This sine has threee errors.

Warm-Up

1. Multiply your age by 12.
2. Add your friend’s age.
3. Divide by 7.
4. Divide by 11.
5. Divide by 13.
6. Add the first six digits after the decimal point.

   e.g., 0.123456 → $1 + 2 + 3 + 4 + 5 + 6 = 21$
7. Finally, multiply by 5…

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What’d You Get?

135

Session **135**

Whoa!

How does that work?

Calculation Nation®:
Game On!

October 12, 2012
G. Patrick Vennebush
pvennebush@nctm.org

ILLUMINATIONS
Resources for Teaching Math

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Dollar Nim

- Start with a dollar
- Remove any coin:
    - Penny
    - Nickel
    - Dime
    - Quarter
- Player to take the last coin wins

IN GOD WE TRUST
LIBERTY
2010
S
LIBERTY
IN GOD WE TRUST
S
2012
IN GOD WE TRUST
LIBERTY
2004
UNITED STATES OF AMERICA
IN GOD WE TRUST
LIBERTY
S
QUARTER DOLLAR

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Dollar Nim

- What is the winning strategy for this game?
- How could you modify this game for use with your students?

Eleven Nim

- Start with a dollar
- Remove any coin:
  - Penny
  - Dime
- Player to take the last coin wins

John Mason, Math 2.0 Listserve

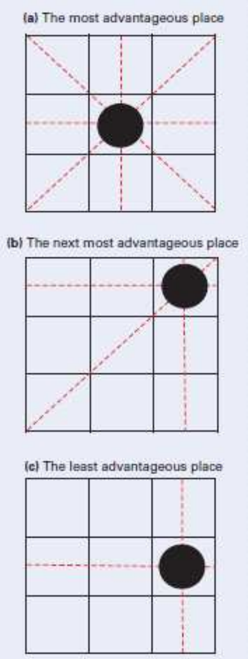
“Just because I play a game, it does not follow that I become aware of what I am doing [or the] underlying mathematical thinking. …the value of playing a ‘mathematical game’ may lie not in the playing so much as in the reflective consideration of effective and ineffective actions.”

Three C’s of Game Play

- Competition
- Collaboration
- Communication

Even one-player games can spark rich discussion of strategy.

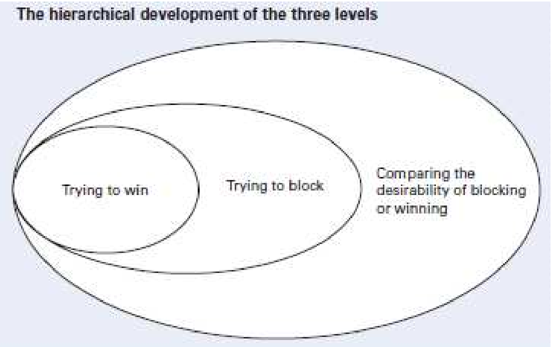
Tic Tac Toe



(a) The most advantageous place

(b) The next most advantageous place

(c) The least advantageous place



The hierarchical development of the three levels

Trying to win
Trying to block
Comparing the desirability of blocking or winning

Kamii, C. The Educational Value of Tic-Tac-Toe for Four-to Six-Year-Olds. *Teaching Children Mathematics,* May 2008.

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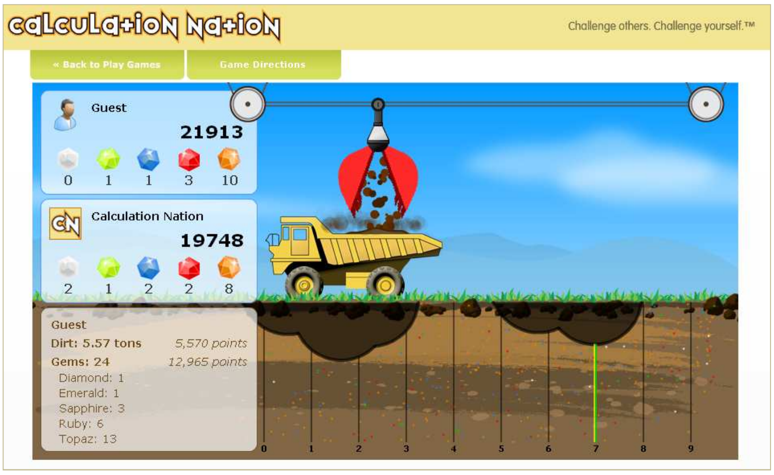
Dig It



Calculation Nation
Challenge others. Challenge yourself.™
« Back to Play Games
Game Directions
0
1
2
3
4
5
6
7
8
9

Guest 21913

0 1 1 3 10

Calculation Nation 19748

2 1 2 2 8

Guest

Dirt: 5.57 tons 5,570 points

Gems: 24 12,965 points

Diamond: 1

Emerald: 1

Sapphire: 3

Ruby: 6

Topaz: 13

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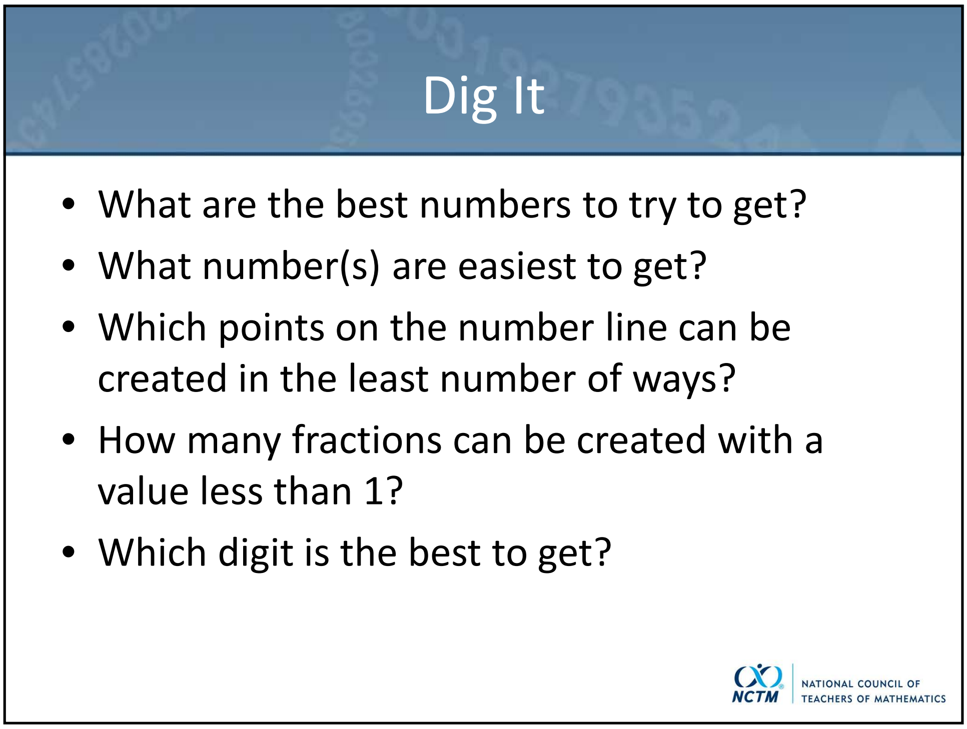

Dig It

- What are the best numbers to try to get?
- What number(s) are easiest to get?
- Which points on the number line can be created in the least number of ways?
- How many fractions can be created with a value less than 1?
- Which digit is the best to get?

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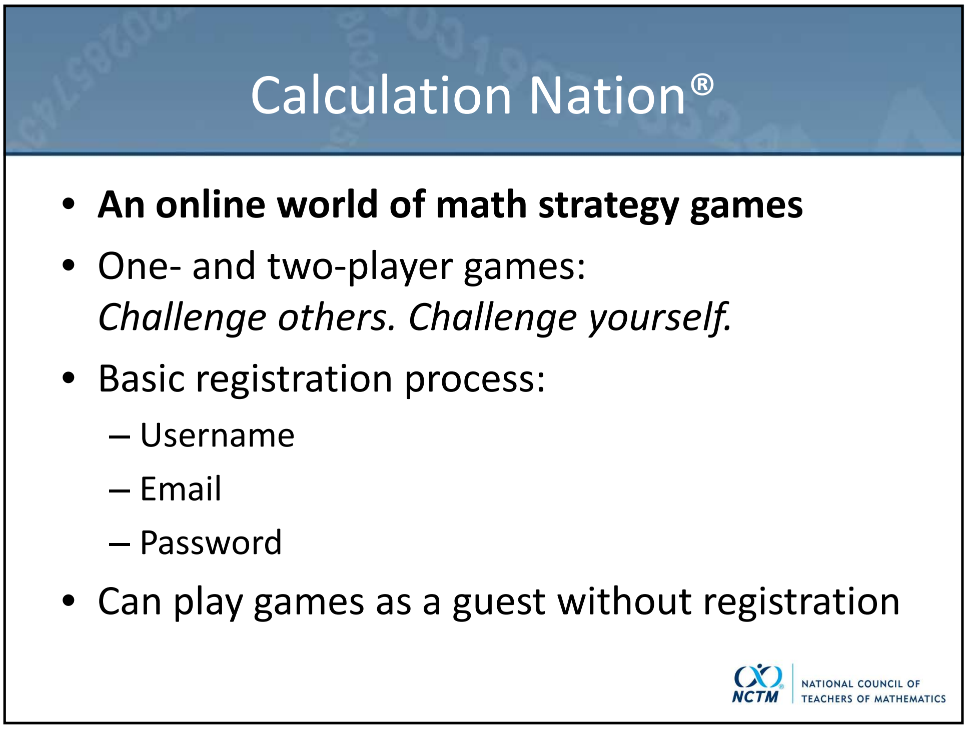

Calculation Nation®

- **An online world of math strategy games**
- One- and two-player games:
  Challenge others. Challenge yourself.
- Basic registration process:
  - Username
  - Email
  - Password
- Can play games as a guest without registration

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Calculation Nation™

Challenge others. Challenge yourself.™
Calculation Nation
illuminations.nctm.org

Official Launch

April 22, 2009

To Date: 1,209,527 Visitors

September 2012: 1,500 Visitors/Day

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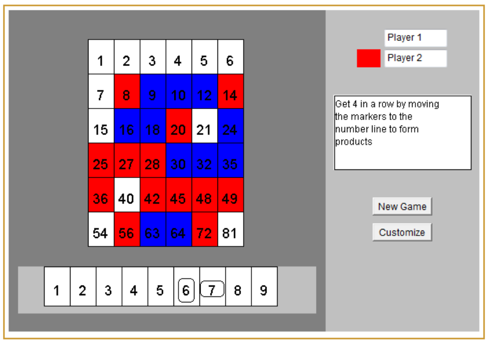
Calculation Nation™

- Idea Inspired by Teachers
- Played the "Product Game" Online Using Instant Messenger



1	2	3	4	5	6
7	8	9	10	12	14
15	16	18	20	21	24
25	27	28	30	32	35
36	40	42	45	48	49
54	56	63	64	72	81

1	2	3	4	5	6	7	8	9

Player 1
Player 2

Get 4 in a row by moving the markers to the number line to form products

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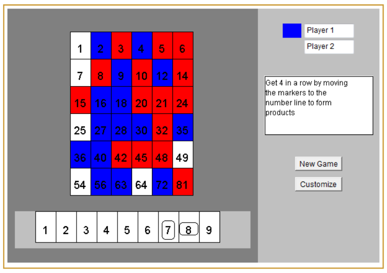
History

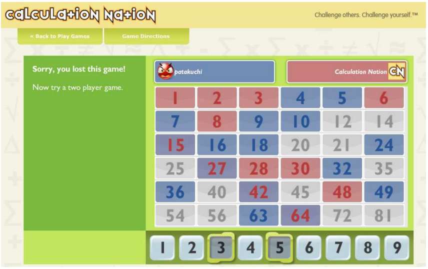
- Two teachers in Wyoming


Player 1
Player 2
New Game
Customize

Get 4 in a row by moving the markers to the number line to form products

1	2	3	4	5	6
7	8	9	10	12	14
15	16	18	20	21	24
25	27	28	30	32	35
36	40	42	45	48	49
54	56	63	64	72	81

1	2	3	4	5	6	7	8	9


calculation nation
Challenge others. Challenge yourself.™
« Back to Play Games
Game Directions
patakuchi
Calculation Nation
CN

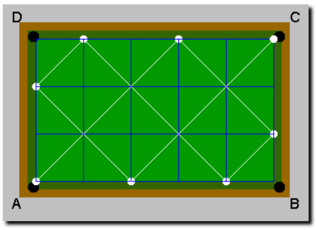
Sorry, you lost this game!

Now try a two player game.

1	2	3	4	5	6
7	8	9	10	12	14
15	16	18	20	21	24
25	27	28	30	32	35
36	40	42	45	48	49
54	56	63	64	72	81

1	2	3	4	5	6	7	8	9

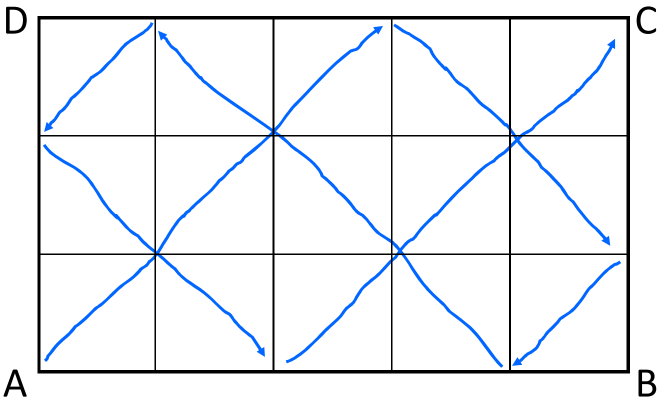
Paper Pool



How to Play Paper Pool

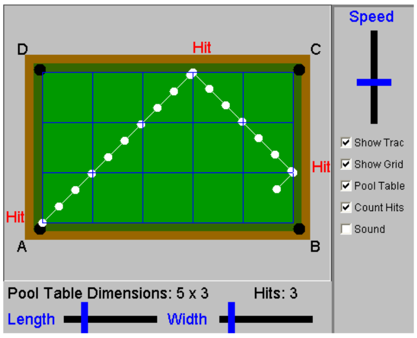
- The ball starts in corner A.
- The ball is **hit** with an imaginary stick so that it travels at a 45° diagonal across the grid.
- If the ball **hits** a side of the table, it bounces off at a 45° angle and continues its travel.
- The ball continues to travel until it **hits** a pocket.

Paper Pool



From Paper Pool…

- Online Version of the Paper Pool Lesson http://illuminations.nctm.org/LessonDetail.aspx?ID=U165

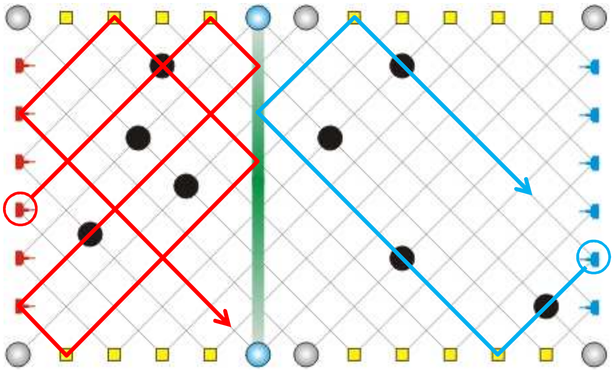

Speed
D
Hit
C
Hit
Hit
A
B
Length
Width

☑ Show Trac
☑ Show Grid
☑ Pool Table
☑ Count Hits
☐ Sound

Pool Table Dimensions: 5 x 3

Hits: 3

…to Slam Ball



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Game Design

- **Other Games:** Do the math, then you can do something fun.
- **Our Games:** Doing the math **IS** something fun.

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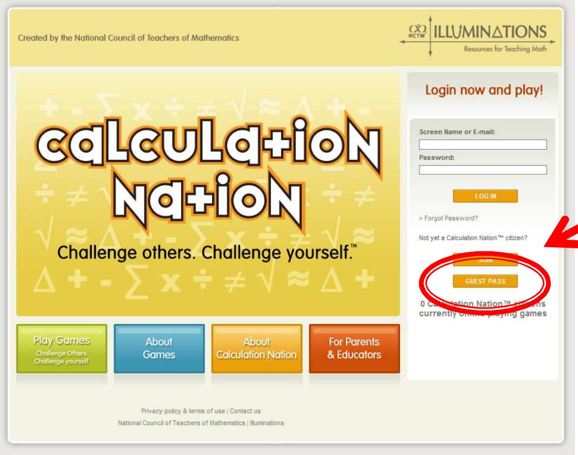
http://calculationnation.nctm.org


Created by the National Council of Teachers of Mathematics
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Resources for Teaching Math
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Challenge others. Challenge yourself.™
Login now and play!
Screen Name or E-mail:
Password:
LOG IN
> Forgot Password?
Not yet a Calculation Nation™ citizen?
JOIN
GUEST PASS
0 Calculation Nation™ citizens currently online playing games
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Challenge Others
Challenge yourself
About Games
About Calculation Nation
For Parents & Educators
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Click here:
“Guest Pass”

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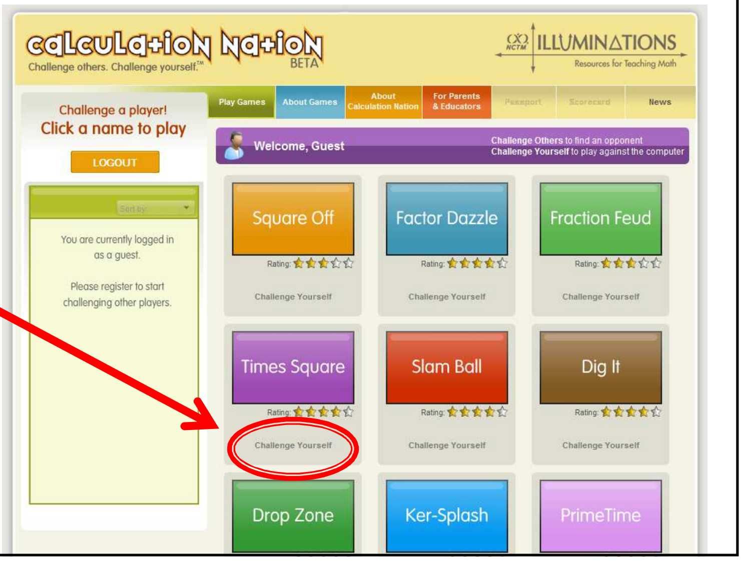
Play a Game!

Click here:
“Challenge Yourself”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Click a name to play
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Passport
Scorecard
News
Welcome, Guest
Square Off
Factor Dazzle
Fraction Feud
Times Square
Slam Ball
Dig It
Drop Zone
Ker-Splash
PrimeTime
Rating:
Challenge Yourself

You are currently logged in as a guest.

Please register to start challenging other players.

Challenge Others to find an opponent
Challenge Yourself to play against the computer

Ker-Splash

- Choose an expression:

$17x + 29y + 43$

$24x + 22y + 39$

- The values of *x* and *y* are unknown… but you can choose to increase one of them by 1, and decrease the other by 1. Which would you like to increase and which to decrease?
- Now, here are the values: $x = 6$, $y = 4$

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Ker-Splash

Your Equation	$x + 1$, $y - 1$	$x - 1$, $y + 1$
$17x + 29y + 43$	249	273
$24x + 22y + 39$	273	269

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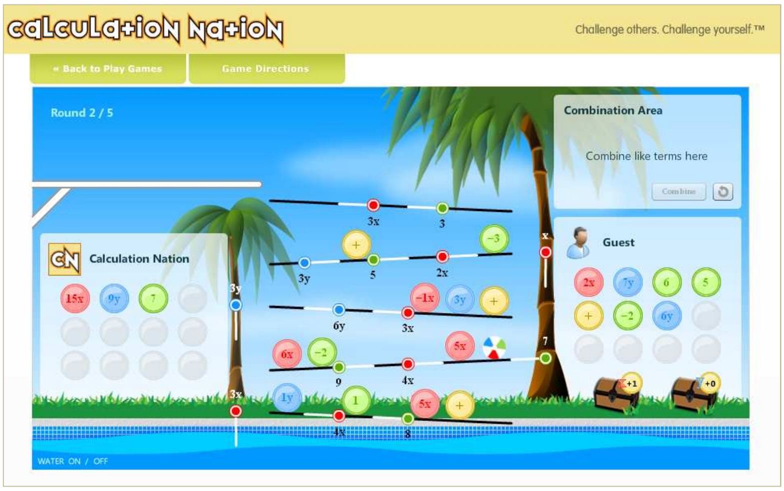
Ker-Splash


Calculation Nation
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« Back to Play Games
Game Directions
Round 2 / 5
Combination Area
Combine like terms here
Combine
Calculation Nation
Guest
WATER ON / OFF

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Tips for Teaching with Games

- Do not show children how to play at a higher level. Instead, encourage them to do their own thinking.
- Do not reinforce “correct” behaviors or try to correct “wrong” ones.
- Play with individual children whenever possible.

Kamii, C. The Educational Value of Tic-Tac-Toe for Four-to Six-Year-Olds. *Teaching Children Mathematics,* May 2008.

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Prime Time

Which is most likely to give an outcome of 4?

- Roll one die
- Roll two die, sum
- Roll two die, difference
- Spinner with four consecutive integers (your choice)
- Flipping *n* coins, number of heads

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Prime Time

Roll one die

P(4) = 1/6

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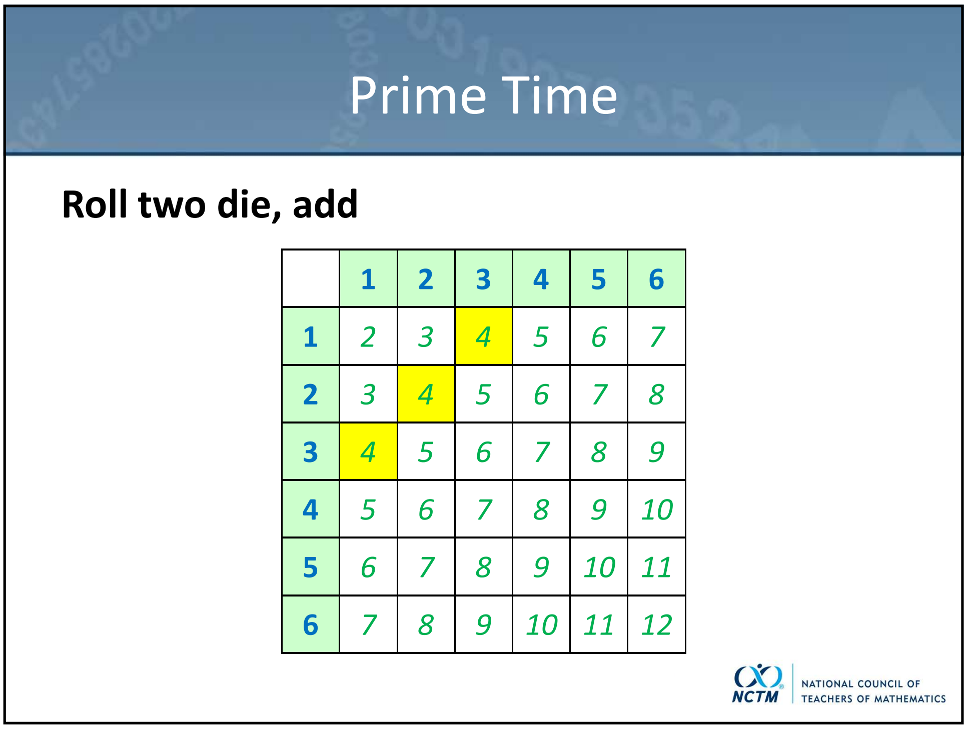

Prime Time

Roll two die, add

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

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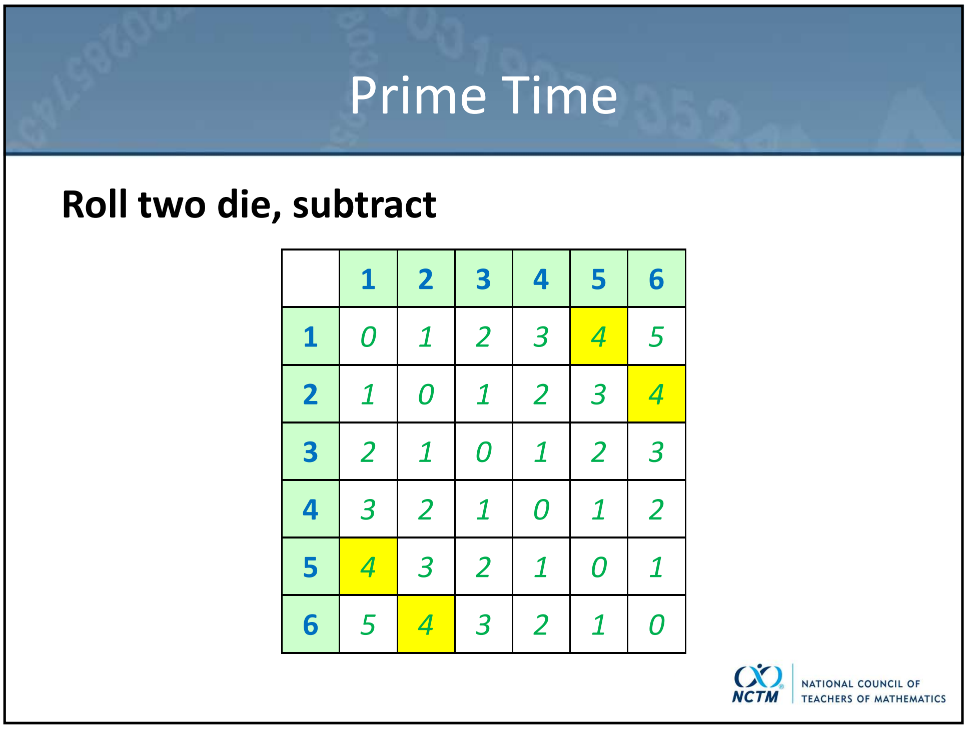

Prime Time

Roll two die, subtract

	1	2	3	4	5	6
1	0	1	2	3	4	5
2	1	0	1	2	3	4
3	2	1	0	1	2	3
4	3	2	1	0	1	2
5	4	3	2	1	0	1
6	5	4	3	2	1	0

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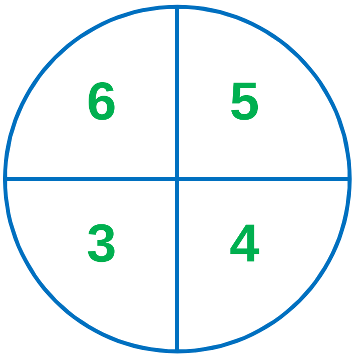
Prime Time

Spinner with four consecutive integers (your choice)



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Prime Time

Flipping *n* coins, number of heads

n	P(exactly 4 heads)
1	0
2	0
3	0
4	1/16
5	5/32
6	15/64

n	P(exactly 4 heads)
7	35/128
8	70/256
9	126/512
10	210/1024
11	330/2048
12	495/4096

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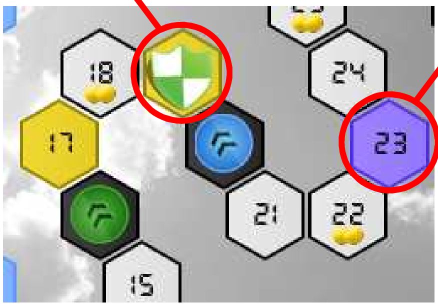
Prime Time

Current Location: 19

Desirable Location: 23



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illuminations.nctm.org

NCTM ILLUMINATIONS
Resources for Teaching Math

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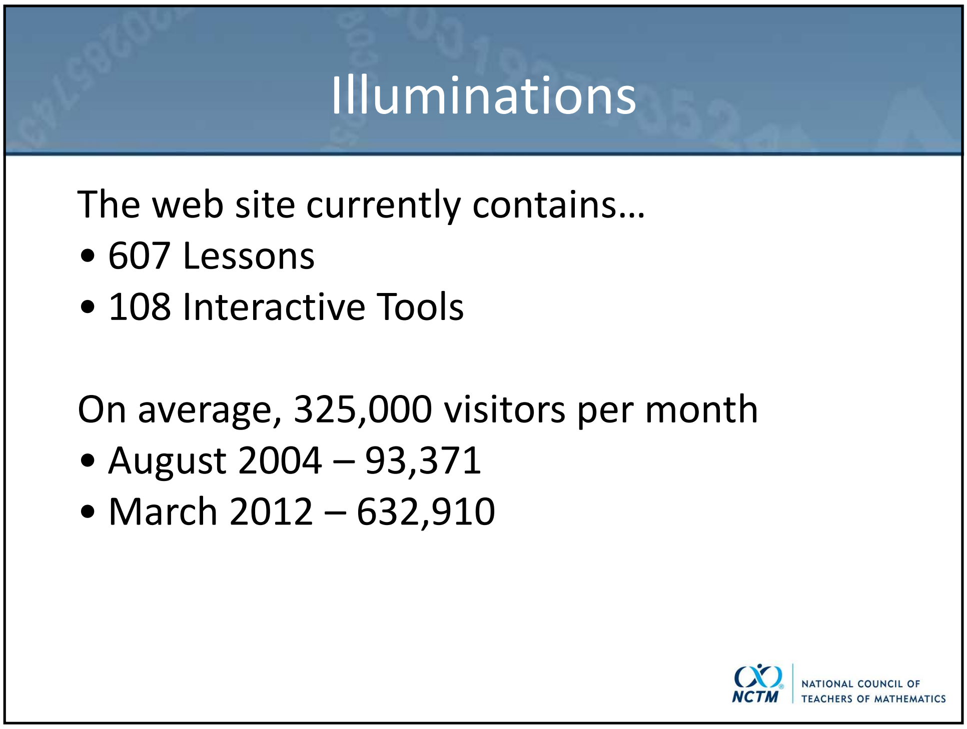

Illuminations

The web site currently contains…

- 607 Lessons
- 108 Interactive Tools

On average, 325,000 visitors per month

- August 2004 – 93,371
- March 2012 – 632,910

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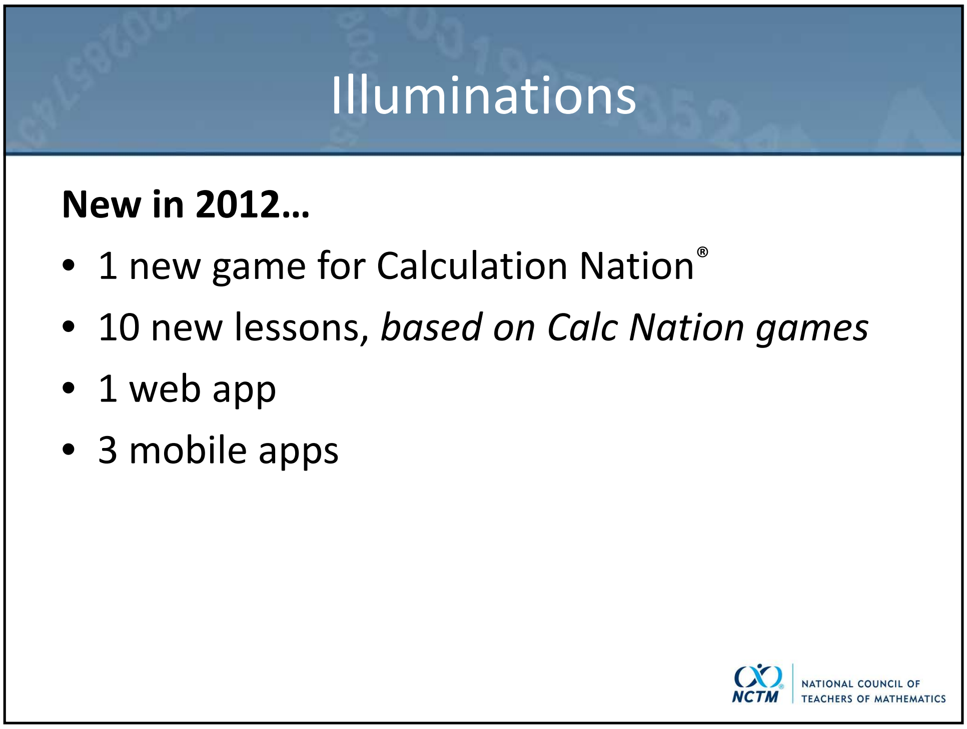

Illuminations

New in 2012…

- 1 new game for Calculation Nation®
- 10 new lessons, *based on Calc Nation games*
- 1 web app
- 3 mobile apps

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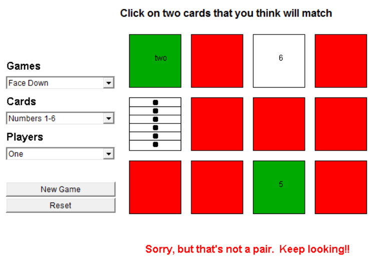
Illuminations



Click on two cards that you think will match

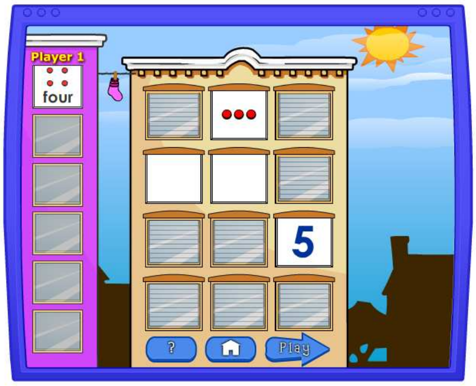
Games
Face Down

Cards
Numbers 1-6

Players
One

Sorry, but that's not a pair. Keep looking!!

two
6
5
New Game
Reset


Player 1
four
5
?
Play

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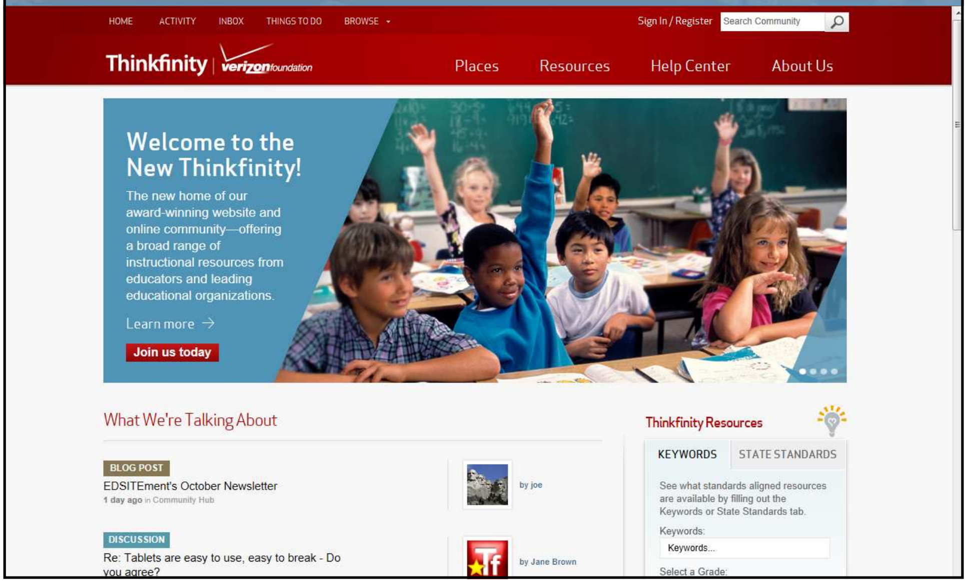
www.thinkfinity.org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ACTIVITY
INBOX
THINGS TO DO
BROWSE
Sign In / Register
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Places
Resources
Help Center
About Us
Learn more
Join us today
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KEYWORDS
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The new home of our award-winning website and online community—offering a broad range of instructional resources from educators and leading educational organizations.

What We're Talking About

EDSITEment's October Newsletter
1 day ago in Community Hub
by joe

Re: Tablets are easy to use, easy to break - Do you agree?
by Jane Brown

Thinkfinity Resources

See what standards aligned resources are available by filling out the Keywords or State Standards tab.

Keywords:
Keywords...

Select a Grade:

Thinkfinity

- Provides standards-based content and professional development for K–12 teachers
- Supported by the Verizon Foundation
  - – NCTM received a three-year, $1.4M grant for Illuminations
- Consortium of content partners across all disciplines
  - – science, arts, humanities, geography, economics, language arts, math, and history

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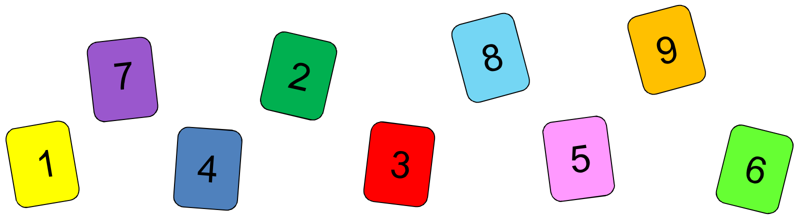
Game of Nine Cards

- **Materials:** Nine cards numbered 1–9
- **Object:** To have any three cards in your hand that add up to 15



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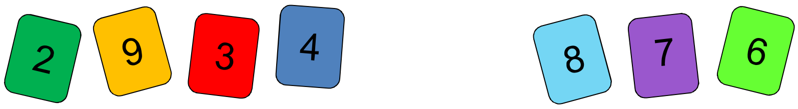
Game of Nine Cards

- Sample Game:



Player 1

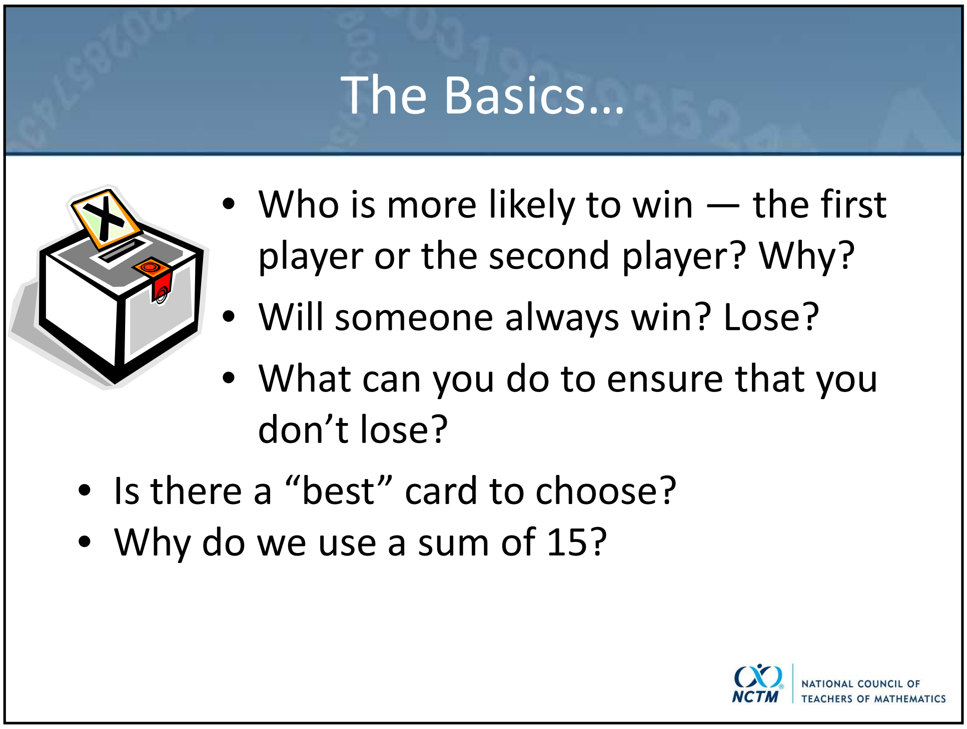
Player 2

Player 1 Wins: 2 + 9 + 4 = 15

Game of Nine Cards

Now what?

You Play!



The Basics…

- Who is more likely to win — the first player or the second player? Why?
- Will someone always win? Lose?
- What can you do to ensure that you don't lose?
- Is there a "best" card to choose?
- Why do we use a sum of 15?

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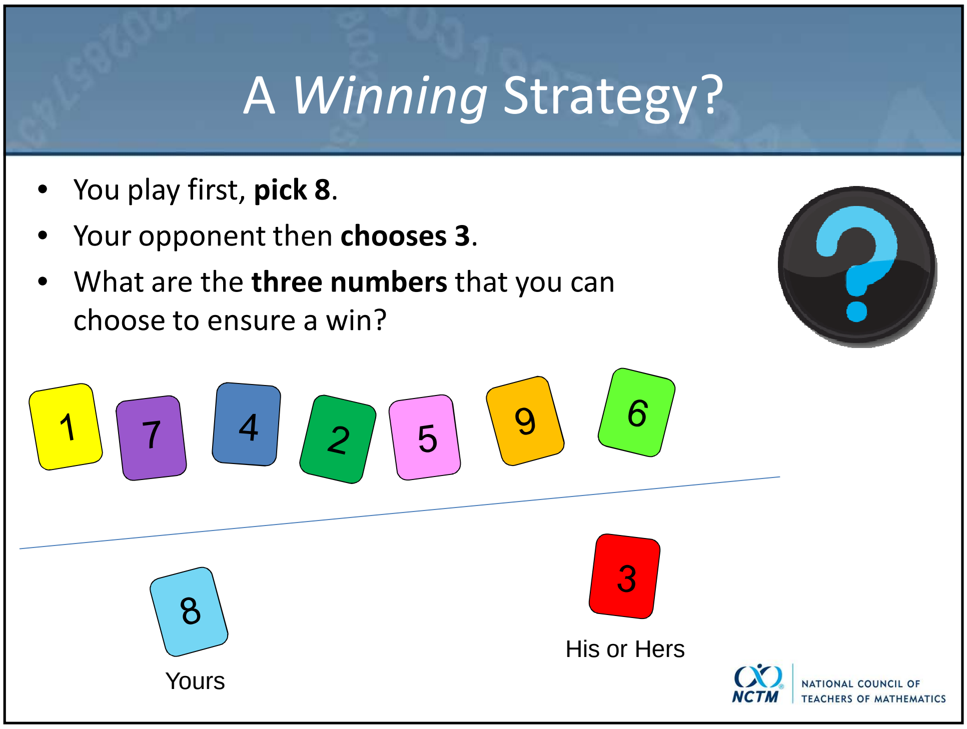

A *Winning* Strategy?

- You play first, **pick 8**.
- Your opponent then **chooses 3**.
- What are the **three numbers** that you can choose to ensure a win?

1
7
4
2
5
9
6
8
Yours
3
His or Hers

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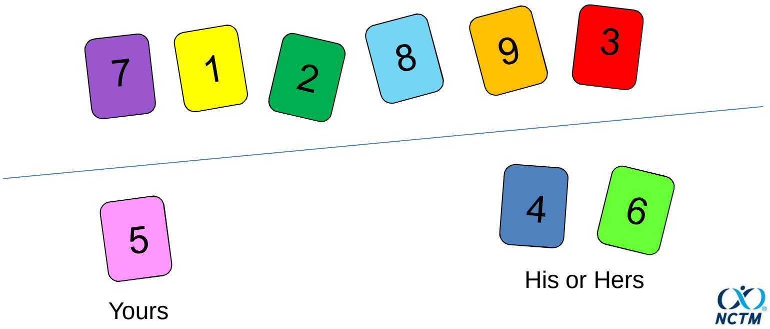
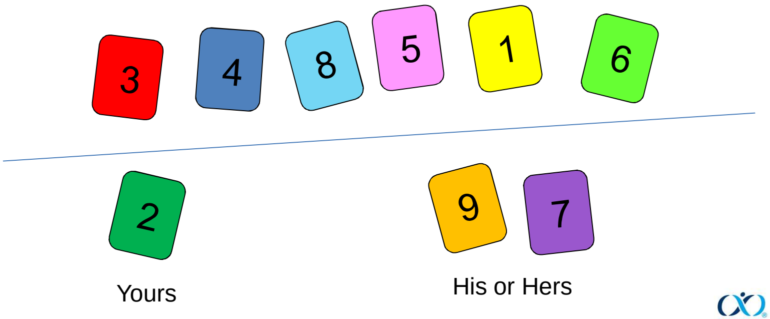
A *Winning* Strategy?

- Your opponent plays first, **picks 6**.
- You **choose 5**.
- Your opponent **picks 4**.
- Which **two numbers** should you ***not*** pick?


7
1
2
8
9
3
5
Yours
4
6
His or Hers

A *Winning* Strategy?

- Your opponent plays first, **picks 7**.
- Then you **choose 2**.
- Your opponent **picks 9**.
- Which **three numbers** should you ***not*** pick?

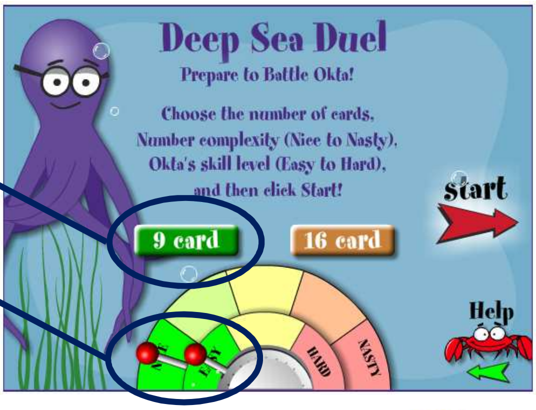

3
4
8
5
1
6
2
Yours
9
7
His or Hers

More Sophisticated Yet?

- If your opponent plays first and picks an **even** number, what number should you choose to **avoid a loss**?

Another App from Under the Sea

Deep Sea Duel



Deep Sea Duel

Prepare to Battle Okta!

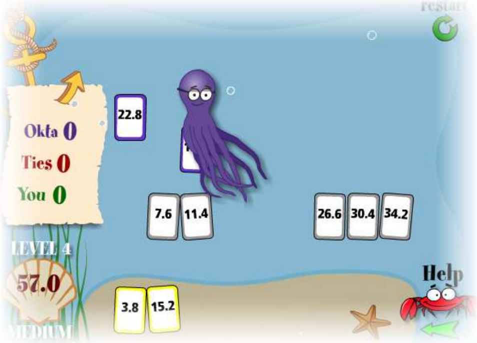
Choose the number of cards, Number complexity (Nice to Nasty), Okta's skill level (Easy to Hard), and then click Start!

9 Card

Nice / Easy

Game of Nine Cards

- **Deep Sea Duel** is online!
  - http://illuminations.nctm.org/deepseaduel


restart
Okta 0
Ties 0
You 0
22.8
7.6
11.4
26.6
30.4
34.2
LEVEL 4
57.0
MEDIUM
3.8
15.2
Help

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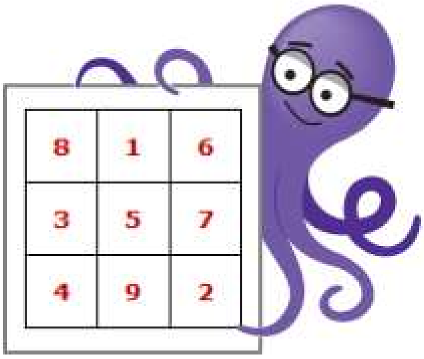
A Hint from Under the Sea



8	1	6
3	5	7
4	9	2

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Game of Nine Cards

- Why is this game mathematically significant? Consider the following chart:

8	1	6
3	5	7
4	9	2

Modifying the Game of Nine Cards

- Label the nine cards as follows:

5, 12, 19, 26, 33, 40, 47, 54, 61

The winner must get three cards that total 99.

Mahoney, John. What Is the Name of This Game?
Mathematics Teaching in the Middle School, October 2005.

Modifying the Game of Nine Cards

- Label the nine cards with fractions:

1/6, 5/24, 1/4, 7/24, 1/3, 3/8,
5/12, 11/12, 1/2

The winner must get three cards that total 1.

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Game of Nine Cards

- Use exponents! Label the cards as follows:

  x, x^2, x^3, …, x^9

  This time we want the *product* of three cards (which means we must *add* the exponents). The winner must get x^{15}.

- Have your students come up with the Magic Square for each of these modifications!

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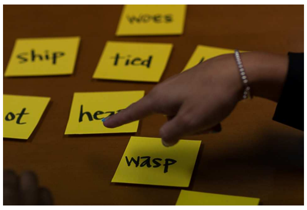
Modification: Words

- Use words!

  TIED, HOT, HEAR,
  TANK, WASP, WOES,
  SHIP, HORN, BRIM


woes
ship
tied
ot
he
wasp

- Winner needs three cards with same letter.

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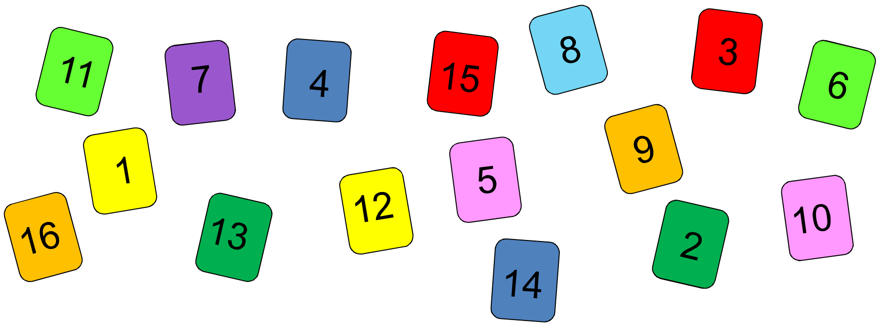
Modification: Sixteen Cards

The winner needs a sum of *four* cards to win.


11
7
4
15
8
3
6
1
9
16
13
12
5
10
2
14

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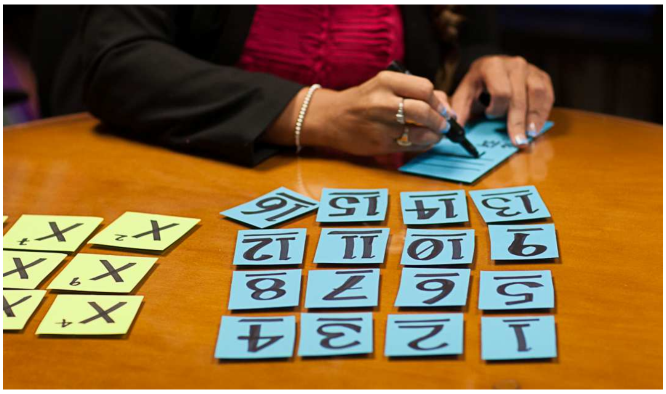
Modification: Sixteen Cards

- **What sum would the winner need?**



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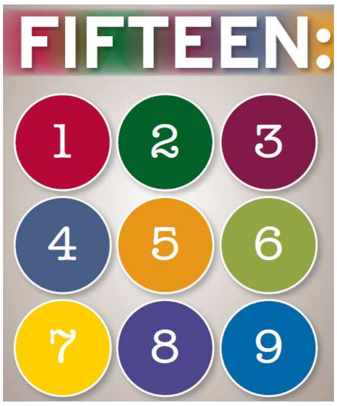
Modification: Two or More

- The winner is the first player to obtain the sum of exactly 15 from any **TWO OR MORE** cards.
- How does your strategy change?



Yeo, Joseph. [Title removed in order to not give away punch line of strategy.] *Mathematics Teacher,* August 2012.

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Can You Use This Game?

- How could you modify the game of nine cards to fit the needs of your students?

Reminder: What is the Goal?

- How does your strategy from the first version of the game compare to the strategy for these modifications?
- *Reflect:* How did you come up with these other versions for the game of nine cards?

Game of Nine Cards

nctm.org/gameofnine

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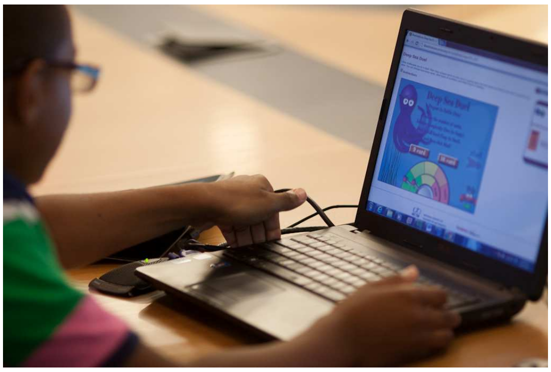
iStuff. Android. Computer.

illuminations.nctm.org/deepseaduel



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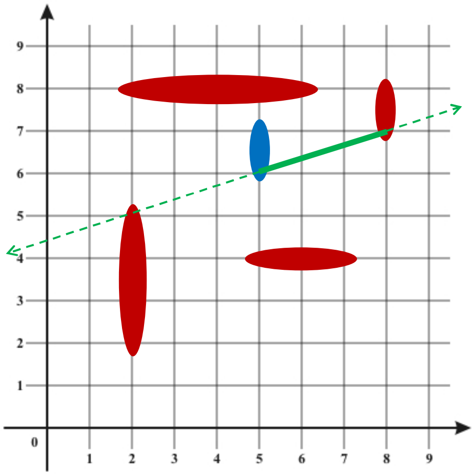
Equations of Attack

- Original Game:
  - Both players place ships of length 2, 3, 4, and 5 on game board
  - Other player guesses location of your ships
  - Then, draw a line connecting each of your ships to each of your opponent’s ships of the same length
  - Determine the equation of the lines connecting the ships

Equations of Attack



Equations of Attack

- Revised Game:
  - – Players take turns placing ships at lattice points
  - – One player gets all of the cannons at the even locations along the *y*-axis — that is, (0,0), (0,2), (0,4), (0,6), and (0,8)
  - – Other player gets all cannons at the odd locations
  - – Player rolls dice; roll both, use them to make a fraction representing the slope

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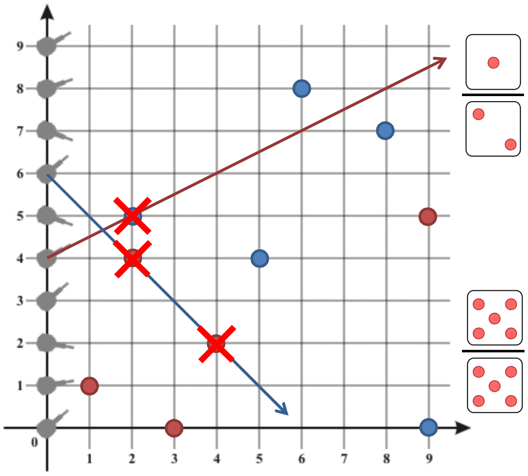
Equation of Attack



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Equations of Attack

- The Algebra Standard states that “in grades 6–8 all students should explore relationships between symbolic expressions and graphs of lines, paying particular attention to the meaning of intercept and slope” (NCTM, 2000, p. 222).
- How does the activity help with understanding of those concepts?

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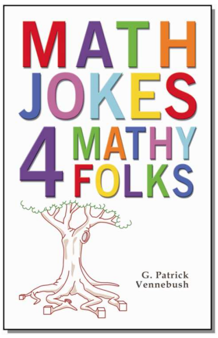
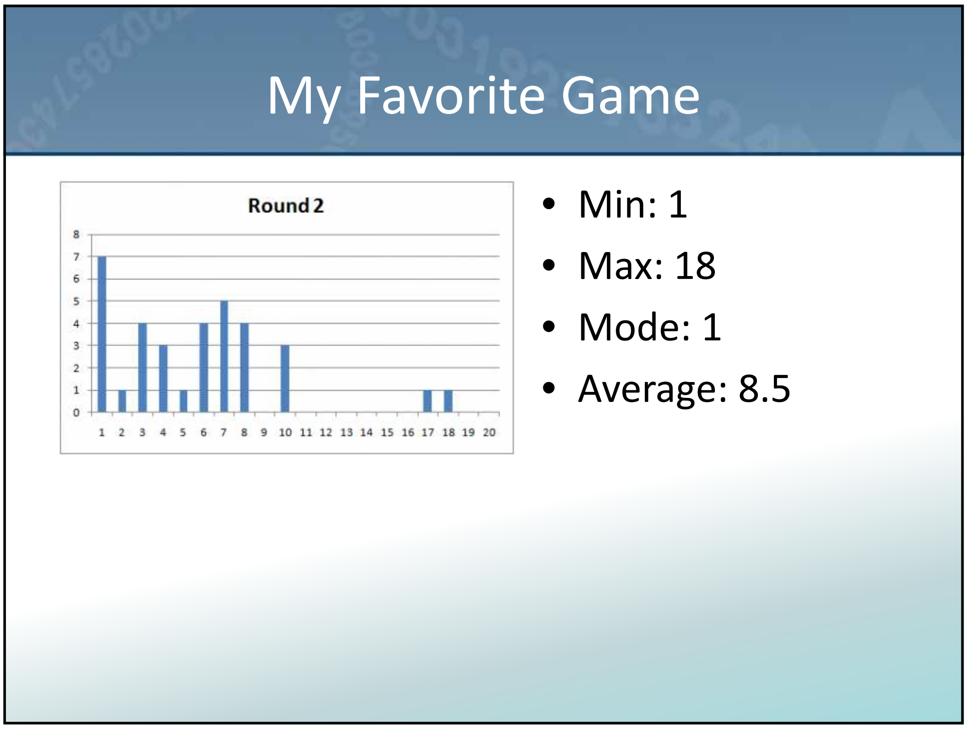
My Favorite Game…


MATH JOKES 4 MATHY FOLKS

G. Patrick Vennebush

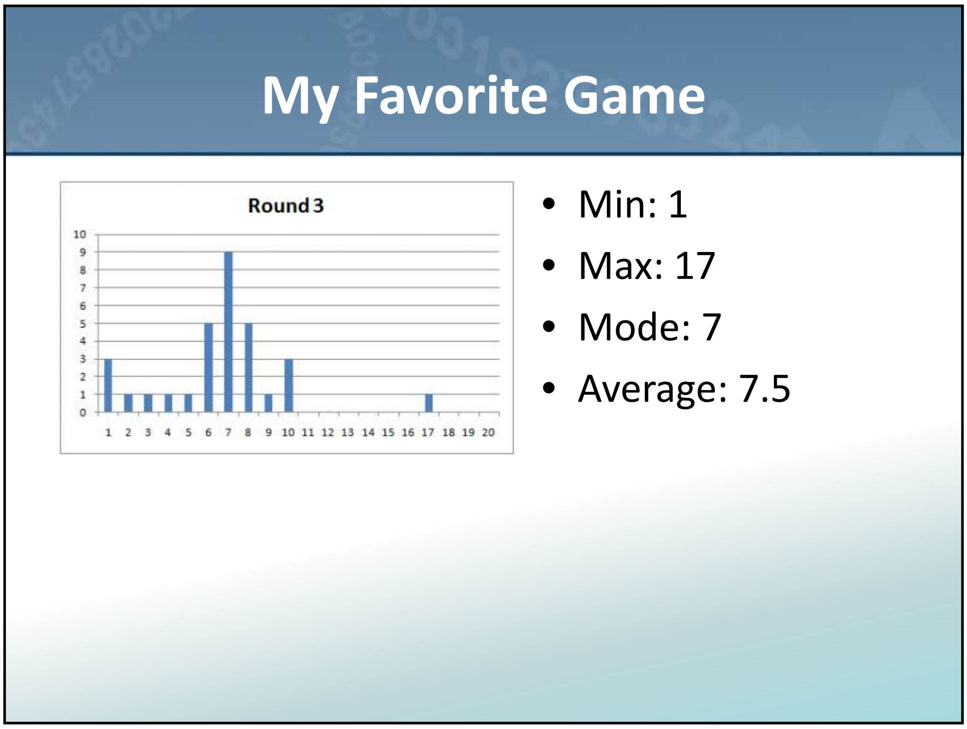
- Write a positive integer on a piece of paper.
- Show it to your neighbor.
- The winner is…

Whoever wrote the smallest integer NOT written by anyone else.



My Favorite Game

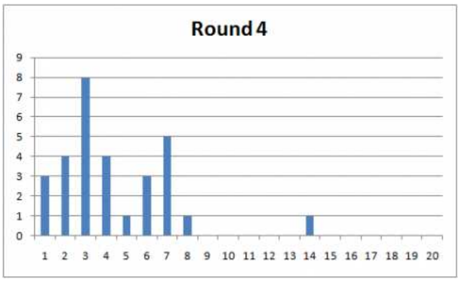
- Min: 1
- Max: 18
- Mode: 1
- Average: 8.5



My Favorite Game

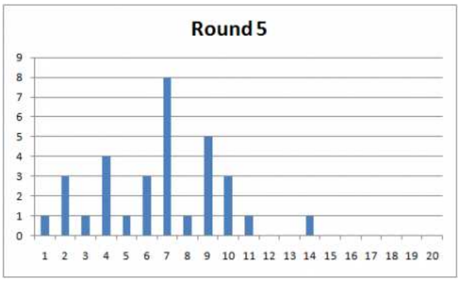
- Min: 1
- Max: 17
- Mode: 7
- Average: 7.5

My Favorite Game



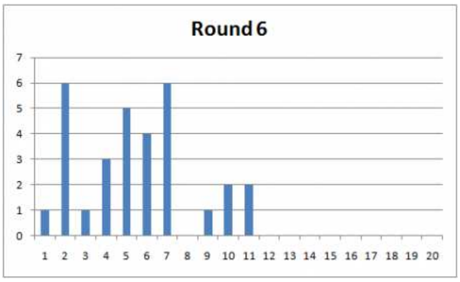
- Min: 1
- Max: 14
- Mode: 3
- Average: 5.3

My Favorite Game



- Min: 1
- Max: 14
- Mode: 7
- Average: 7.7

My Favorite Game



- Min: 1
- Max: 11
- Mode(s): 2, 7
- Average: 6.4

11Q – Q

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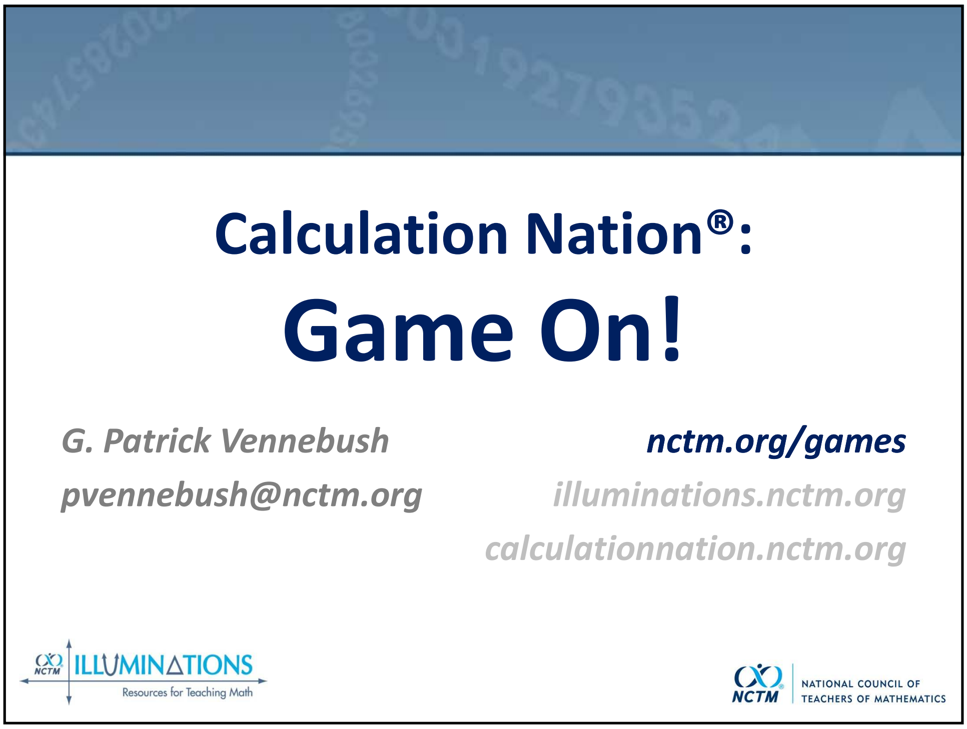

Calculation Nation®:

Game On!

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