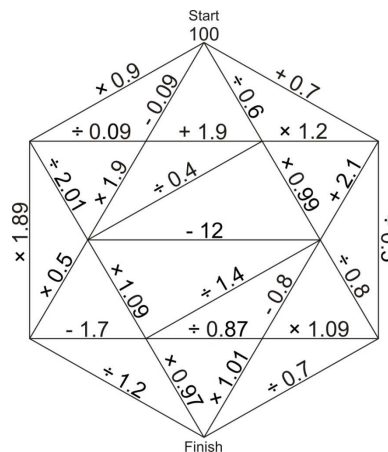


Decimal Maze

- Begin with a value of 100.
- Move down or sideways from **Start** to **Finish**.
- As you cross a segment, perform the indicated operation.
- You may not go up. You may not cross a segment more than once.
- *What is the largest possible value when you reach **Finish**?*



Welcome

12	15	16	8
8	11	12	5
6	8	10	8
11	14	15	8

What'd You Get?

41

Session 41

Whoa!

How does that work?



Developing Reasoning and Sense-Making with NCTM's Free Online Resources

G. Patrick Vennebush, Online Projects Manager
National Council of Teachers of Mathematics



Today's Agenda

- Investigate Problems that Promote RSM
 - Online Classroom Activities
 - Lessons
 - Mobile Apps
 - Online Games
- Demonstrate Good Teaching Practices for Using Technology
- Have Some Fun!



Multiplication Array

In the 3×2 multiplication table below, the numbers 2, 3, 5, 7 and 11 are used to replace the variables a , b , c , d , and e .

\times	a	b	c
d			
e			

What is the maximum possible sum of the six products?

Multiplication Array

\times			

[Excel](#)

Multiplication Array

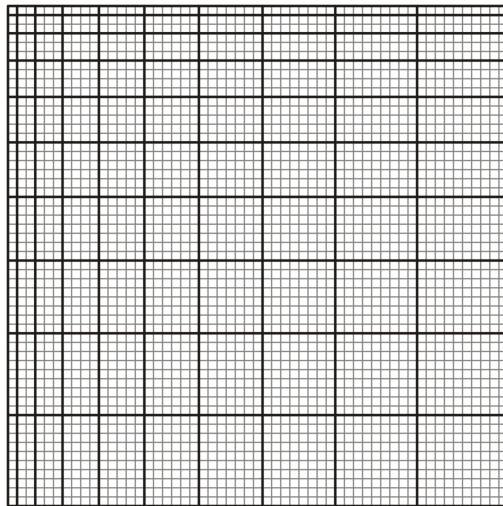
\times	a	b	c
d	ad	bd	cd
e	ae	be	ce

$$(a + b + c)(d + e) = ad + bd + cd + ae + be + ce$$

Multiplication Array

\times	2	5	7
11			
3			

Multiplication Array



Multiplication Array

A handout with the Spatial Multiplication Table that appears on the previous slide can be found at:

<http://mathjokes4mathyfolks.com/mj4mf-spatialmulttable.pdf>

Reasoning and Sense Making

- **Reasoning** involves drawing conclusions on the basis of evidence or assumptions. It often begins with explorations, conjectures, or false starts. [...] **Sense making** involves developing an understanding of a situation, context, or concept by connecting it with existing knowledge. Reasoning and sense making are closely intertwined and interdependent.

NCTM. (2010). Executive Summary: Focus in High School Mathematics: Reasoning and Sense Making. Reston, VA: author.



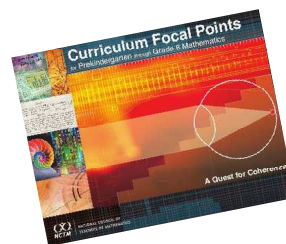
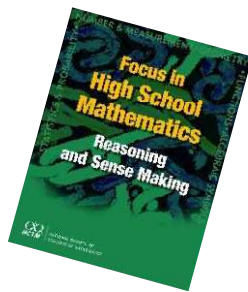
Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.



Reasoning and Sense Making

- *Focus in High School Mathematics: Reasoning and Sense Making* (2009)
- *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics* (2006)



Reasoning and Sense Making

- It is very important for teachers to lead scholars into the habit of **attending to the process going on in their own minds** while solving questions, and of **explaining how they solve them**. [...] It is next to impossible for a person to direct another's thoughts unless he knows the channel in which they are already flowing.

– Warren Colburn, *Teaching Arithmetic in the Method of Pestalozzi*, 1830

Reasoning and Sense Making

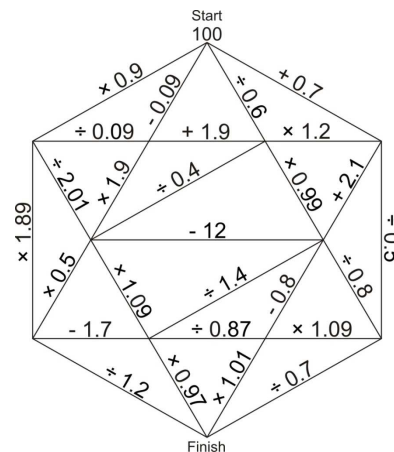


Reasoning and Sense Making



Decimal Maze

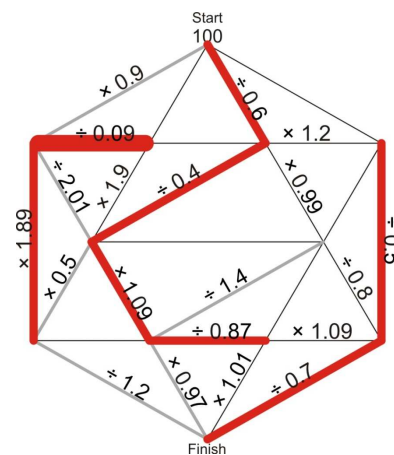
- Begin with a value of 100.
- Move down or sideways from **Start** to **Finish**.
- As you cross a segment, perform the indicated operation.
- You may not go up. You may not cross a segment more than once.
- *What is the largest possible value when you reach **Finish**?*



Decimal Maze

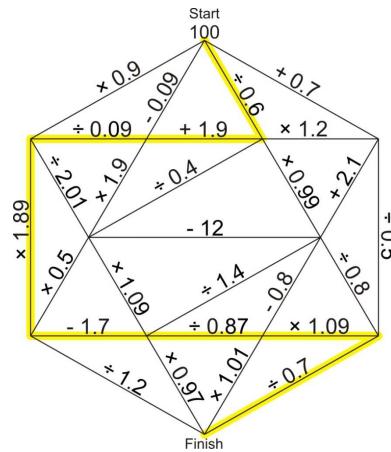
Here's some help:

- The **red lines** are beneficial.
- The **gray lines** are detrimental.
- Addition and subtraction are inconsequential.



Decimal Maze

- Maximum value: **6332**
- Minimum value?
- Finish value closest to 100?
- How many paths from Start to Finish?
- How else might you modify this activity?



Decimal Maze

Too Big or Too Small?

In this lesson, students develop number sense through a series of three hands-on activities. Students explore the following concepts: the magnitude of a million, fractions between 0 and 1, and the effect of decimal operations.

Learning Objectives

Materials

Instructional Plan

Included here is a selection of problems and activities, appropriate for the middle grades classroom, for which the use of decimal operations is essential. These activities can be used in varied ways to generate discussion and to extend the number-related concepts. The discussion that arises as students describe their thinking will certainly give insight into students' understanding of the magnitude of a million.

Activity 1: Exploring the Size of a Million Dollars

This activity explores whether one million dollars will fit in a standard suitcase. If so, how large would the suitcase have to be? You may have students work in small groups (2 or 3 members per group) to explore these questions.

Begin the investigation by asking the following questions:

Just as you decide to go to bed one night, the phone rings and a friend offers you a chance to be a millionaire. You are offered \$1 million. The money will be sent to you in the next few days, each containing \$5 million or more. Will you accept the million? If not, how much more money would you need to accept to just "play it safe"? Could your friend give you the million? Can he make you a millionaire?

Involve students in formulating and exploring questions to investigate the truth of this claim. For example:

- Can \$1,000,000 in one-dollar bills fit in a standard-sized suitcase? If not, what is the smallest denomination of bill that would fit in a standard-sized suitcase?
- Could you fit the number of \$1 bills in a standard-sized suitcase in a standard-sized suitcase?

Provide follow-up activities including finding the path that leads to the smallest finish number or finding a path that leads to a finish number as near the start number (100) as possible.

NCTM Standards and Expectations

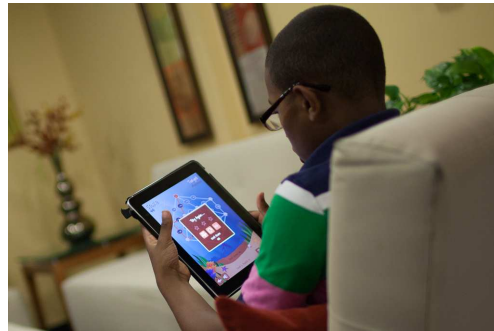
References

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Play Anywhere. Learn Everywhere.



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Illuminations

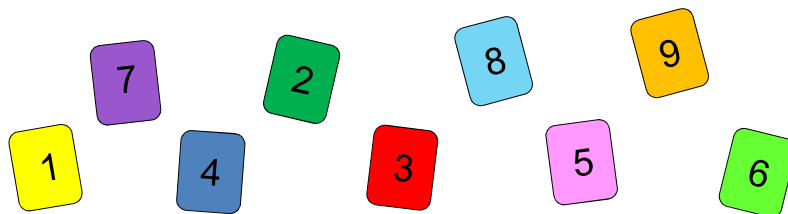
Illuminations has two primary goals:

- Provide Standards-based resources that improve the teaching and learning of mathematics for all students.
- Provide materials that **illuminate** the vision for school mathematics set forth in the *Principles and Standards for School Mathematics* and *Curriculum Focal Points*.



Game of Nine Cards

- **Materials:** Nine cards numbered 1–9
- **Object:** To have any three cards in your hand that add up to 15



Game of Nine Cards

- Sample Game:



Player 1



Player 2

Player 1 Wins: $2 + 9 + 4 = 15$

Game of Nine Cards

Now what?

You Play!

The Basics...



- Who is more likely to win — the first player or the second player? Why?
- Will someone always win? Lose?
- What can you do to ensure that you don't lose?
- Is there a “best” card to choose?
- Why do we use a sum of 15?

A Winning Strategy?

- You play first, **pick 8**.
- Your opponent then **chooses 3**.
- What are the **three numbers** that you can choose to ensure a win?




Yours





His or Hers

A Winning Strategy?


- Your opponent plays first, **picks 6.**
- You **choose 5.**
- Your opponent **picks 4.**
- Which **two numbers** should you **not** pick?








Yours



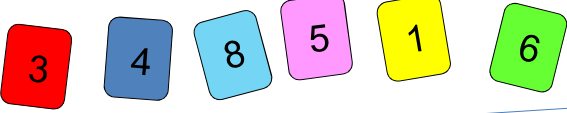
His or Hers





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A Winning Strategy?


- Your opponent plays first, **picks 7.**
- Then you **choose 2.**
- Your opponent **picks 9.**
- Which **three numbers** should you **not** pick?








Yours



His or Hers



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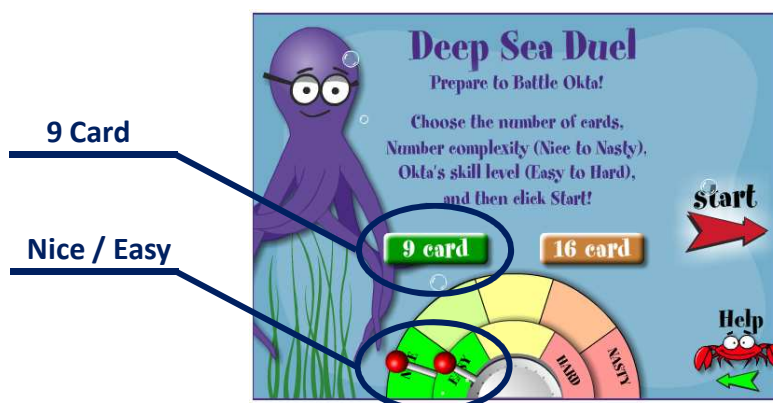
More Sophisticated Yet?

- If your opponent plays first and picks an **even** number, what number should you choose to **avoid a loss**?



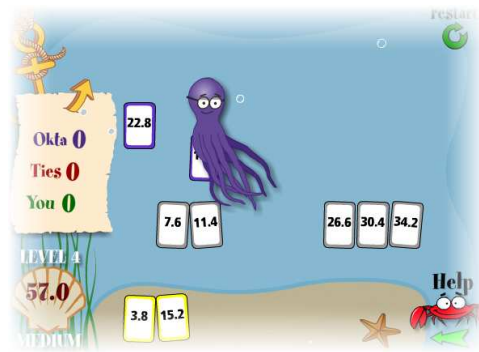
Another App from Under the Sea

Deep Sea Duel

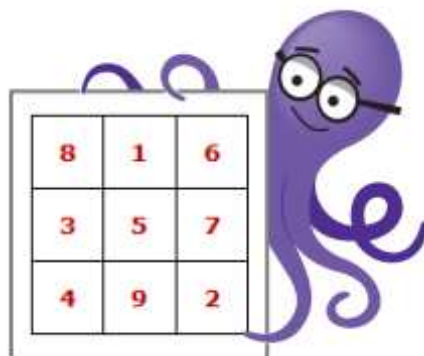


Game of Nine Cards

- **Deep Sea Duel** is online!
– <http://illuminations.nctm.org/deepseaduel>



A Hint from Under the Sea



Game of Nine Cards

- Why is this game mathematically significant?
Consider the following chart:

8	1	6
3	5	7
4	9	2

Modifying the Game of Nine Cards

- Label the nine cards as follows:
5, 12, 19, 26, 33, 40, 47, 54, 61

The winner must get three cards that total 99.

Modifying the Game of Nine Cards

- Label the nine cards with fractions:

$\frac{1}{6}$, $\frac{5}{24}$, $\frac{1}{4}$, $\frac{7}{24}$, $\frac{1}{3}$, $\frac{3}{8}$,
 $\frac{5}{12}$, $\frac{11}{12}$, $\frac{1}{2}$

The winner must get three cards that total 1.



Game of Nine Cards

- Use exponents! Label the cards as follows:

x , x^2 , x^3 , ..., x^9

This time we want the *product* of three cards (which means we must *add* the exponents).

The winner must get x^{15} .

- Have your students come up with the Magic Square for each of these modifications!

Modification: Words

- Use words!

**TIED, HOT, HEAR,
TANK, WASP, WOES,
SHIP, HORN, BRIM**

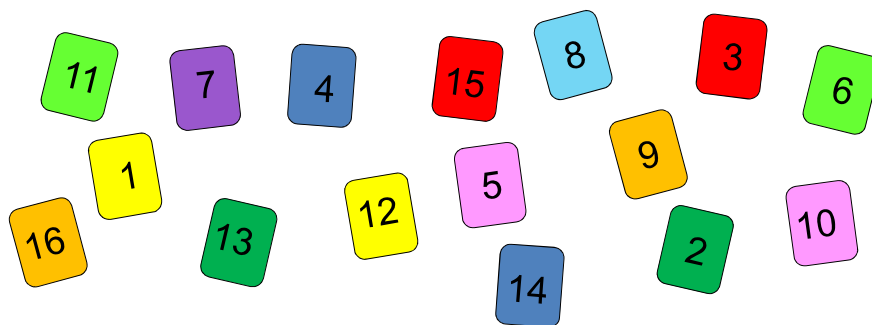


- Winner needs three cards with same letter.



Modification: Sixteen Cards

The winner needs a sum of *four* cards to win.



Modification: Sixteen Cards

- What sum would the winner need?



Modification: Two or More

- The winner is the first player to obtain the sum of exactly 15 from any **TWO OR MORE** cards.
- How does your strategy change?



Can You Use This Game?

- How could you modify the game of nine cards to fit the needs of your students?



Reminder: What is the Goal?

- How does your strategy from the first version of the game compare to the strategy for these modifications?
- *Reflect:* How did you come up with these other versions for the game of nine cards?



Game of Nine Cards

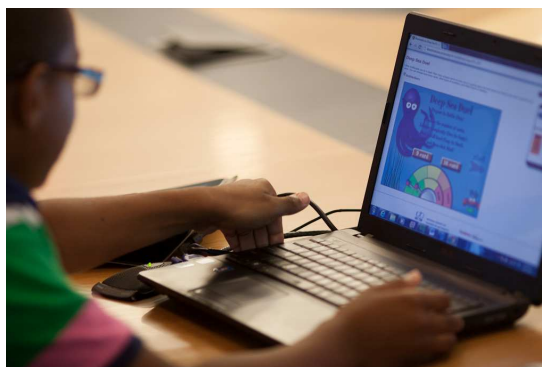
nctm.org/gameofnine



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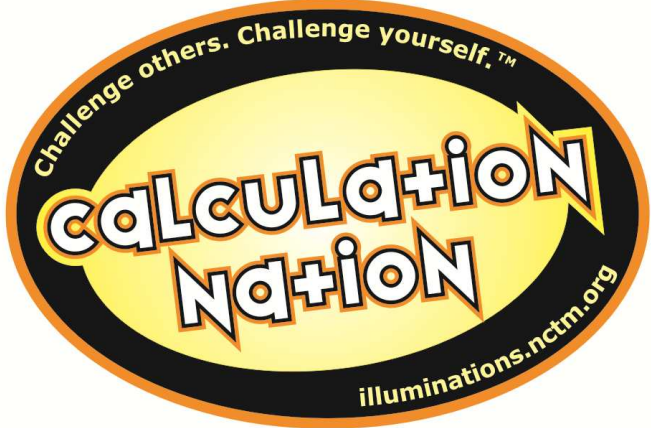
iStuff. Android. Computer.

illuminations.nctm.org/deepseaduel




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Resources for Teaching Math

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Challenge yourself

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About Calculation Nation

For Parents & Educators

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Password:

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JOIN

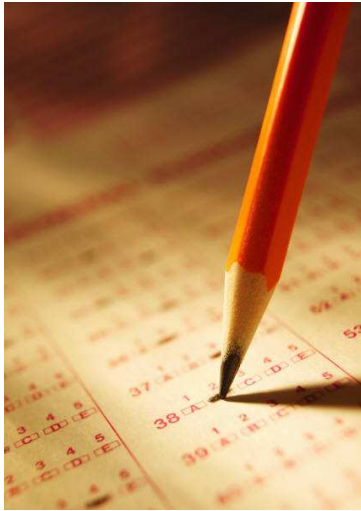
GUEST PASS

1 Calculation Nation™ citizens currently online playing games

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Calculation Nation®



“Psychological Moratorium” Principle

- Learners can take risks in a space where real-world consequences are lowered

— James Gee

Square Off

Calculation Nation® - Challenge others. Challenge yourself.® - Mozilla Firefox
calculationnation.nctm.org/Games/Game.aspx?GameId=A907366A-99F1-419A-B8CF-0CCB393D3A71

calculation nation Challenge others. Challenge yourself.®

← Back to Play Games Game Directions

It's your turn.

Draw a force field with one of the four possible perimeters. Then click on the perimeter of your rectangle. The force field must capture at least one ship or you'll lose your turn. Points are earned based on the area of your rectangle and the number of spaceships you capture.

You can click Get New Numbers

Guest: 660 1750 Calculation Nation.

CLICK THE PERIMETER OF YOUR RECTANGLE TO COMPLETE YOUR TURN

6 20 74 26

GET NEW NUMBERS

USE NUMBERS

24

STATISTICS

WIDTH: 6 HEIGHT: 6 AREA: 36

CURRENT EFFICIENCY: 100%

OVERALL EFFICIENCY: 100%

Square Off

Square Off NAME: _____

In the grid to the right, dots represent spaceships and your goal is to capture them in rectangles to earn points. You cannot overlap rectangles on the grid.

Area Points:
Rectangle Area = 10

Spaceship Points:

# Captured	Points
2	50
3	150
4	300
5	500
6	1000

Total Points = Area Points + Ship Points

For this turn, your rectangle must have a perimeter of 28 units.

1. What are all the possible dimensions of rectangles you could draw? Find area and points too.

Dimensions	Area	Points

2. What move would capture the largest area? What is the area? Draw the rectangle above.

3. What move would capture the most spaceships? How many ships? Draw the rectangle above.

4. Of all possible moves, what is your best move? Draw the rectangle on the grid above and show your calculation for the points earned. Explain why this is the best move.

Free Online at: calculationmation.actm.org

www.actm.org



- Refer to your handout.
- It is your turn. Create a rectangle with a perimeter of 28 units.
- **What is your best move?**

Considerations for Best Move

- After noting the scheme used for scoring and the locations of the spaceships remaining:

1. What are all the possible dimensions of rectangles you could draw? Find area and points too.

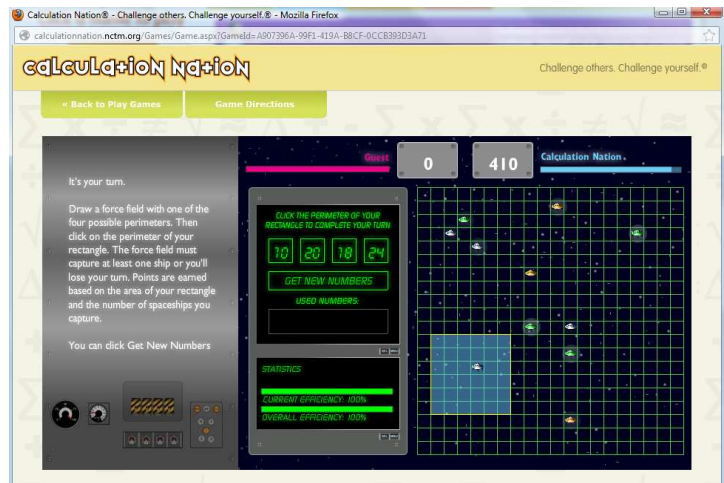
Dimensions	Area	Points

2. What move would capture the largest area? What is the area? Draw the rectangle above.

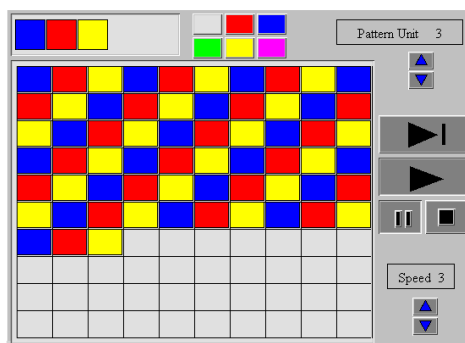
3. What move would capture the most spaceships? How many ships? Draw the rectangle above.



What's Your Efficiency?



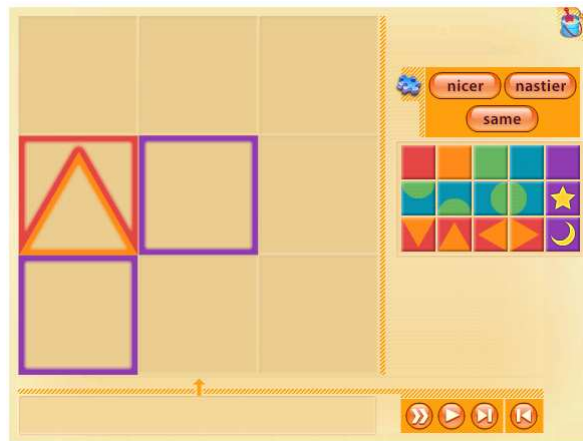
Patterns



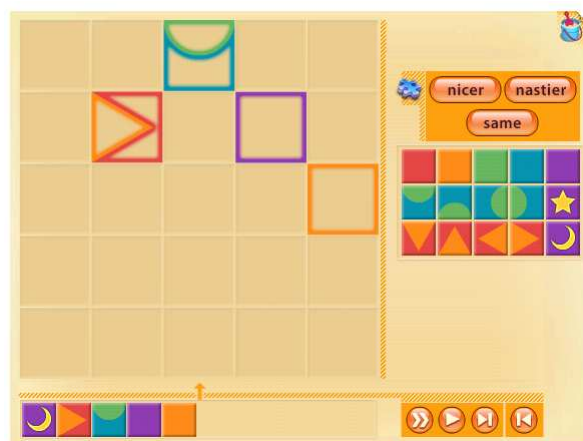
Create pattern units of two to five squares and display them on the grid.

Can you visualize how the grid will look when your pattern is repeated?

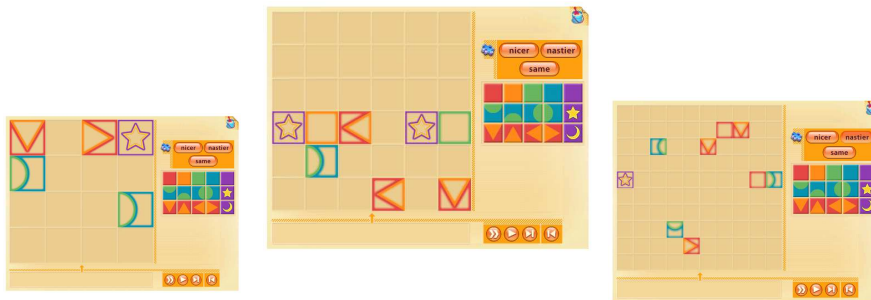
Patterns (with a Challenge!)



Patterns (with a Challenge!)



Let's Play!

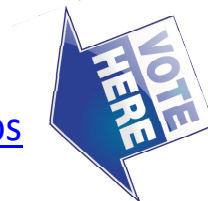


Updated E-Example: 4.1 Patterns

Patterns

- How would you use this app in your classroom?
- Do you have suggestions for change?
- How could you modify the use of this app to better fit the needs of your students?

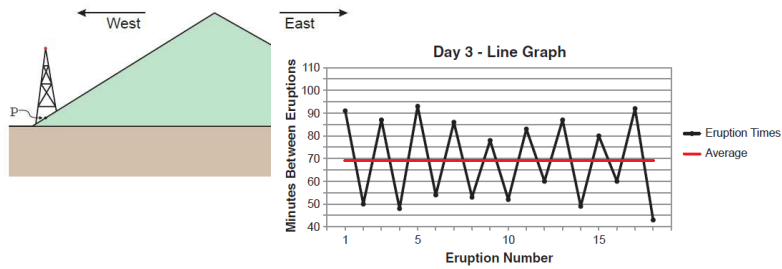
PLEASE: Offer feedback!
www.nctm.org/betaapps



RSM Tasks

- www.nctm.org/rsmtasks

A cell phone tower will be built somewhere on the west side of the hill pictured in figure 1. How far up the hill must the tower be placed to provide a signal to anyone on the east side of the lake?



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An Example



2	12x		
1-		3-	2÷
7+			
3-		6x	

www.kenken.com print this puzzle 00:00:19



Welcome – Number Trick

	4	7	8	1
8	12	15	16	9
4	8	11	12	5
2	6	9	10	3
7	11	14	15	8



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