcakes. He baked 4 times as many cupcakes as he did cakes. How many cakes did he bake?





© SMARTTraining, LLC 2010



Comparison-Division

24

cupcakes 

cakes 

?

A baker baked 24 cupcakes. He baked 4 times as many cupcakes as he did cakes. How many cakes did he bake?

$24 \div 4 = 6$. The baker baked 6 cakes.

© SMARTTraining, LLC 2010

Practice Problems

Harry weighs 36 kg. He is 3 times as heavy as his brother. How heavy is his brother? (3rd grade)

You choose: Concrete or Pictorial?

© SMARTTraining, LLC 2010

Practice Problems

A farmer had twice as many ducks as chickens. After he had sold 413 ducks and another 19 ducks had flown away, he had half as many ducks as chickens left. How many ducks did he have left? (4th grade Challenging Word Problems)

You choose: Concrete or Pictorial? (Hint: if you choose concrete, I suggest Cuisenaire rods.)

© SMARTTraining, LLC 2010

How To Use The Part Whole Bar Model And The Comparison Model Methods In **Fraction** problems

© SMARTTraining, LLC 2010

Part Whole-Fraction

12

4 units = 12
1 unit = $12/4 = 3$
3 units = $3 \times 3 = 9$

?

FIND THE VALUE OF ONE UNIT FIRST

?

3 units = 9
1 unit = $9/3 = 3$
4 units = $4 \times 3 = 12$

9

© SMARTTraining, LLC 2010

Part Whole-Fraction

7/8 of a number is 63. What is 2/3 of that same number? (4th grade)

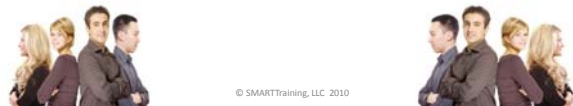
Let's use paper strips for this one.

© SMARTTraining, LLC 2010

Comparison-Fraction

Let's Try It:


There are $\frac{4}{5}$ as many girls as boys in a club. If there are 30 boys, how many girls are there?




© SMARTTraining, LLC 2010

Comparison-Fraction

girls

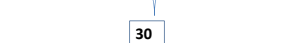


boys



?

5 units = 30
1 unit = $30/5 = 6$
4 units = 24
There are 24 girls.




30

There are $\frac{4}{5}$ as many girls as boys in a club. If there are 30 boys, how many girls are there?

© SMARTTraining, LLC 2010

Practice Problems



Tyrone bought a bag of marbles. $\frac{1}{4}$ of the marbles were blue, $\frac{1}{8}$ were green and $\frac{1}{5}$ of the remainder were yellow. If there were 24 yellow marbles, how many marbles did he buy? (5th grade)

Let's use paper strips on this one.

© SMARTTraining, LLC 2010

 SMARTTraining: Singapore Math Training

Teaching Tips, Product discounts, Workshop Updates & More

 SingaMathExpert
MathSingapore

(c) SMARTTraining, LLC 2010

Sources:

Knowing and Teaching Elementary Mathematics,
Liping Ma

Elementary Mathematics for Teachers, Thomas
Parker and Scott Baldrige

Primary Mathematics Standards Edition,
published by Marshall Cavendish Education

The Singapore Model Method for Learning
Mathematics, Ministry of Education of
Singapore

© SMARTTraining, LLC 2011
