Hooking Students on Algebra: Activities That Make It Fun

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Activity: Equations of Attack
Lesson Plan

NCTM Standards:

Content Standards:

Numbers and Operations

- Use fractions and integers to plot points and slope.

Algebra

- Use symbolic algebra to represent situations and to solve problems, especially those that involve linear relationships.
- Identify functions as linear or nonlinear and contrast their properties from tables, graphs, or equations.

Process standards include:

Problem Solving

- Solve problems that arise in mathematics and in other contexts
- Apply and adapt a variety of appropriate strategies to solve problems

Communication

- Organize and consolidate their mathematical thinking through communication;
- Communicate their mathematical thinking coherently and clearly to peers, teachers, and others

Representations

- Select, apply, and translate among mathematical representations to solve problems

Learning Objectives

Students will:

- Plot and name points on a coordinate grid using correct coordinate pairs
- Graph lines given slope and *y*-intercept
- Practice writing equations given slope and *y*-intercept
- Determine algebraically if a point lies on a line

Students will plot points on a coordinate grid to represent ships before playing a graphing equations game with a partner. Points along the *y*-axis represent cannons and slopes are chosen randomly to determine the line and equation of attacks. Students will use their math skills and strategy to sink their opponent's ships and win the game. After the game, an algebraic approach to the game is investigated.

Materials

- Colored pencils or markers
- Two Colored dot stickers
- Slope Card Activity Sheet
- Equations of Attack Activity Sheet

Instructional Plan

Tell students that they will be playing a strategy game in which they must sink their opponent's ships. To win the game students will need to use their knowledge of graphing and linear equations.

Break the class up into pairs. Depending on the ability levels of your students, you may choose to allow them to pick their own partners or separate them into pre-determined pairs that are matched for mathematical ability. Distribute the Equations of Attack activity sheet, Slope Cards activity sheet, 2 different-colored pencils, a coin, and scissors to each pair of students. **Note:** If your class is just beginning to explore linear equations, you may wish to create your own set of slope cards with only integers (e.g., 2 and –3) and unit fractions (e.g., $^1/_4$, but not $^3/_4$). Likewise, to challenge more advanced students, consider including decimal slopes (e.g., 1.5).

Read through the questions and game rules with students. You may also want to go through an example of the game on the board before students begin. Draw a ship at (1,5) and one at (2,7). Tell students to assume that you drew a slope card with a value of 3, and that you have the odd-numbered cannons. Ask students which cannon would be the best to use given the location of the ships and the slope. Show students that if you choose 1 as your cannon location, the line you draw intersects (and sinks!) the ship at (2,7).

Since students will have to write the equations for their lines of attack, you may wish to write the equation for this line on the board:

$$y = 3x + 1$$

If students struggle with the example, you may choose to do another example or two. Since the player has the odd-numbered cannons, the other possible lines of attack would be:

- $y = 3x + 3$
- $y = 3x + 5$
- $y = 3x + 7$
- $y = 3x + 9$

If some students in your class seem to understand while others continue to struggle, have a student who understands come to the board and draw the line of attack to determine whether either of the two ships is sunk.

In general, the equations will be:

$$y = (\text{slope})x + (\text{cannon position})$$

However, try not to share this with students. They should discover this pattern and its meaning on their own.

Playing the Game

Have students start by stacking the slope cards face down. As students move on to plotting their ships, walk around and make sure they plot them correctly. They may try to color in blocks or choose locations between points rather than at the lattice points. You may choose to check the game boards (ship and cannon locations) before students start, to ensure the desired results.

In playing the game, students should use their color to draw their line on the game board, from their cannon and using the correct slope, to see if the line intersects any of their opponent's ships. However, if you notice a large number of students drawing lines incorrectly, you may choose to pause play and do another example or two on the board. Encourage students to use vocabulary words, such as *slope* and y-*intercept*, and to name the points as (x, y) coordinate pairs. As students are playing their games, remind them to list their equations in Question 1. Student pairs can share an activity sheet or record their answers separately.

Discussion Questions

- Is there a ship placement that is totally safe from the cannons? If so, where? If not, why?
- Do you prefer to find if the ships are sunk by graphing or by using algebraic substitution?

Resources

Hendrickson, K. Equations of Attack. *Illuminations: Resources for Teaching Math.* Retrieved
from http://illuminations.nctm.org/LessonDetail.aspx?id=L782

BALLOON POWERED CARS

Names:________________________________

Test Results:
Distance Traveled FORWARD______
Total Money Spent_______

Balloon Powered Cars!

Group Roles:

Materials Manager	________________	Designer	________________
Time Keeper	________________	Builder	________________

Situation:

Your company has been invited to create a balloon powered car prototype. Each company must make a car that will travel a long distance along a straight path. Speed is not an issue. As always, expense is a factor in the final consideration of the prototype. Due to the environmental laws in the area, there is a limit on the materials that can be used so select your supplies carefully and with great consideration. In addition to designing your car, each team will also be responsible for renting track time in order to test their prototypes. You and your team must determine which track offers the better deal: The Balloon Racer Track or The Power Racer Track. Have fun and good luck!

The Balloon Racer Track has an initial down payment fee of $100 and an additional $30 fee for every 30 minutes on the track. The Power Racer Track has an initial down payment fee of $200 and an additional $20 fee for every 30 minutes on the track. Which track offers the better deal? Be sure to represent your conclusion by writing both equations, creating tables, and graphing.

Time:

15 minutes—Planning time used to name company, think of advertisement, plan construction, and determine and purchase needed materials.
15 minutes—Constructing and naming the prototype.
10 minutes—Calculating the total price of the car and the cost of the rental time for the two race tracks (equations, tables, and graphs)
Time to test THE BALLOON POWERED CARS!

Materials:

5"x 8" index card……….= $ 20
4"x 6"index card………..= $15
3"x 5"index card………..= $10
Straw…………….......... = $20
1" of tape……………..... = $5
Large paper clip……….. = $8
Small paper clip………..= $5
Pencil…………………...= $20
Tack/brad……………….= $10
Large paper plate……… = $15
Small paper plate……….= $10
Construction paper……..= $3 per square inch

Clothes pin……………..= $15
Craft stick……..………= $8
Balloon………………… = $35

Order Form for all Needed Supplies

Materials	Cost per item	Quantity Needed	Algebraic Expression	Total Cost

Race Track Information and Conclusions

Number of Intervals (30 minutes each)	The Balloon Racer Track	The Power Racer Track
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		
11		
12		

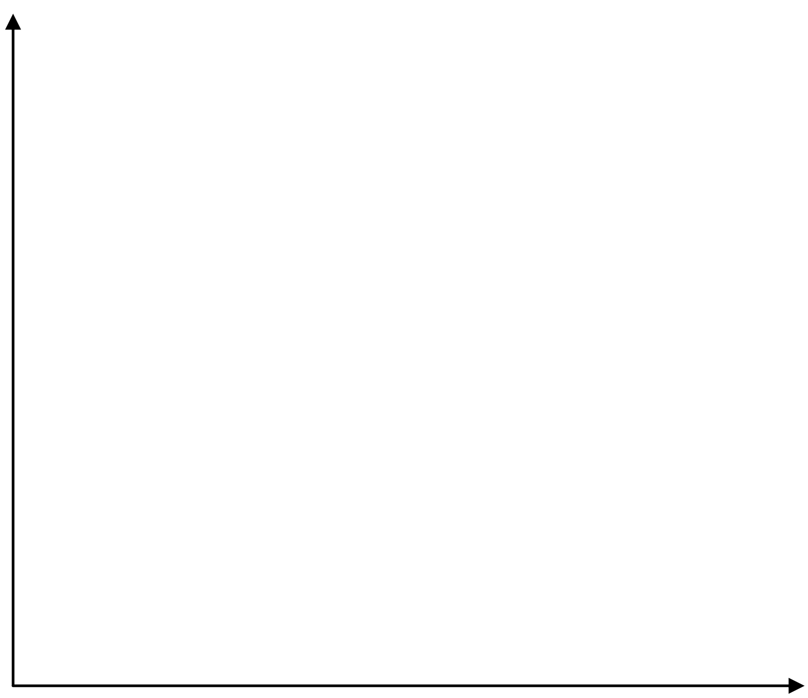
1) Write an equation for The Balloon Racer Track.

2) Write an equation for The Power Racer Track.

3) Graph each equation on Quadrant I (provided below). Be sure to label your graph.

4) Do these lines ever interest? What does this mean?

5) Which race track is a better deal? Why?



Activity: Rise of the Water

NCTM Standards:

Content Standards:

Numbers and Operations

- Develop a deeper understanding of very large and very small numbers and of various representations of them.
- Develop fluency in operations with real numbers, vectors, and matrices, using mental computation or paper-and-pencil calculations for simple cases and technology for more-complicated cases.
- Judge the reasonableness of numerical computations and their results.

Measurement

- Make decisions about units and scales that are appropriate for problem situations involving measurement.
- Analyze precision, accuracy, and approximate error in measurement situations.

Algebra

- Analyze functions of one variable by investigating rates of change, intercepts, zeros, asymptotes, and local and global behavior.
- Interpret representations of functions of two variables.
- Use symbolic algebra to represent and explain mathematical relationships.
- Identify essential quantitative relationships in a situation and determine the class or classes of functions that might model the relationships.
- Draw reasonable conclusions about a situation being modeled.
- Approximate and interpret rates of change from graphical and numerical data.

Process Standards

Problem Solving

- Build new mathematical knowledge through problem solving;
- Solve problems that arise in mathematics and in other contexts;
- Apply and adapt a variety of appropriate strategies to solve problems;
- Monitor and reflect on the process of mathematical problem solving.

Communication

- Organize and consolidate their mathematical thinking through communication;
- Communicate their mathematical thinking coherently and clearly to peers, teachers, and others;
- Analyze and evaluate the mathematical thinking and strategies of others;
- Use the language of mathematics to express mathematical ideas precisely.

Connection

- Recognize and use connections among mathematical ideas;

- Understand how mathematical ideas interconnect and build on one another to produce a coherent whole;
- Recognize and apply mathematics in contexts outside of mathematics.

Representation.

- create and use representations to organize, record, and communicate mathematical ideas;
- select, apply, and translate among mathematical representations to solve problems;
- use representations to model and interpret physical, social, and mathematical phenomena.

Learning Objectives:

- Students will use measurement to determine the height of water in a container when pouring an amount of water from a cup.
- Students will communicate using a table and graph to describe the increase of height when filling water into the glass container.
- Students will form a linear equation using the data from their table and graph.

Materials:

- Yard stick
- Ruler
- Markers
- Two or more different sized glass containers
- Three or more different sized plastic cups
- Recording sheet Table 1 and 2
- Recording chart
- Easel graph pad
- 2 sets of large colored dot stickers
- Jugs of water
- Graph and cup labels: Graph A,B,C,D,E,F Cup 1,2,3

Instructional Plan:

Part 1:

Students can be grouped into small groups through this activity. Each group will receive 1 yard stick, 1 box of markers, one container, one size cup, recording sheet, graph paper, graph label for their graph, and at least five of each colored dot stickers.

The students will start with the cup they received and will begin filling the glass container, one cup at a time. After each cup of water is poured into the container, the students will measure the height of the water to the nearest .1 cm on their Table 1 recording sheet.

Students will use the sticky dots to plot the corresponding points in the coordinate system. After completing the table or filling the container, whichever comes first, students will use a marker and a meter stick to connect the first dot (when $x = 0$) and the last dot to form a linear graph. (Line should come close to passing through all the other dots, as well.)

Each group will find an equation for the line they just drew, which will be linear. Students will write their equation beside the line it represents. Students will bring their graph to the front of the classroom to be posted and will be led into a discussion (see Questions/Discussion below for Part1).

<u>Part 2:</u>

Students will be given a challenge in part 2. Students will have to determine the height of the water without pouring water into the container.
Students will be given this scenario: Suppose that when you started pouring water from your cup into the container, the container already had some water in it. Furthermore, suppose that the height of this water already in the container was 10 cm.
Students will then complete Table 2, showing the height of the water after 0, 1, 2, . . . , 10 cupfuls of water have been added to the container.
For each pair of numbers in the table, students will plot a point in the coordinate plane to represent the height and the number of cupfuls. They will use the same coordinate system as before. Students will use a marker and meter stick to connect the dots to form a linear graph.
Students will find an equation for the new graph, afterwards will post their graphs in front of the classroom, and will be led into a discussion. (see Questions/Discussion below for Part2).

Questions/Discussion:

After completing Part 1 of the activity, as a class the students will observe each graph of all groups from the classroom. The students will receive a chart to record which cup and container matches the linear graphs from each group. The teacher will ask students how they decided which cups and containers matched the graphs.
Questions to consider as students look at all the tables and graphs:

1. What do all the tables and graphs have in common?

2. What is different when comparing the tables/graphs?

3. Can we determine which containers go with each of the tables/graphs?

4. Can you find an equation for the table/graph that your group created?
   (Hint: Try a linear equation of the form $y = mx$.)

5. What does the slope, *m*, tell you about how the water level in your container changed?

Part 2:
When all graphs have been posted, consider the following questions.

1. What does each new graph have in common with the original graph? What is different?
2. What does the slope, *m*, of each equation represent?
3. What does the *y*-intercept, *b*, of each equation represent?

Directions

1) Use the cup that you were given to begin filling the glass container, one cup at a time.
2) After each cup of water is poured into the container, measure the height of the water to the nearest .1 cm.
3) Record your results in the table and then use the sticky dots to plot the corresponding points in the coordinate system.
4) After completing the table or filling the container, whichever comes first, use a marker and a meter stick to connect the first dot (when $x = 0$) and the last dot to form a linear graph. (Your line should come close to passing through all the other dots, as well.)
5) Find an equation for the line you just drew. (Hint: Try a linear equation of the form $y = mx$.) Write your equation beside the line it represents.
6) Bring your graph to the front of the classroom to be posted.

Graph A Container ____ Cup ____	Graph B Container ____ Cup ____
Graph C Container ____ Cup ____	Graph D Container ____ Cup ____
Graph E Container ____ Cup ____	Graph F Container ____ Cup ____
Graph G Container ____	Graph H Container ____

Cup ____	Cup ____

TABLE 1

x = number of cupfuls	y = height of the water
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	

Discussion Questions

Questions to consider as you look at all the tables and graphs:

1) What do all the tables and graphs have in common?

2) What is different when comparing the tables/graphs?

3) Can we determine which containers go with each of the tables/graphs?

4) Can you find an equation for the table/graph that your group created?
(Hint: Try a linear equation of the form $y = mx$.)

5) What does the slope, m, tell you about how the water level in your container changed?

Part 2

See if you can complete each part below without actually pouring any water into your container.

1) Suppose that when you started pouring water from your cup into the container, the container already had some water in it. Furthermore, suppose that the height of this water already in the container was 10 cm. Complete another table, showing the height of the water after 0, 1, 2, . . . , 10 cupfuls of water have been added to the container.
2) For each pair of numbers in the table, plot a point in the coordinate plane to represent the height and the number of cupfuls. Use the same coordinate system as before. Use a marker and meter stick to connect the dots to form a linear graph.
3) Can you find an equation for the new graph that your group created?

(Hint: Try a linear equation of the form $y = mx + b$.) Write this equation beside the line it represents.

4) Once again, when you are finished, bring your graph to the front of the room to be posted. When all graphs have been posted, consider the following questions.

   What does each new graph have in common with the original graph?

   What is different?

   What does the slope, m, of each equation represent?

   What does the y-intercept, b, of each equation represent?

TABLE 2

x = number of cupfuls	y = height of the water
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	