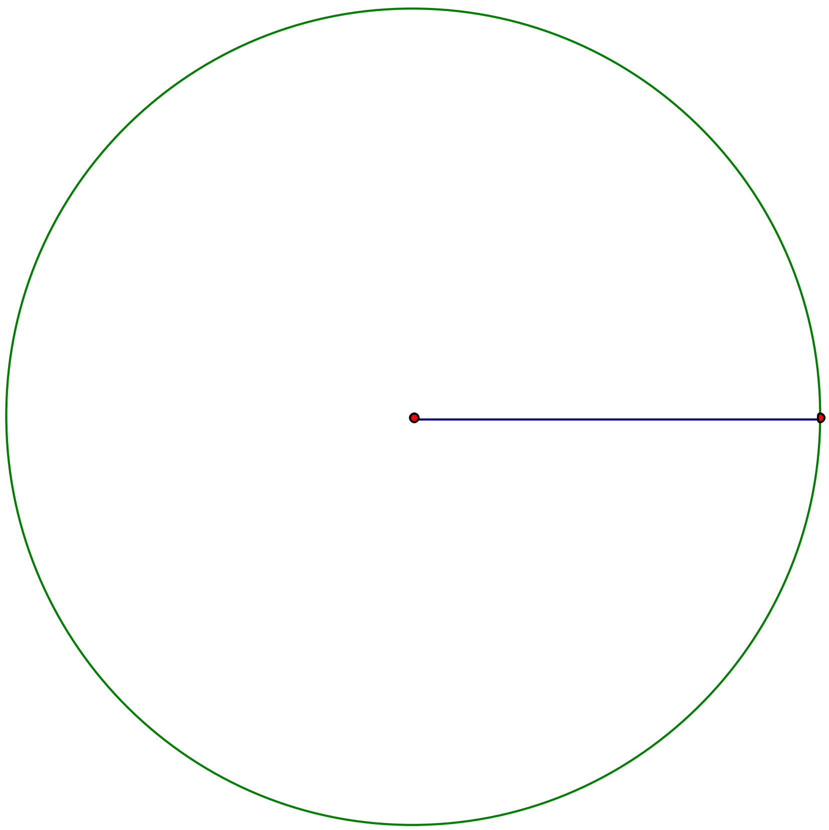
Optimization

Cut out the circle below. Also cut along the radius. The circle has a radius of 5.4 cm. Find the cone of largest volume that can be formed by overlapping sectors of the circle. Then answer the following questions.

1. What are the dimensions of your cone of largest volume?
2. How large is the angle θ of overlap?
3. What is the volume of the largest cone?

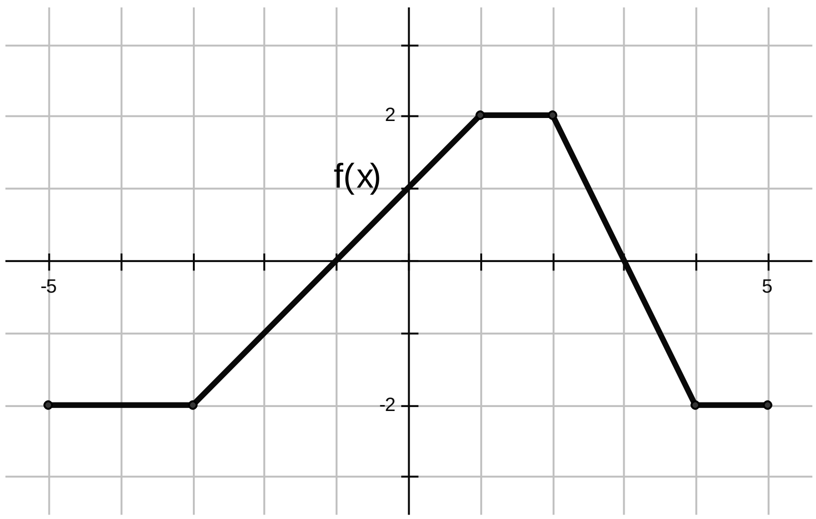


Activity designed by Kevin A. Ford

The Area Function

Now that we've learned what an integral is, let's develop a new type of function: the area function.

Consider the function $f(x)$ defined on the interval [-5,5] shown below:



Now, a new function $A(t)$ is defined as the area under f from -3 to t:

$$A(t) = \int_{-3}^{t} f(x)dx$$

Find the following:

$A(-3) =$

$A(-1) =$

$A(1) =$

$A(3) =$

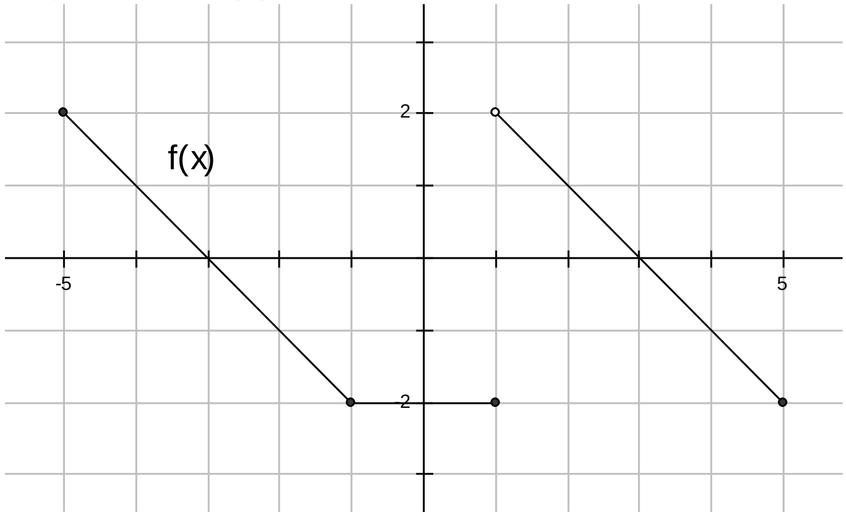
$A(5) =$

$A(-4) =$

$A(-5) =$

Sketch a graph of $A(t)$ on the grid above using the x-axis as the t-axis.

Now let's look at a different function. Suppose that a function $f(x)$ defined on the interval [-5,5] is given by the following graph:



Suppose that we define an area function as: $A(x) = \int_{-5}^{x} f(t)dt$

Why did I write $f(t)$ instead of $f(x)$?

Determine the following values of A.

$A(-5)$ $A(-4)$ $A(-3)$

$A(1)$ $A(3)$ $A(5)$

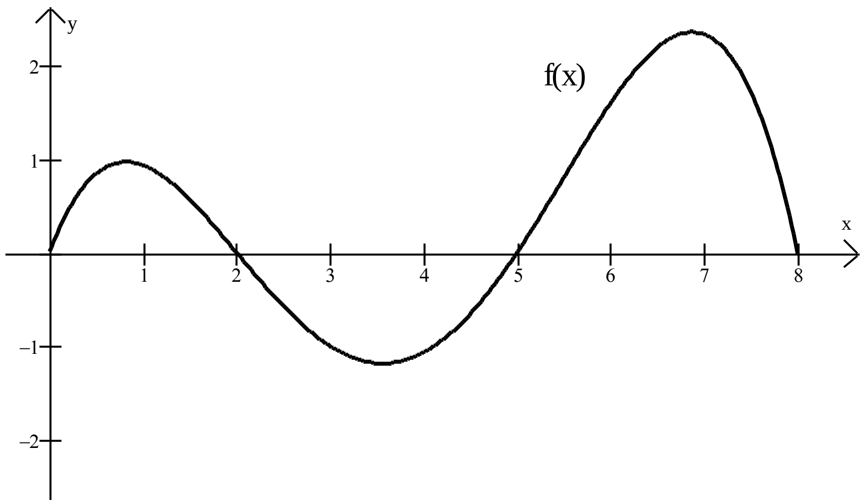
Sketch the graph of A on the grid above.

On what intervals is A increasing?

On what intervals is A decreasing?

At what values of x does A have local maximums or minimums?

For a new problem, consider the curve $f(x)$ defined on the interval [0,8] shown below.



Let g be defined by the integral $g(x) = \int_0^x f(t)\, dt$

What is $g(0)$?

Is $g(5) > 0$ or is $g(5) < 0$? Explain.

On what interval(s) is g increasing?

On what interval(s) is g decreasing?

Make a rough sketch of g over the graph of f above.

At what value(s) of x does it appear that $g'(x) = 0$?

At what value of x on (0,8) does g have a local minimum? A local maximum?

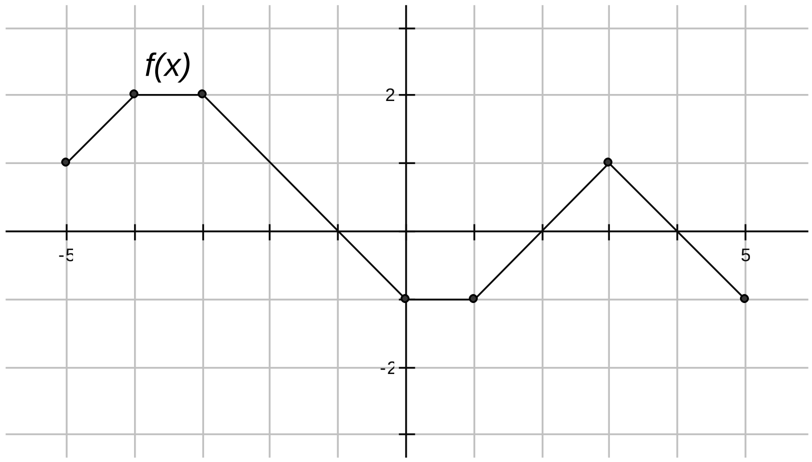
Does there appear to be a familiar relationship between g and f? What is this relationship?

Area Functions, a comparison

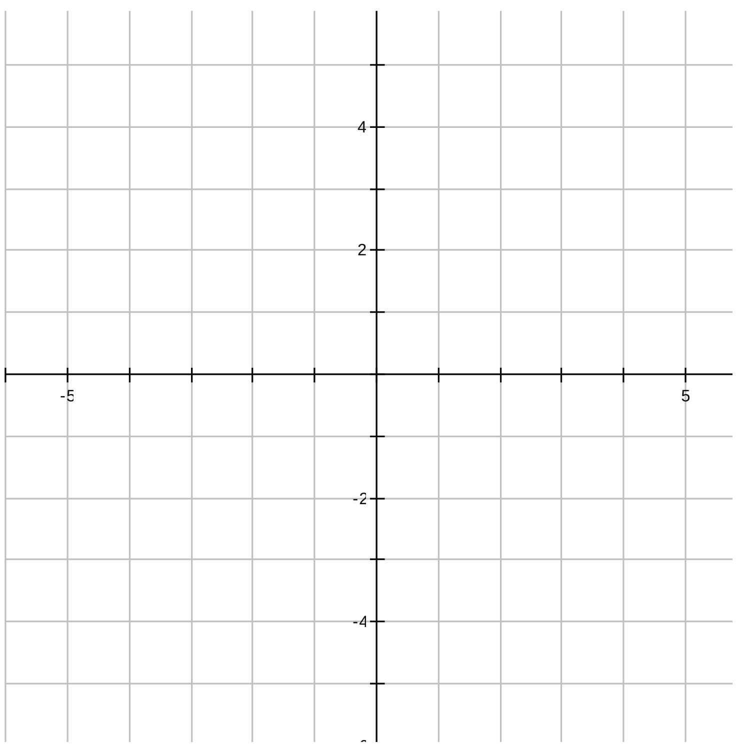
Question 1

Let's use a function *f* to create an area function: $A(x) = \int_{-5}^{x} f(t)\,dt$. Use geometry to complete the table below by finding values of *A* for integer values of *x* on the interval $-5 \le x \le 5$.

x	*A*(*x*)
-5	
-4	
-3	
-2	
-1	
0	
1	
2	
3	
4	
5	



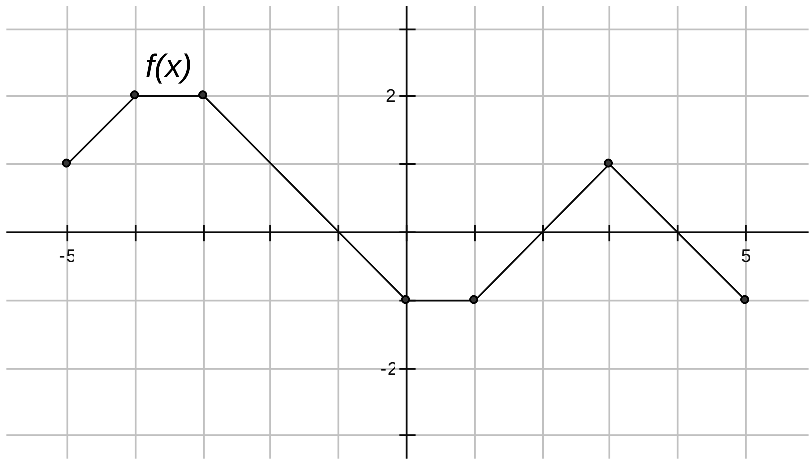
Now, on the grid below, sketch a graph of *A*(*x*) on the interval $-5 \le x \le 5$.



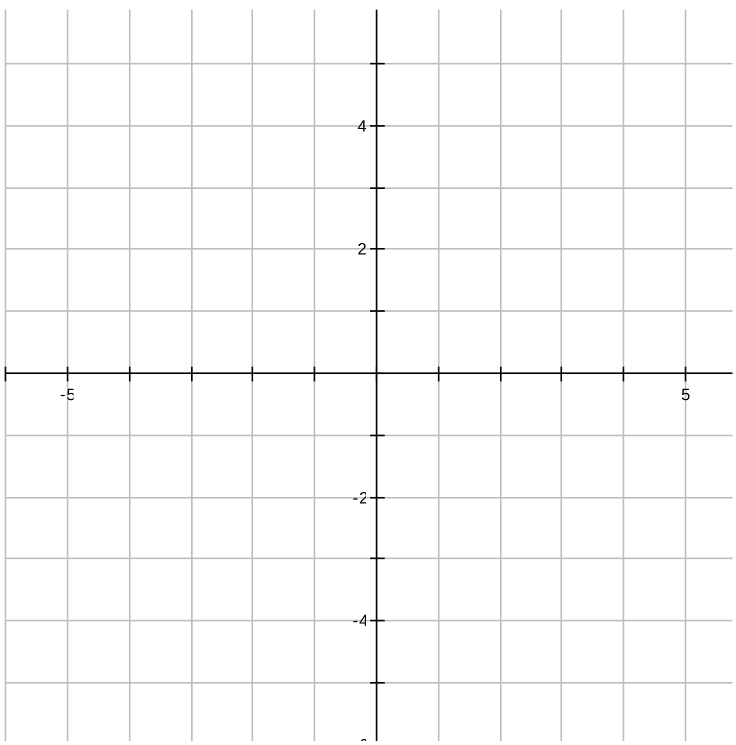
Question 2

Now, let's define a new area function: $B(x) = \int_{-3}^{x} f(t)\,dt$, where *f* is the same function as in question 1. Complete the table below by finding values of *B* for integer values of *x* on the interval $-5 \le x \le 5$.

x	*B*(*x*)
-5	
-4	
-3	
-2	
-1	
0	
1	
2	
3	
4	
5	



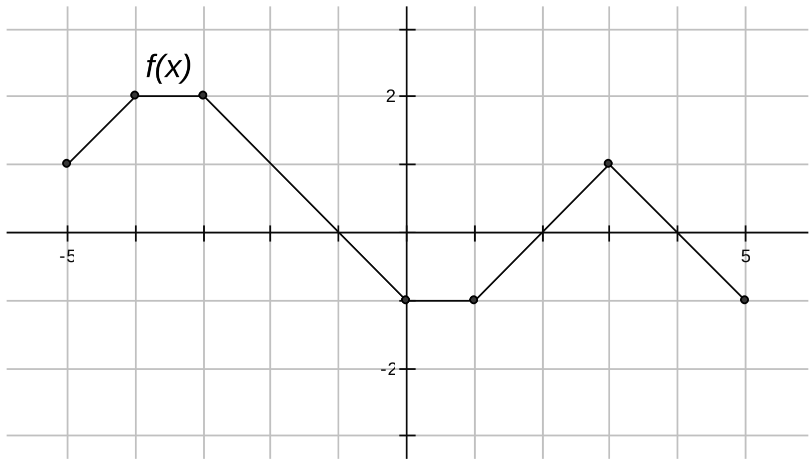
Now, on the grid below, sketch a graph of *B*(*x*) on the interval $-5 \le x \le 5$.



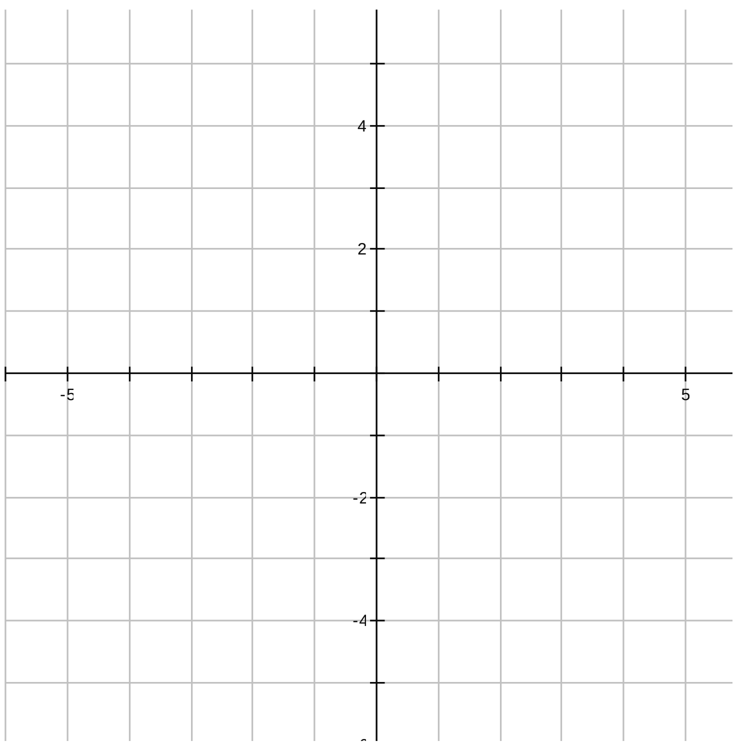
Question 3

Now, let's define a new area function: $C(x)=\int_0^x f(t)\,dt$, where *f* is the same function as in question 1. Complete the table below by finding values of *C* for integer values of *x* on the interval $-5 \le x \le 5$.

x	*C*(*x*)
-5	
-4	
-3	
-2	
-1	
0	
1	
2	
3	
4	
5	



Now, on the grid below, sketch a graph of *C*(*x*) on the interval $-5 \le x \le 5$.



4. Look at your graphs for the two area functions, *A*, *B* and *C*. How are the graphs similar?

5. Look at your graphs for the two area functions, *A*, *B* and *C*. How are the graphs different?

6. How is the graph of *f* related to the graph of *A*?

7. How is the graph of *f* related to the graph of *B*?

8. How is the graph of *f* related to the graph of *C*?