PLACE VALUE

BASE-10

BASE-5

PLACE HOLDERS

STRUCTURE AND PATTERNS

APPLICATION AND DISCOVERY

Apply: One-Handed Tribe

A closer look at place value

A Formative Assessment Lesson

Intended for use within grades 3-5 to meet CCSS: Numbers and Operations in Base Ten.

For more information, details, ideas and help using this lesson within your classroom visit:
https://sites.google.com/site/moreygoodman/

Morey Goodman

Adopted from 2012 MARS, Shell Center, University of Nottingham

Applying: One-Handed Tribe

MATHEMATICAL GOALS

This lesson unit is intended to help you assess how well students are able to:

- Solve problems related to the understanding of the place value system.
- Use their understanding of base-10 to explore base-5.
- Communicate their reasoning clearly.

COMMON CORE STATE STANDARDS

This lesson relates to the following *Mathematical Practices* in the *Common Core State Standards for Mathematics:*

1. (1) Make sense of problems and preserver in solving them.
2. (3) Construct viable arguments and critique the reasoning of others.
3. (6) Attend to precision.
4. (7) Look for and make use of structure.

This lesson gives students the opportunity to apply their knowledge of the following *Standards for Mathematical Content* in the *Common Core State Standards for Mathematics:*

3. NBT: Use place value understanding and properties of operations to perform multi-digit arithmetic.

INTRODUCTION

This lesson is designed to enable students to develop strategies for solving problems. The problem is characterized by students needing to use their understanding of base-10 and place value to solve problems in a base-5 system.

- Before the lesson, students are asked to count dots in base-5 within a group. You assess their conversations, noting misconceptions about place value, vocabulary already in use, and then create a common classroom dialogue.
- Students will tackle the formative assessment individually. You will assess their responses and formulate questions that will require them to review their work.
- At the start of the lesson, students respond individually to the question set, and then work in groups to combine their thinking and create a place value chart in base-5 based on the base-10 place value chart.
- In the same small groups, students work on a collaborative task which asks them to match pictures with the corresponding base-5 numbers and expanded form.
- As a whole class, three generalizable methods are offered that they might use to solve addition and subtraction problems in base-5.
- Returning to their groups they will use the methods to solve the addition/subtraction card set and match it to the appropriate number.
- Finally, students work individually to review their initial solutions, and then use what they have learned to revise the same formative assessment.

MATERIALS REQUIRED

Each student will need a copy of *One-Handed Tribe Initiation, One-Handed Tribe,* and *Some Different Approaches*.

Each small group of students will need a copy of *Card Set A-D,* cubes, base-5 manipulatives, large sheet of paper, glue, scissors, and markers.

There are some projector resources to help you with whole-class discussions.

TIME NEEDED
20 minutes before the lesson, 15 minutes on the formative assessment, a 90 minute lesson, and 15 minutes for follow-up lesson. Timings given are only approximate. Exact timings will depend on the needs of the class.

BEFORE THE LESSON

Task: *One-Handed Tribe Initiation* (20 minutes)

Have the students complete this task, in a group, before the formative assessment. This will give you an opportunity to assess misconceptions about place value and regrouping and listen to existing vocabulary (digits vs. numbers and place holders) to find out the kinds of difficulties student have with it. You will then be able to target your help more effectively in the next part of the lesson.

Give out the *One-Handed Tribe: Initiation* and unix-cubes.

> *This activity is about counting in base-5. Apply your understanding of base-10 and place value to count the dots in base-5.*
>
> *Read through the task carefully and remember to show all your work, so that I can understand your reasoning.*
>
> *As well as trying to solve the problem, try to present your work in an organized and clear way.*

What happens when the students are unable to move forward?

Discuss digits as they relate to the alphabet and words. Or review the digits in base-10.

It is important that, as far as possible, students are allowed to complete the task without your assistance. Take note of vocabulary students already have and any misconceptions they may have. Pick 2-3 groups who have different approaches or vocabulary to share with the class.

After 10 minutes, have one member of the groups chosen by you to share out the strategies for their findings. Ask students to be aware of how the groups discuss the numbers (i.e. do students say, “one-two” or “twelve” when they are discussing a base-5 number)

Last 5 minutes, have a whole group discussion about norms for tribal discussions on how we are going to say base-5 numbers.

Task: *One-Handed Tribe* (15 minutes)

Have the students complete this task, in class or for homework, after your whole group discussion of the *One-Handed Tribe: Initiation*. This will give you the opportunity to assess the work and to find out the kinds of difficulties students have with it. You will then be able to target your help more effectively in the next parts of the lesson.

Give out the task *One-Handed Tribe* formative assessment. It is important that the students ONLY spend 15 minutes working on this. Do not expect them to finish or do well.

> *Use what we talked about with the imitation to complete what you can. Don’t worry if you don’t finish or don’t understand everything. We will be working on this tomorrow.*

Assessing student’s responses

Collect students' responses to the task. Make some notes on what their work reveals about their current levels of understanding and their different problem solving approaches.

We suggest that you do not score students' work. The research shows that this will be counterproductive, as it will encourage students to compare their scores and will distract their attention from what they can do to improve their mathematics.

Instead, help students to make further progress by summarizing their difficulties as a series of questions. Some suggestions for these are given on the next page. These have been drawn from common difficulties observed in trials of this unit.

We recommend that you write a selection of questions on each student's work. If you do not have time, select a few questions that will be of help to the majority of students. These can be written on the board at the beginning of the lesson. You may also want to note students with a particular issue, so that you can ask them about their difficulties in the formative lesson.

Common Issues:	**Suggested questions and prompts:**
Student has difficulty getting started	*What do you know?* *How can this information be useful?* *What do you need to find out?* *If you have 49 and one more, what do you have?*
Student ignores place value	*What is the difference between a digit and a number?* *What is the purpose of a place value holder?* *What is the difference between the numbers 9 and 10?*
Students make incorrect assumptions	*Is "10" a digit in base-10?* *A group of how many, forces you to move into the next place value holder?*
Students' work is unsystematic	*Would someone in your class who has not used this method be able to follow your work?* *Can you describe your method as a series of logical steps?* *Could you think only in base-5?*
Students' work is poorly presented For example: Student circles groups of five and it is left to the reader to decide why this is the answer.	*Can you explain each part of your solution?* *Can you justify the choices you have made?*
Student completes the task	*Can you count in base-2?* *How can you check what you have used will work in other bases?*

SUGGESTED LESSON OUTLINE

Reviewing individual solutions to the problem **(10 minutes)**

Return the task to the students.

If you have not added questions to individual pieces of work, write your list of questions on the board. Students should select questions appropriate to their own work and spend a few minutes thinking about their responses to them.

Recall the One-Handed Tribe. What was the task about?

I have looked at your work and I have some questions about it.

I would like you to think, on your own, about my questions and how your work could be improved.

When a list of questions is written on the board rather than on individual pieces of work, some students may struggle to identify which questions they should be considering. If this is the case, it may be helpful to give students a printed version of the list of questions with the relevant ones highlighted.

Collaborative Work: ***Making a base-5 place value chart and matching Card sets.*** **(30 minutes)**

Ask students to work in small groups of two or three students.

Display the base-10 place value chart and ask the groups to create the corresponding base-5 place value chart on the top of their poster paper. If your students seem to struggle with the base-10 chart, use manipulatives to help them understand the importance of each place value location.

Give each group the Card Set with a large sheet of paper, and a glue stick for making a poster after they have completed their place value chart. *Note: Instructions are given as if you have cut out the cards and grouped them for the students.*

You are now going to match first the base-5 number with the corresponding dot representation. You will be given six dot cards and six number cards. In your group take a number and find a dot card that matches it.

Take turns at matching pairs of cards. Each time you do this, explain your thinking clearly and carefully to your group mates. Place your cards side by side on your large sheet of paper, not on top of one another, so that everyone can see them.

Write your reasons for the match on the cards or the poster just as we did with the example in class.

Make sure you leave plenty of space around the cards as, eventually, you will be adding 2 more cards to each matched pair.

The purpose of this structured group work is to encourage students to engage with each other's explanations and take responsibility for each other's understanding.

You have two tasks during the small-group work: to make a note of student approaches to the task, and to support student reasoning.

Give out Expanded Form Card Set and ask students to match these with the cards already on the poster.

You are now going to match the expanded form with the cards already on your poster. In your group, take an expanded form card and try to find a pair that matches it (number card and dot card).

Again take turns at matching cards you think belong together. Each time you do this, explain your thinking clearly and carefully.

Write your reasons for the match on the poster.

Whole Class Discussion: Sample student work (10 minutes)

Show or give out a copy of *Some Different Approaches*.

Allow students a few minutes to read through the problem and attempt it on their own.

Then show each of the methods used by Jason, Kate and Simon. Organize a whole-class discussion of these approaches.

These might be helpful approaches.

Can you use these methods to solve the Addition/Subtraction Card Set?

Do you think they will give an accurate answer? Why or why not?

Do you think one is faster than the others?

Collaborative Work: Finishing card set (10 minutes)

Hand out addition and subtraction cards and ask students to match them with the cards already on their poster.

Sharing Posters (5 minutes)

When students have completed the task they are to share their reasoning as they did earlier in the lesson unit.

Improving individual solutions to the assessment task (10 minutes)

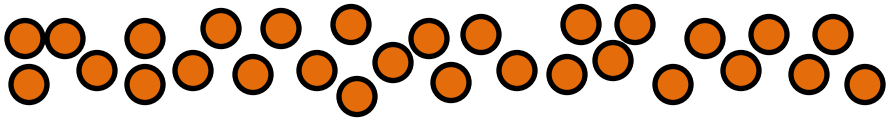
Give students their response to the first *One-Handed Tribe* formative assessment with a second blank copy of the assessment.

Look at your original responses and think about what you have learned this lesson.

Using what you have learned try to improve you work.

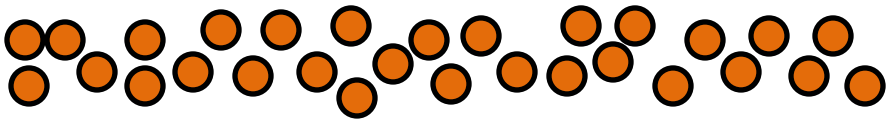
One Handed Tribe
- Initiation -

Imagine an isolated tribe of people who only use one hand to count. With only one hand, they have only five fingers available. This is called a base-5 number system. How would this tribe write the number of dots on the figure below?



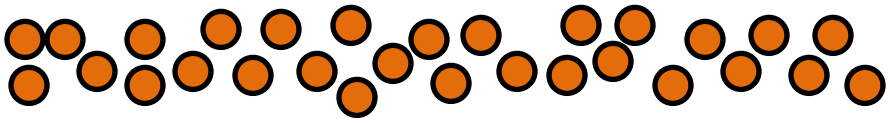
One Handed Tribe
- Initiation -

Imagine an isolated tribe of people who only use one hand to count. With only one hand, they have only five fingers available. This is called a base-5 number system. How would this tribe write the number of dots on the figure below?



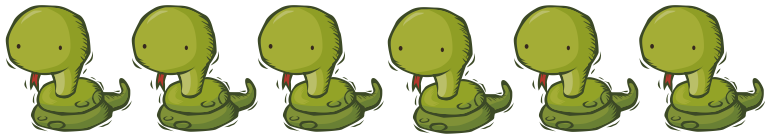
One Handed Tribe
- Initiation -

Imagine an isolated tribe of people who only use one hand to count. With only one hand, they have only five fingers available. This is called a base-5 number system. How would this tribe write the number of dots on the figure below?



One-Handed Tribe

1. Count the number of vipers shown below in base-5. Explain your answer.



_____

_____

_____

_____

2. The Tribe Leader says that without a doubt, 12+3=20. The rite of passage to become a level 2 member for this tribe is to prove that this is true. Can you become a level 2 tribe member?

3. One of your friends says that 41-14=32. If you think your friend is correct explain why. If you think your friend in incorrect replace the statement with one that is correct and then explain why your statement is correct.

   _____

   _____

   _____

   _____

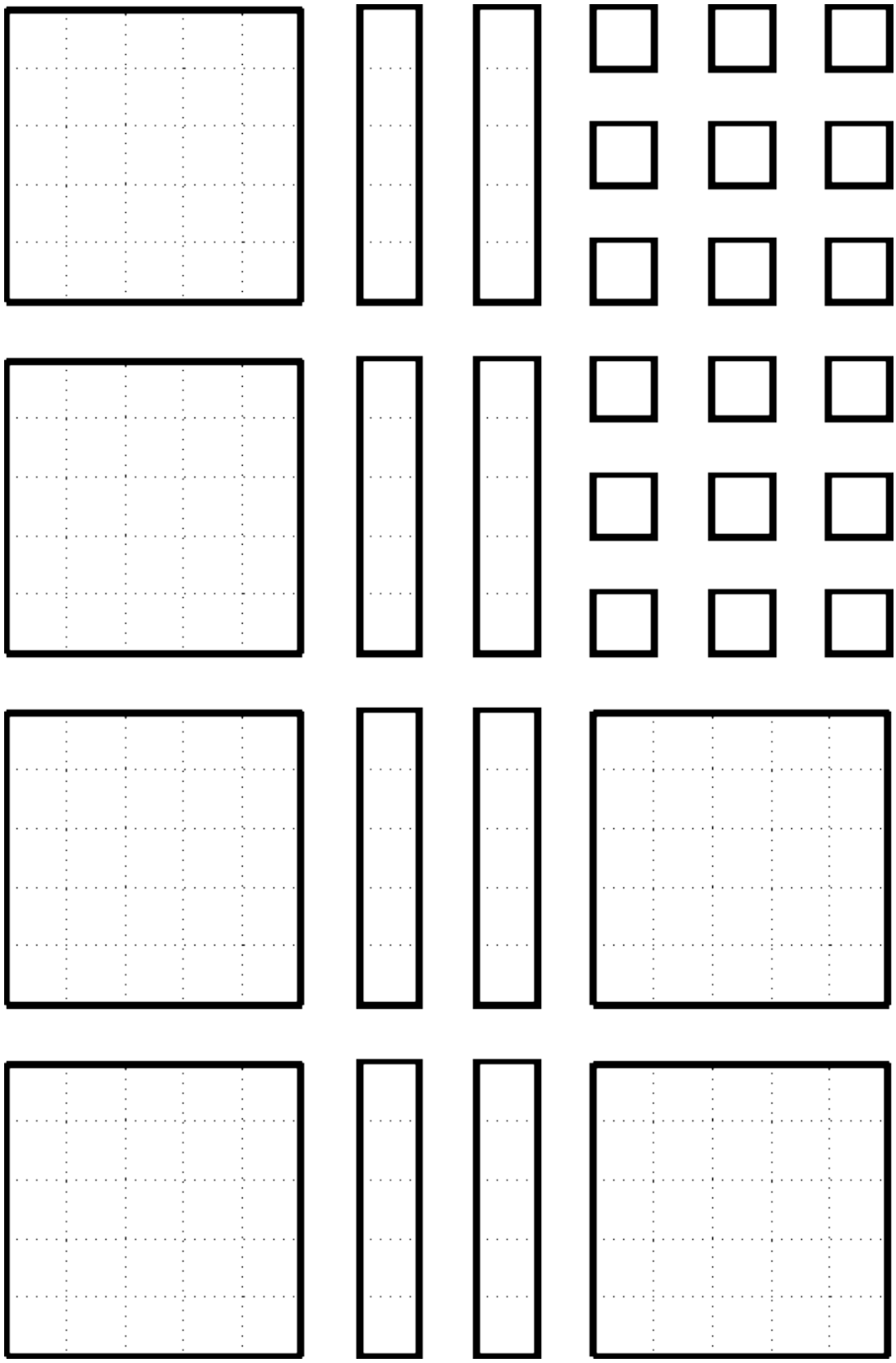
   _____

   _____

Place Value Chart - Base Ten

Thousands	Hundreds	Tens	Ones	Tenths	Hundredths	Thousandths
1000	100	10	1	1/10	1/100	1/1000
10^3	10^2	10^1	10^0	10^{-1}	10^{-2}	10^{-3}

Base 5 Manipulatives



Matching Cards and Working Together

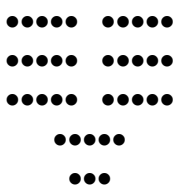
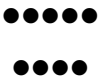
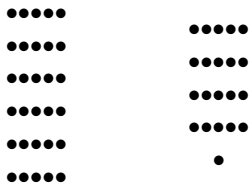
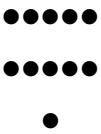
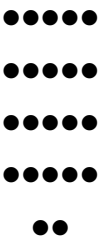
1. **Take turns matching pairs of cards.**

2. **Each time you match, explain your thinking clearly and carefully to your group members.**

3. **Place your cards side by side on your poster—leave room to explain your reasoning.**

4. **Make sure you leave plenty of space next to your cards because you will be adding more cards.**

Make sure that everyone in the group can explain the reasons for the set of cards you have matched together.

Number Card	Expanded Form Card	Dot Card	Addition/Subtraction
123	(1x25)+(2x5)+(3x1)		101+22= ☐
14	(1x5)+(4x1)		30-11= ☐
201	(2x25)+(0x5)+(1x1)		123+23= ☐
Your Choice	Your Choice	Your Choice	Your Choice
21	(2x5)+(1x1)		30-4= ☐
42	(4x5)+(2x1)		104-12= ☐

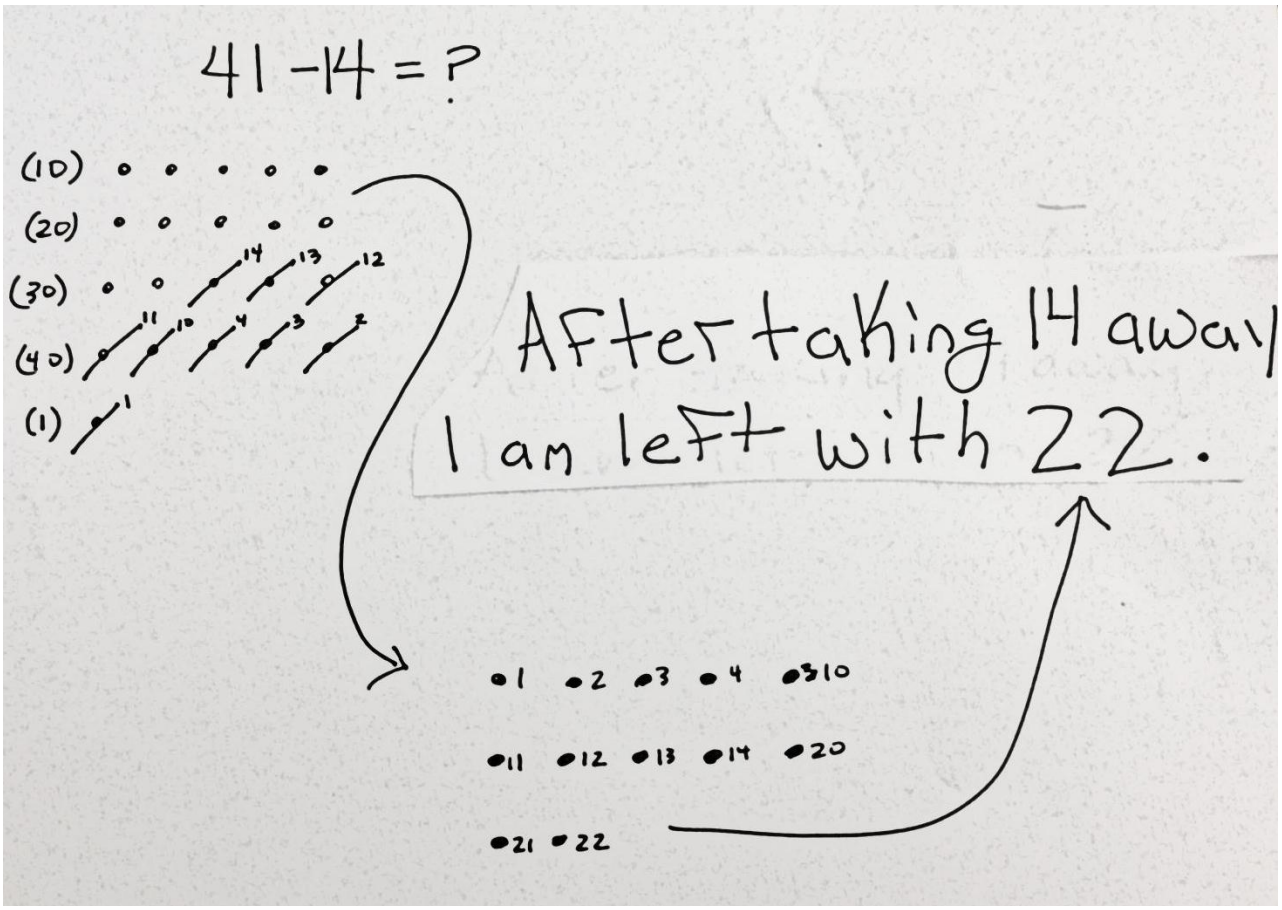
Some Different Approaches

Kate, Simon and Jason were asked to solve the following in base-5.

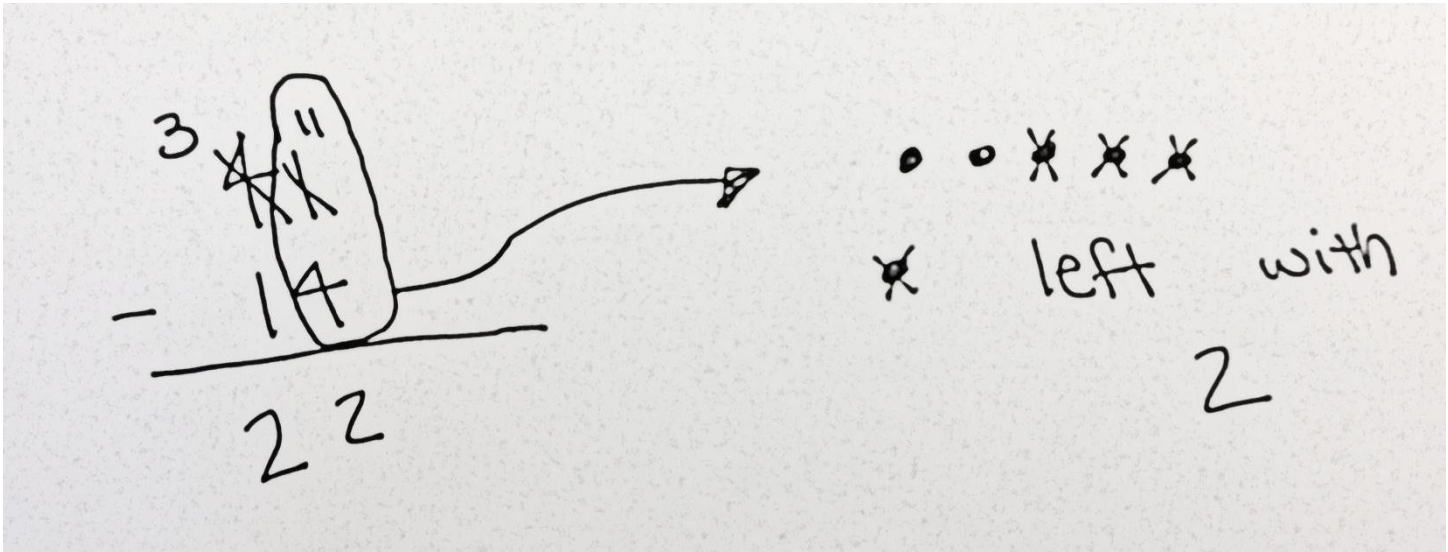
$$41 - 14 = ?$$

Take a few minutes to try this problem on your own.

Jason's Method



Kate's Method



Simon's Method