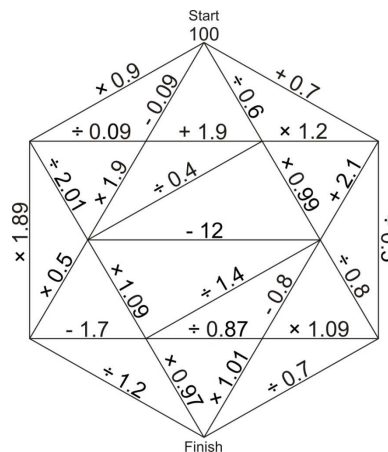## Decimal Maze

- Begin with a value of 100.
- Move down or sideways from **Start** to **Finish**.
- As you cross a segment, perform the indicated operation.
- You may not go up. You may not cross a segment more than once.
- *What is the largest possible value when you reach **Finish**?*

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Welcome

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What'd You Get?

41

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Session 41

Whoa!

How does that work?

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Developing Reasoning and Sense-Making with NCTM's Free Online Resources

G. Patrick Vennebush, Online Projects Manager
National Council of Teachers of Mathematics

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Today's Agenda

- Investigate Problems that Promote RSM
  - Online Classroom Activities
  - Lessons
  - Mobile Apps
  - Online Games
- Demonstrate Good Teaching Practices for Using Technology
- Have Some Fun!

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Multiplication Array

In the 3 × 2 multiplication table below, the numbers 2, 3, 5, 7 and 11 are used to replace the variables *a*, *b*, *c*, *d*, and *e*.

×	*a*	*b*	*c*
d			
e			

What is the maximum possible sum of the six products?

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Multiplication Array

×

Excel

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Multiplication Array

×	**a**	**b**	**c**
d	ad	bd	cd
e	ae	be	ce

$$(a + b + c)(d + e) = ad + bd + cd + ae + be + ce$$

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Multiplication Array

×	2	5	7
11			
3			

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Multiplication Array
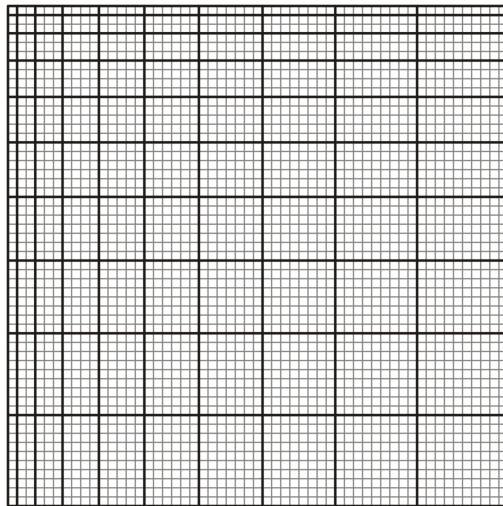
Multiplication Array

A handout with the Spatial Multiplication Table that appears on the previous slide can be found at:

http://mathjokes4mathyfolks.com/mj4mf-spatialmulttable.pdf

Reasoning and Sense Making

- **Reasoning** involves drawing conclusions on the basis of evidence or assumptions. It often begins with explorations, conjectures, or false starts. […] **Sense making** involves developing an understanding of a situation, context, or concept by connecting it with existing knowledge. Reasoning and sense making are closely intertwined and interdependent.

NCTM. (2010). Executive Summary: Focus in High School Mathematics:
Reasoning and Sense Making. Reston, VA: author.

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Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

COMMON CORE
STATE STANDARDS INITIATIVE
PREPARING AMERICA'S STUDENTS FOR COLLEGE & CAREER

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Reasoning and Sense Making

- *Focus in High School Mathematics: Reasoning and Sense Making* (2009)
- *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics* (2006)

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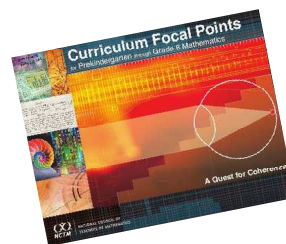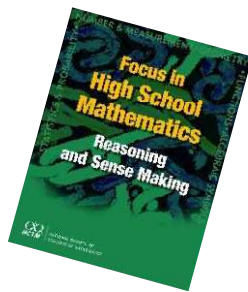
Reasoning and Sense Making

- It is very important for teachers to lead scholars into the habit of **attending to the process going on in their own minds** while solving questions, and of **explaining how they solve them**. […] It is next to impossible for a person to direct another's thoughts unless he knows the channel in which they are already flowing.
  - Warren Colburn, *Teaching Arithmetic in the Method of Pestalozzi*, **1830**

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Reasoning and Sense Making

Reasoning and Sense Making

Decimal Maze

- Begin with a value of 100.
- Move down or sideways from **Start** to **Finish**.
- As you cross a segment, perform the indicated operation.
- You may not go up. You may not cross a segment more than once.
- *What is the largest possible value when you reach **Finish**?*
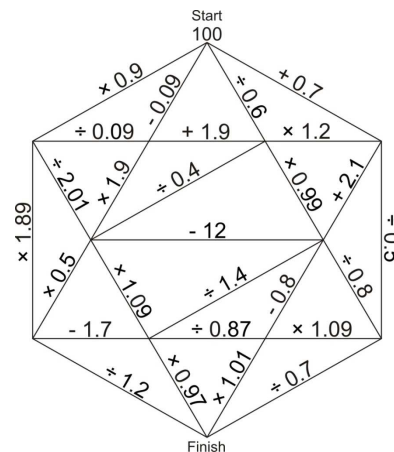
Decimal Maze

Here's some help:

- The **red lines** are beneficial.
- The gray lines are detrimental.
- Addition and subtraction are inconsequential.
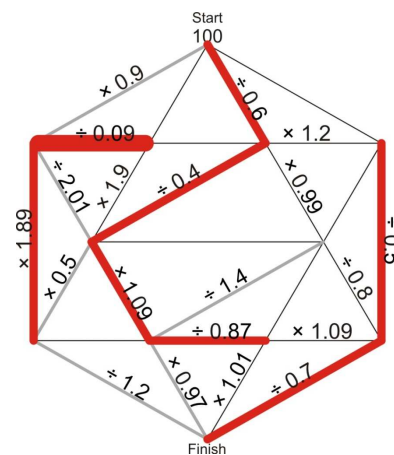
Decimal Maze

- Maximum value: **6332**

- Minimum value?

- Finish value closest to 100?

- How many paths from Start to Finish?

- How else might you modify this activity?

Decimal Maze

Pick-a-Path

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Pick-a-Path

- http://illuminations.nctm.org/pickapath

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Play Anywhere. Learn Everywhere.
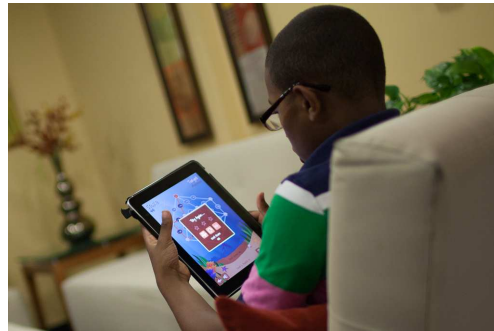

illuminations.nctm.org

Illuminations

Illuminations has two primary goals:

- Provide Standards-based resources that improve the teaching and learning of mathematics for all students.

- Provide materials that **illuminate** the vision for school mathematics set forth in the *Principles and Standards for School Mathematics* and *Curriculum Focal Points*.

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Game of Nine Cards

- **Materials:** Nine cards numbered 1–9
- **Object:** To have any three cards in your hand that add up to 15

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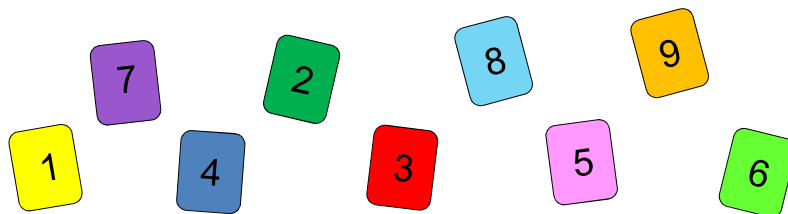
Game of Nine Cards

- Sample Game:

Player 1   Player 2

Player 1 Wins: 2 + 9 + 4 = 15

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Game of Nine Cards

Now what?

You Play!

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The Basics…

- Who is more likely to win — the first player or the second player? Why?
- Will someone always win? Lose?
- What can you do to ensure that you don't lose?
- Is there a "best" card to choose?
- Why do we use a sum of 15?

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A *Winning* Strategy?

- You play first, **pick 8**.
- Your opponent then **chooses 3**.
- What are the **three numbers** that you can choose to ensure a win?

1 7 4 2 5 9 6

8
Yours

3
His or Hers

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A *Winning* Strategy?

- Your opponent plays first, **picks 6**.
- You **choose 5**.
- Your opponent **picks 4**.
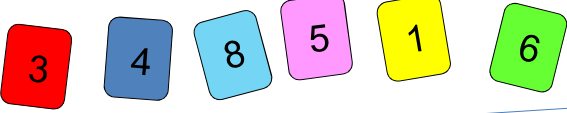- Which **two numbers** should you *not* pick?

7 1 2 8 9 3

5

4 6

Yours

His or Hers

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A *Winning* Strategy?

- Your opponent plays first, **picks 7**.
- Then you **choose 2**.
- Your opponent **picks 9**.
- Which **three numbers** should you *not* pick?

3 4 8 5 1 6

2

9 7

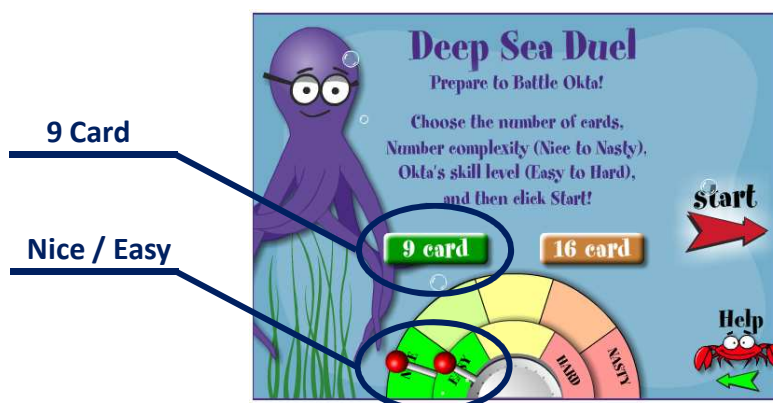Yours

His or Hers

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More Sophisticated Yet?

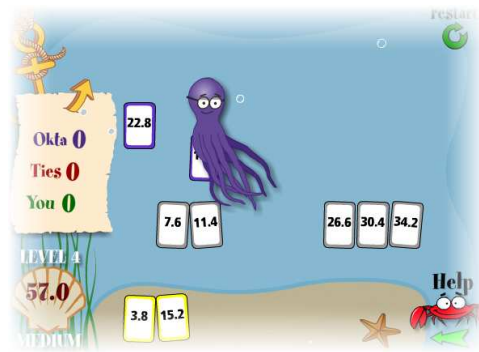- If your opponent plays first and picks an **even** number, what number should you choose to **avoid a loss**?

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Another App from Under the Sea

Deep Sea Duel

9 Card

Nice / Easy

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Game of Nine Cards

- **Deep Sea Duel** is online!
  - http://illuminations.nctm.org/deepseaduel

A Hint from Under the Sea
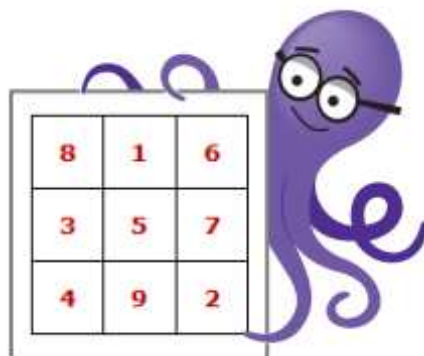
Game of Nine Cards

- Why is this game mathematically significant?
  Consider the following chart:

8	1	6
3	5	7
4	9	2

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Modifying the Game of Nine Cards

- Label the nine cards as follows:

  5, 12, 19, 26, 33, 40, 47, 54, 61

  The winner must get three cards that total 99.

Mahoney, John. What Is the Name of This Game?
Mathematics Teaching in the Middle School, October 2005.

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Modifying the Game of Nine Cards

- Label the nine cards with fractions:

1/6, 5/24, 1/4, 7/24, 1/3, 3/8,

5/12, 11/12, 1/2

The winner must get three cards that total 1.

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Game of Nine Cards

- Use exponents! Label the cards as follows:

$$x, \ x^2, \ x^3, \ ..., \ x^9$$

This time we want the *product* of three cards (which means we must *add* the exponents). The winner must get x^{15}.

- Have your students come up with the Magic Square for each of these modifications!

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Modification: Words

- Use words!

  TIED, HOT, HEAR, TANK, WASP, WOES, SHIP, HORN, BRIM

- Winner needs three cards with same letter.

Modification: Sixteen Cards

The winner needs a sum of *four* cards to win.

11   7   4   15   8   3   6

1   9

16   13   12   5   10

14   2
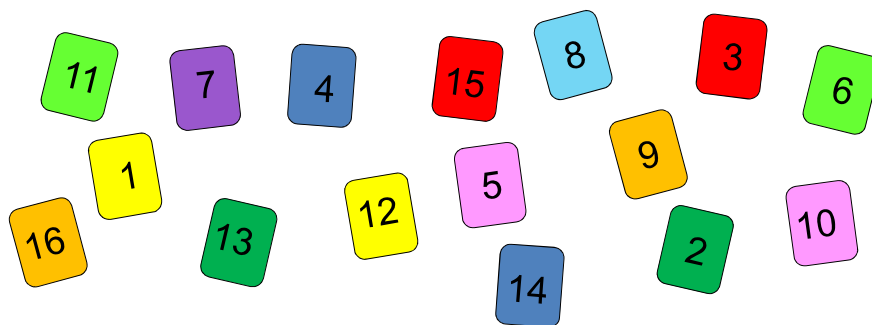
Modification: Sixteen Cards

- **What sum would the winner need?**

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Modification: Two or More

- The winner is the first player to obtain the sum of exactly 15 from any **TWO OR MORE** cards.

- How does your strategy change?

Yeo, Joseph. [Title removed in order to not give away punch line of strategy.] *Mathematics Teacher,* August 2012.

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Can You Use This Game?

- How could you modify the game of nine cards to fit the needs of your students?

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Reminder: What is the Goal?

- How does your strategy from the first version of the game compare to the strategy for these modifications?
- *Reflect:* How did you come up with these other versions for the game of nine cards?

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Game of Nine Cards

nctm.org/gameofnine

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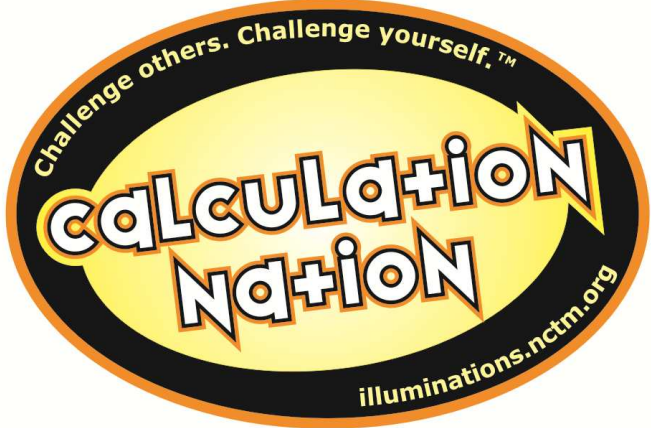
iStuff. Android. Computer.

illuminations.nctm.org/deepseaduel
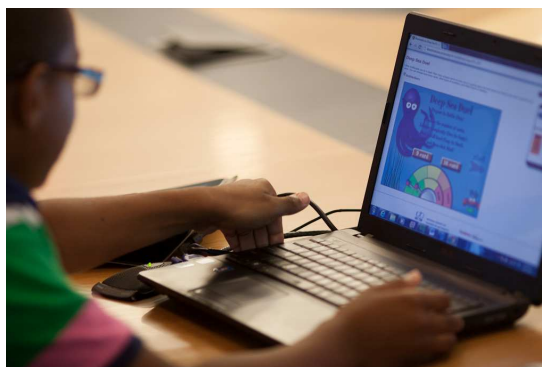
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Calculation Nation™

http://calculationnation.nctm.org

Calculation Nation®

"Psychological Moratorium" Principle

- Learners can take risks in a space where real-world consequences are lowered

— James Gee

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Square Off

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Square Off

- Refer to your handout.
- It is your turn. Create a rectangle with a perimeter of 28 units.
- **What is your best move?**

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Considerations for Best Move

- After noting the scheme used for scoring and the locations of the spaceships remaining:

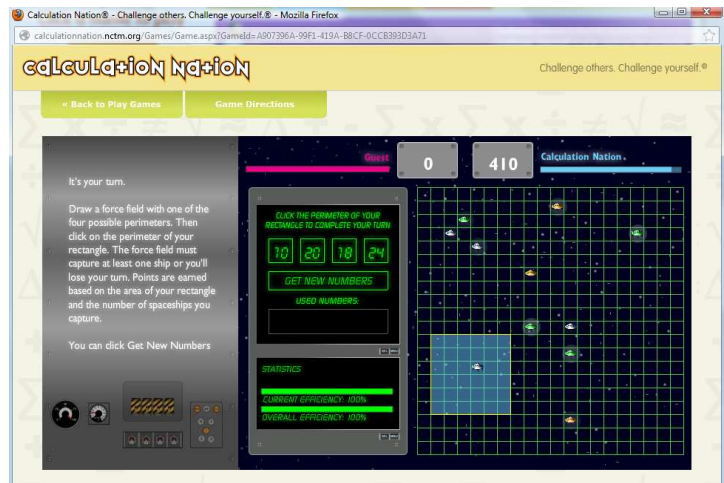1. What are all the possible dimensions of rectangles you could draw? Find area and points too.

Dimensions						
Area						
Points						

2. What move would capture the largest area? What is the area? Draw the rectangle above.

3. What move would capture the most spaceships? How many ships? Draw the rectangle above.

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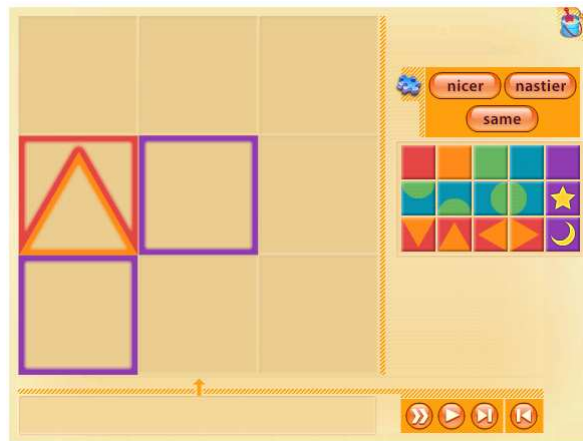
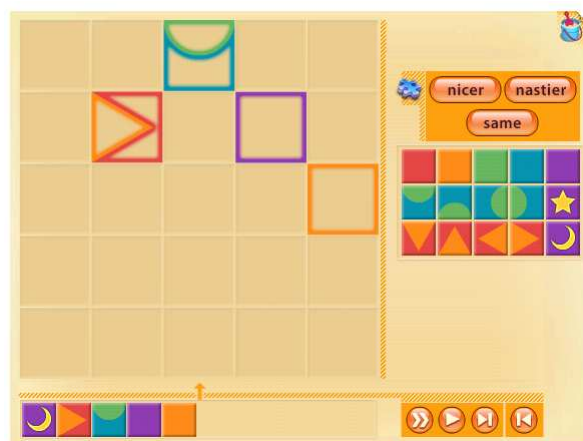## What's Your Efficiency?

Patterns
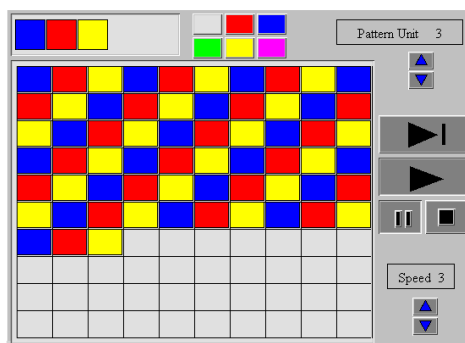
Create pattern units
of two to five squares
and display them on
the grid.

Can you visualize how
the grid will look
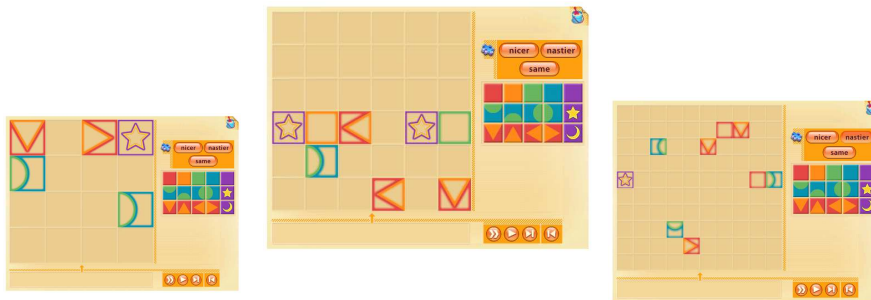when your pattern is
repeated?

Patterns (with a Challenge!)


Patterns (with a Challenge!)

Let's Play!

Updated E-Example: **4.1 Patterns**

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Patterns

- How would you use this app in your classroom?
- Do you have suggestions for change?
- How could you modify the use of this app to better fit the needs of your students?

PLEASE: Offer feedback!
  www.nctm.org/betaapps

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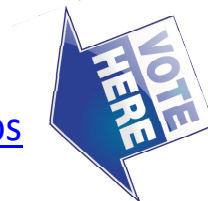
RSM Tasks

- www.nctm.org/rsmtasks

A cell phone tower will be built somewhere on the west side of the hill pictured in figure 1. How far up the hill must the tower be placed to provide a signal to anyone on the east side of the lake?
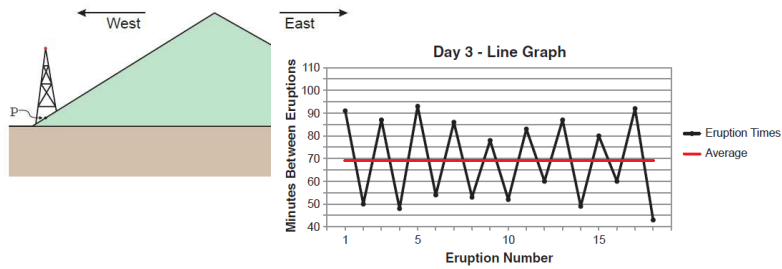
KenKen®

illuminations.nctm.org/kenken

An Example

Welcome – Number Trick

	4	**7**	**8**	**1**
8	**12**	**15**	**16**	**9**
4	**8**	**11**	**12**	**5**
2	**6**	**9**	**10**	**3**
7	**11**	**14**	**15**	**8**

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