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50 Ways to Teach Them Algebra

1. **Stuck on an Escalator:** This short, humorous Youtube video emphasizes the need for students to take control of their own learning, to walk off their own escalator when it gets stuck.

2. Use **dry erase** boards to encourage participation, review concepts and vocabulary, catch common errors, and assess understanding.

3. **First-day hook:** ask students to write on their dry erase boards what they consider the greatest time-saving device ever invented. Then use a magic trick to prove that Algebra is truly the greatest time-saving invention! Here it is:

  a) Write your age b) Double it.   c) Add 15.   d) Triple that.
  e) Add 33.   f) Divide by 6.   g) Subtract your age.

To prove that the magic trick always produces the number 13, we could perform the calculations with every real number (which would literally take forever!), or we could prove it for every real number in one single calculation using Algebra. Using x to represent any age, the steps produce the number 13 as follows:

  a) x b) $2x$ c) $2x + 15$ d) $3(2x + 15) = 6x + 45$
  e) $6x + 78$ f) $x + 13$ g) $x + 13 - x =$ **13**

This is from the article "The Use of Intrigue to Enhance Mathematical Thinking and Motivation in Beginning Algebra," by Lewkowicz, *The Mathematics Teacher*, February, 2003.

4. **FOUR SQUARE**: Students fold a small sheet of paper into four sections. In the center, they write two algebraic expressions such as x^2 and x. In the top left section, they find the sum of the two expressions. In the top right, they find the difference. In the bottom left, they find the product. In the bottom right, they find the quotient. This highlights the fact that there are four basic operations that must be performed on all types of expressions. I use the FOUR SQUARE format to quickly practice and review operations on fractions, integers, decimals, monomials, polynomials, rational expressions, functions, and complex numbers.

5. **Happy Birthday** verse n, LIKE TERMS (see song sheet!)

6. **Calendar Magic** from "Reasoning with Algebra," by Friedlander and Hershkowitz, *The Mathematics Teacher*, September, 1997.

    a) Choose any 2x2 block of dates.   Justify using algebra:
    b) Add along one diagonal.   The dates are   x   x+1

      c) Add along the other diagonal.   x + 7   x + 8
        The sums are equal.   Adding each diagonal gives 2x + 8.

7. **Open-ended questions on dry erase boards:** questions like these give me a quick assessment of students' level of understanding. Students who offer very simple answers are tentative in their understanding. Confident students will give complex, but correct, responses. Incorrect responses can be quickly noted and addressed.

a) Write an equation whose solution is 5. (most students begin with x+ 2 = 5, which shows a fundamental misconception of the nature of solving equations)

b) Write an expression that simplifies to x^2. (if a student writes " $x^{100} \div x^{98}$", things are going well. If a student writes x • x, you know it's tentative. If a student writes x + x, address the issue!)

c) Write a fraction that's equivalent to 1/3 (How many possibilities are there???)

d) Write a number between .6 and .7 (How many possibilities are there???)

e) Write two numbers whose sum is 1 (complements)

f) Write two numbers whose sum is 0 (opposites)

g) Write two numbers whose product is 0 (zero product rule)

h) Write two numbers whose product is 1 (reciprocals)

i) Write a second-degree trinomial (vocabulary practice)

8. **Shoes and socks:** To solve 2-step equations like 2x + 7 = 11. When you get dressed, you put on socks, then shoes. To undress, you take off the shoes, then the socks. In order of operations, multiplication comes before addition. To isolate a variable and solve an equation, undo addition before multiplication.

9. **Equation BINGO**: Students draw a 4x4 grid and fill the 16 spaces with the integers from -7 to 7 and one free space. You display one equation at a time to solve. Students cross out the solution to each equation on their BINGO grid, and yell "BINGO" when they have 4 in a row crossed out. Keep playing until most students have had at least one BINGO. Then…

10. In equation BINGO, show an **identity**: As your last equation, use an identity like 3(x + 2) = 3x + 6. Since all real numbers are solutions, students may choose that one number they lacked, and everyone gets BINGO and wins the game!

11. **Add some color:** Fill the spaces of a coloring book page with exercises; provide a key describing what color to fill in each space according to its solution.

Change of pace, fun to discuss and grade as a group. Students tend to work carefully, since colors are difficult to erase or change! (see enclosed sample)

12. **Colorful transformations:** Use colored pencils to graph various transformations with their parent function on the same coordinate grid.

13. **Equivalent expressions:** Use color to show equivalent expressions, as on the enclosed sheet of radical expressions and rational exponents.

14. **Set the mood**: "Aquarius" music and tie-dye outfit to introduce radicals

15. **Sing Happy Birthday** LIKE RADICALS (see song sheet)

16. **Mathematical Faux Pas**: You don't wear white after Labor Day. You don't wear sequins before 6:00. You don't wear white to a wedding unless you are the bride. You just don't. The same person who made up these rules may have been responsible for the rule, "You don't leave radicals in the denominator."

17. **Musical words of wisdom about mathematical faux pas**: You don't tug on Superman's cape, you don't spit into the wind, you don't pull the mask off the old Lone Ranger, and you don't leave radicals in the denominator. (or "you don't divide by 0).

18. **Inequalities**: Emphasize the vocabulary word "inequalities" by asking students for words beginning with the prefix "in". They will realize that "in" means "not"; an inequality is "not" equal (except for "insurance"!).

19. **Parentheses or brackets**? Parentheses mean "Don't touch me!" They are the same shape your body would make if I tried to poke you in the stomach. Brackets are tightened abs and a straight body that is ready to be poked.

20. Play the **Twilight Zone** music as students discover the exception requiring that in inequality symbol be reversed with we multiply or divide both sides by a negative number.

21. Graphing the first linear equation: Offer students yellow butterscotch and "x-tra spicy" mint candies to introduce the Cartesian Coordinate system. Tell them they may take exactly 5 pieces of candy in any combination. Then make a table of the number of **X**-tra spicy candies and the **Y**ellow candies each took. Show how the combinations of x and y values create ordered pairs that satisfy the equation x + y = 5. Graph the ordered pairs and notice the line formed. Note that any combination of candies that does not follow the "rule" x + y = 5 is not on the line.

22. History of Mathematics: Use colorful stories, legends, and historical characters to enliven the class. Rene Descartes, French philosopher ("I think; therefore, I am") and mathematician, supposedly had the idea for the Cartesian

coordinate system when he tried to describe the location of a fly on a tiled ceiling so his servant could swat it. Enquiring minds might be fascinated to learn of Descartes' death as a result of Queen Christina of Sweden sending a battleship to force the reluctant Descartes to come to Sweden to tutor her. The Queen was a morning person; Descartes was not. He lasted only a few weeks of tutoring in the early morning cold, and died of pneumonia.

23. **Graphing horizontal and vertical lines**: Where does a 3000-pound rhinoceros sleep? ANYWHERE IT WANTS TO! In the equation x = 6, the value of x in every ordered pair has to be 6. But y is the 3000-pound rhino—it can be anything it wants to be! Similarly, in the equation y = 3, x is the 3000-pound rhino. So when students look at a **linear equation with one variable** in confusion, I ask who the 3000-pound rhinoceros is in the equation.

24. **Tinkertoy slope**: Create a Tinkertoy ladder, with rungs about 3 inches apart, to introduce **slope**. Run a Matchbox car up an 18-inch board "road," showing the rise and run components of slope as the car climbs steeper and steeper hills. A road that is all run and no rise (horizontal) has a slope of zero. A road that is all rise and no run (vertical) has an undefined slope.

25. **Mnemonic for the slope of a horizontal line**: hOrizOntal.

26. **Mnemonic for the slope of a vertical line**: at Six Flags, I never let my own children ride anything with an UNDEFINED slope!

27. **10-Second graphing:** Put a paper with a coordinate grid in a page protector. Display a linear equation, have students graph it with dry erase markers.

28. **Slope of perpendicular lines**: Have students graph a line, say, y = ¾ x – 2. Then have them trace that line and the axes onto a piece of tracing ("patty") paper. Turn the patty paper 90 degrees so that the x-axis becomes the y, and the y-axis coincides with the x. The patty paper line and the original line are perpendicular. The slope of the rotated line is now -4/3.

29. **Matching game**: Index cards are now sold with graph grids on one side! Create decks of cards so that students must match each equation with its graph.

30. **Sorting matched cards:** When a group has finished matching equations with graphs, have them sort the cards according to categories: Negative vs. positive slope, passes through the origin vs. does not contain the origin, linear vs. non-linear, etc. Have them verbalize what is significant about the graphs and about the equations.

31. **Other card ideas**: You can make matching card sets of equivalent fractions (students would match 50/100 with 3/6), equivalent equations (students would

match 3x + 2 = 5 with x + 7 = 8), equivalent logarithmic expressions (students would match log 10 with ln e)—the possibilities are endless! These cards take time to make, but can be used again and again. Sets of cards can be combined for review. Rather than straight matching, students could play the game "Memory" or "Concentration", in which cards are laid out face down and two are turned over at a time. If the two match, the student picks them up and takes another turn. If they do not, the student replaces the cards face down and the next student takes a turn. You can also lay out all the cards and have the group pick matches in a free-for-all competitive style, like the game of "Set".

Finally, you can use a single set of matching cards to determine who should work together on an activity—each student must work with the person whose card matches their own.

32. **Graphing calculator activity:** This can be done with the teacher holding a single graphing calculator displayed on a view screen for all students to see, or with each student or pair of students holding their own calculators. Begin with a simple equation like y = ½ x + 3. Ask students to …

Tell me one characteristic of this graph.

Give me the equation of a graph that
        is a steeper hill to climb.
        is a less steep hill to climb.
        hits the y-axis somewhere else.
        is perpendicular to the one we have now.
        is parallel to the one we have now.
        contains the origin.
        goes up hill.
        is horizontal.
        is not a line.

33. **Point-Slope Form song**: I have found that students tend to forget that this form of a linear equation even exists. They all remember y = mx + b, but this poor step-sibling gets forgotten. See song sheet for a song to "Ghostbusters" that might help!

34. Use the analogy of the **Jeopardy** game show, where the answers are given and contestants must supply the questions, to introduce factoring. We are given the product, and factoring is the process of finding the factors that made that product. The Jeopardy Theme Song is available on line.

35. Matching cards for factoring: students match each polynomial to its factored form. I always include an "orphan" prime polynomial which has no match in factored form.

36. **Factoring the difference of two squares:** Arrange 25 Starburst candies in a 5x5 array. Slide a 2x2 array off to the side. Students should arrange the remaining Starbursts into a rectangle. The only one they can make is 3 x 7. Thus, $5^2 - 2^2 = (5 - 2)(5 + 2)$.

37. **The Quadratic Formula song**(s): You've probably sung it one way, but did you know that the Quadratic formula can be sung to the tune of "Row, Row, Row your Boat", "Pop goes the Weasel", "Jingle Bells", and my favorite, the "Macarena"?

38. **Youtube videos for the Quadratic Formula**: There is a great Lego video of the quadratic formula song (it contains gratuitous violence, but on Lego characters, it's not too disturbing!). Also, there are many adorable Youtube videos of toddlers singing the Quadratic formula song.

39. **Introducing Functions:** the song "Matchmaker, Matchmaker" from Fiddler on the Roof introduces a function as being the matchmaker, matching every value of x from the domain with one and only one special member of the range!

40 and 41: **Function example and counterexample**: I show pairs of photographs of people and their dogs who resemble them. The people are members of the domain, while the dogs are members of the range. Each person is matched with one dog, so we have a function. Then I show a photo of myself with my two dogs. Do we still have a function? No—I would be matched with one dog, and then with the other.

Then we switch the domain and range, foreshadowing the idea of inverses. The domain is now the set of dogs. The range is the set of people. Each dog is matched with its owner. Do we have a function? Students generally say that we don't, but we actually do. Each of my dogs gets matched with me, but this does not violate the definition of function!

42. **Function or Not a Function?** Instead of "Deal or No Deal," I show a relation on the overhead. Students work in pairs. Each pair has one dry erase board that says "FUNCTION" and another that says "NOT A FUNCTION". Each pair discusses, then chooses which board to hold up.

43. The **Conic Section song**… See song sheet!

44. **Patty Paper Parabola**: Each student has a piece of patty paper. On the paper is drawn one dot, and near the bottom of the sheet is a line segment with numerous dots on it. One by one, each dot on the line is matched to the single dot not on the line, and a crease made. The creases will create a parabola. The line is the directrix, the single point off the line is the focus. All the conic sections can be made with patty paper, and instructions can be found on line.

45. **Celebrity Cubic Function**—the cubic function is often called the "John Travolta" function, after the disco moves made by Travolta in Saturday Night Fever (check out the Youtube video!).

46. **The Leading Coefficient Test**: This activity gets students out of their seats. Show a polynomial function and have students point their right hand to where the function is heading as x approaches infinity. Have them point their left hand to where the function is heading as x approaches negative infinity. E-mail me at dchar@stlcc.edu if you'd like the Powerpoint I made!

47. **Paper-folding exponential:** It has been said that humanity's most grievous miscalculation is failing to understand the nature of the exponential function. We must consider what will happen to a finite planet in the face of exponential growth of population and pollution. To make this point, have students fold a piece of paper in half. This doubles its thickness. Have them fold it again, which quadruples the thickness. Again, and it is 8 times as thick. Again, and it is 16 times as thick. Ask what would happen if we folded the paper 100 times. How thick would it be? The answer is shocking: the thickness would reach the other limits of the known universe! Google: paper folding exponential for the details!

48. **Word Problems**: **A matter of Translation**! To emphasize the importance of translating word problems correctly, I read some poor translations of instructions on packages of food from other countries (check out a global grocery store for examples!). In college I set my dorm on fire trying to cook shrimp chips when the directions told me to "boil the oil".

49. Play "**Catch Phrase"** (or **Taboo**)! This is a fantastic way to review. Put the necessary vocabulary on cards. Put students in groups. When it is his or her turn, each student must describe the word on his/her card and get the group to say the word.

50. **Imaginary Numbers** (Youtube, Al G Bra). This song says it all:
You may say I'm a math nerd,
But I'm not the only one.
I hope some day you'll join us
And find out that math is fun!

The Happy Birthday Song

You must have like terms
You must have like terms
To add or subtract,
You must have like terms!

You must have common denominators
You must have common denominators
To add or subtract,
You must have common denominators!

You must have like radicals
You must have like radicals
To add or subtract,
You must have like radicals!

THE CONIC SECTION SONG
(to the tune of The Lion Sleeps Tonight)
Words by Debbie Char

For a function to be quadratic, a square is on the x,
With an axis of symmetry and a point called the vertex.

PARABOLA, PARABOLA, PARABOLA, PARABOLA…

There's just one with a minus sign, you should put that in your notes:
With two branches, two foci, and intersecting asymptotes!

HYPERBOLA, HYPERBOLA, HYPERBOLA, HYPERBOLA, …

If the x and the y are squared, equal coefficients, you
Find the diameter by taking the radius times two!

CIRCLE, CIRCLE, CIRCLE, CIRCLE,…

Stretch a circle along an axis, and you will get this shape
Round the foci our gravity keeps the planets from escape!

ELLIPSE, ELLIPSE, ELLIPSE,…

Ellipse, circle, hyperbola, and parabola have shown
Just what happens when you decide to cut up a double cone!

CONIC SECTIONS CONIC SECTIONS CONIC SECTIONS CONIC SECTIONS…

POINT-SLOPE FORM
(to the tune of the GhostBusters Theme song)
lyrics by Debbie Char

When you've got a point
And you know the slope,
Whatcha gonna use?
POINT-SLOPE FORM!!!

When you've got TWO points,
You can find the slope, then
Whatcha gonna use?
POINT-SLOPE FORM!!!

Y minus Y_1, which is the y-coordinate,
Equals m, which is the slope, then in parentheses
X minus X_1, which is the x-coordinate,
That's the Point-Slope form of the equation of a line!

Mood Music:
50 Ways to Leave Your Lover (Paul Simon)

Aquarius, from Hair (to introduce radicals)

Twilight Zone (for unexpected results: contradictions, identities, flipping the
inequality symbol if you multiply or divide by a negative on both sides)

You don't mess around with Jim (by Jim Croce) for the stuff you just DON'T do
(divide by zero, leave radicals in denominators, etc)

Jeopardy Theme (to introduce factoring)

Stayin' Alive (BeeGees) for the cubic "John Travolta" function

Matchmaker, Matchmaker from Fiddler on the Roof to introduce functions

Imaginary Numbers (Al G Bra) on Youtube