The Mathematics Scan (M-Scan):

A Measure of Standards-Based Mathematics Teaching Practices

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Updated: July 2012

The Mathematics Scan (M-Scan)Coding Guide

Introduction to the Measure

The National Council of Teachers of Mathematics (NCTM) provides a vision for high-quality mathematics instruction in its two standards documents *Mathematics Teaching Today: Improving Practice, Improving Student Learning* (NCTM, 2007) and the *Principles and Standards for School Mathematics* (NCTM, 2000). The *Principles and Standards for School Mathematics* (2000)describes standards for teaching mathematics through six principles (Equity, Curriculum, Teaching, Learning, Assessment, and Technology), five content strands (Number and Operations, Algebra, Geometry, Measurement, and Data Analysis and Probability), and five process standards (Problem Solving, Reasoning and Proof, Communication, Connections, and Representation. (See http://standards.nctm.org/).

Although the NCTM standards provide a clear vision for ideal mathematics classroom instruction, they pose challenges to researchers and educators in mathematics education. First, research findings on the low levels of achievement in mathematics among American children suggest that teachers vary widely in the extent to which they provide quality mathematics instruction aligned with the NCTM principles and standards. Second, the principles and standards provide a vision for high quality mathematics but do not provide guidance necessary to measure the principles and standards in practice. The M-Scan measure was developed to address both challenges. The M-Scan provides translation from NCTM standards to classroom practice to measure the quality of standards-based teaching practices in a large number of classrooms. Further, the M-Scan focuses on the implementation of high quality standards-based teaching. The ultimate goal is to use the M-Scan measure to translate from standards to practice in order to improve pre-service and in-service professional development efforts in mathematics.

Two existing measures served as a foundation for M-Scan measurement development. Dimensions were selected, defined, and adapted from the SCOOP measure; which used classroom artifacts such as tests, observations, and instructional materials to measure the quality of mathematics instruction (Borko et al., 2005; Borko, Stecher, & Kuffner, 2007). The SCOOP measure included an observational component to understanding the quality of instruction. Eight of the dimensions included in that measure fit well with the goals of the M-Scan and could be adapted to measure mathematics instructional quality observationally. A ninth dimension was developed based on subsequent pilot work by the measurement development team. The structure of the coding protocol and the 1 to 7 scale was based on the Classroom Assessment Scoring System (CLASS; Pianta, Hamre & LaParo, 2007). Coding guidelines were established with descriptions to correspond to numerical ratings from 1 to 7 (segmented as low [1-2], medium [3-5], and high [6-7]). Procedures were established to train and establish reliable coding following recommendations from the CLASS.

Rationale for the Nine M-Scan Dimensions

The M-Scan measures standards-based mathematics teaching practicesby assessing use of mathematical tasks, mathematical discourse, mathematical representations, and mathematical coherence (as conceptualized by NCTM, 2007; adapted from Walkowiak, 2010). To assess use of tasks, the M-Scan measures the constructs of *cognitive demand*, *problem solving*, and *connections and applications*. To measure discourse, the M-Scan assesses *explanation and justification* and *mathematical discourse community.* To measure representations, the M-Scan taps*use of representations* and *students' use of mathematical tools.* Finally, to measure coherence, the M-Scan examines *structure of the lesson* and *mathematical accuracy*. The nine dimensions were selected because of their link to these principles and standards, but also, because researchers in mathematics education have identified their importance in the classroom (Borko et al., 2005; Stecher et al., 2006; Walkowiak et al., under review).

The relation among these constructs is best described pictorially in Figure 1. The conceptual model describes the umbrella construct of standards-based mathematics teaching practicesas having four components: 1) the tasks that teachers select and the way in which these tasks are enacted in the classroom, 2) the discourse between teachers and children and among children about mathematics, 3) the representations used by teachers and students to represent and translate among mathematical ideas, and 4) the coherence of the lesson focusing on the extent to which the mathematical concepts are presented clearly and accurately and organized in a way that leads to a deeper understanding.

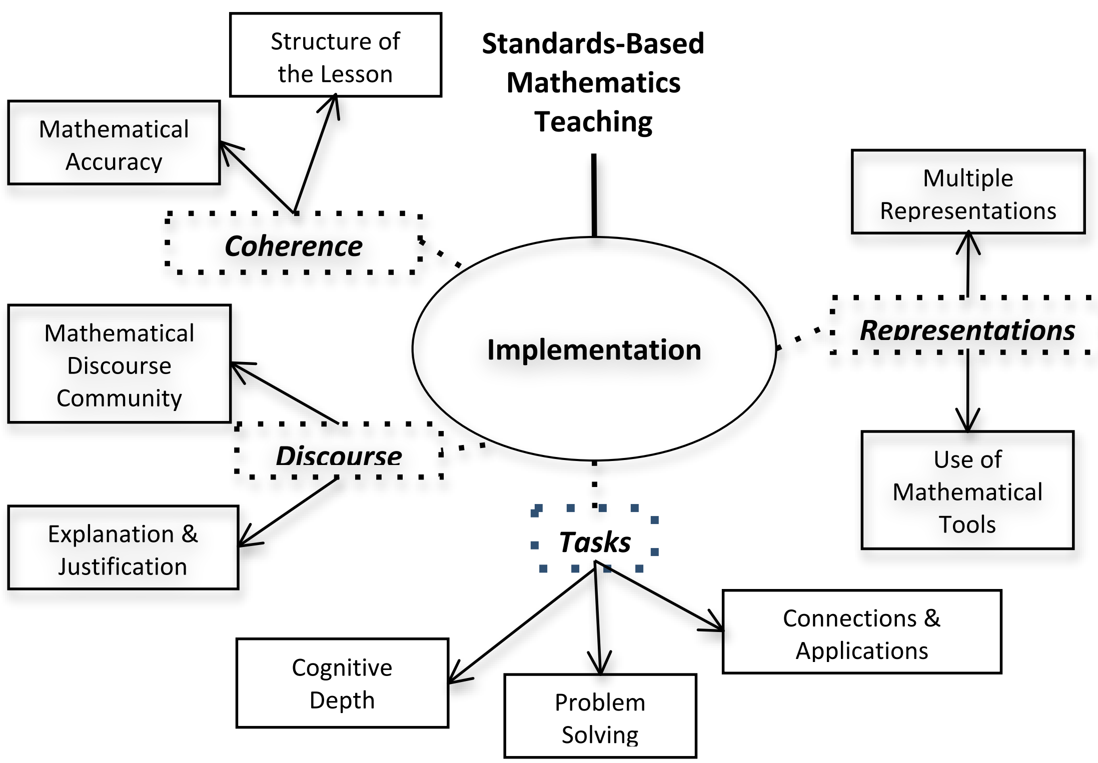

Figure 1. Conceptual model describing Standards-Based Mathematics Teaching Practices

Tasks

Teachers are responsible for engaging students with quality mathematical tasks that deepen understanding and contribute to the development of mathematical fluency. High quality mathematical tasks help students develop mathematics thinking, concepts, and skills. The dimensions of *cognitive demand, problem solving,* and *connections and applications* are intended to reflect the tasks afforded by teachers. *Cognitive Demand*refers to task selection and the enactment tasks during a lesson. It considers the extent to which task selection focuses on cognitively demanding tasks, and the extent to which the teacher consistently and effectively promotes cognitive depth (Stein et al, 1996). Research suggests that lower cognitive demanding tasks are the norms in many mathematics lessons in American classrooms (Fey, 1981; Hiebert et al., 2005; Stodolsky, 1988). *Problem Solving* reflects the teachers' choice of task that enables students to identify, apply and adapt a variety of strategies. Students should work on problem solving tasks with multiple solution paths thus, allowing students to clarify and extend their knowledge (NCTM, 2000). From analyzing the videos of TIMSS, researchers found that United States students spent 66% of their time in mathematics classrooms practicing familiar procedures rather than engaged in problem solving (Hiebert et al., 2005). *Connections and Applications* refers to teachers' choice of tasks that facilitate an understanding of mathematics as a network of ideas in which students connect and apply mathematics to other mathematical concepts, their own experience, to the world around them, and to other disciplines (Boaler, 2002).

Discourse

The discourse of a classroom is central to what students learn about mathematics as a domain of human inquiry (NCTM, 2007). The teacher's role is to initiate and orchestrate discourse and to use it skillfully to foster student learning. Students learn to communicate mathematically when they are encouraged to participate in discourse about mathematical ideas (NCTM, 2000). *Mathematical Discourse Community* reflects the extent to which the classroom social norms foster a sense of community in which students feel free to express their mathematical ideas honestly and openly. And, the extent to which the teacher and students "talk mathematics," and students are expected to communicate their mathematical thinking clearly to their peers and teacher, both orally and in writing, using the language of mathematics. While discourse offers opportunities for engagement and higher cognition, TIMSS results found that most mathematics instruction in the United States does not promote discourse (Hiebert et al., 2003; Hiebert & Stigler, 2000). *Explanation and justification* focuses on the part of discourse that considers reasoning and proving mathematical ideas. It focuses on the extent to which the teacher expects and students provide explanations/justifications, both orally and on written assignments. Teachers who require explanations from students ask many "how?" and "why?" questions to get at the depth of students' understanding. Students not only explain how they obtain their solutions, but they justify why their strategies are appropriate for arriving at such solutions. Studies have shown that students develop understanding when they are required to explain their thinking and justify their strategies (Cobb et al., 1991; Stein & Lane, 1996).

Representations

Representations are necessary to students' understanding of mathematical concepts and relationships. Representations allow students to communicate mathematical approaches, arguments, and understanding to themselves and to others (NCTM, 2007). They allow students to recognize connections among concepts and model mathematical ideas. *Use of representations* focuses on the extent to which teaching and learning promote the use of various representations (e.g., symbols, graphs, pictures, words, charts, diagrams, physical manipulatives) to illustrate ideas and concepts. Additionally, it is the extent to which students select, use, and translate among mathematical representations in an appropriate manner. Simply using mathematical representations does not represent high-quality mathematics instruction. Rather, teachers need to set up a mathematical learning environment that facilitates the students' processes of developing, making sense of, and translating between the representations to add understanding to a mathematical concept (NCTM, 2000). Research has shown that students as young as kindergartners have high competence in representing mathematical concepts when supported by their classroom community (diSessa, Hammer, Sherin, & Kolpakowski, 1991; Greeno & Hall, 1997; Lehrer & Schauble, 2002). *Students' use of mathematical tools* reflects an aspect of representations in whether students are afforded opportunities to use tools (e.g., calculators, pattern blocks, fraction strips, counters, virtual tools, etc.) to represent mathematical ideas. In the elementary grades, students are in the early stages of understanding key mathematical ideas. Their understanding can be enhanced by representing ideas with hands-on tools before using pictures or symbols (Bruner, 1966). While research has supported the use of tools in mathematics classrooms (Raphael & Wahlstrom, 1989; Sowell, 1989; Suydam, 1986), the students' own internal understanding of the mathematics must connect to the representation with the tool, beyond the manufacturer's intent of the tool (Moyer, 2001).

Mathematical coherence

Mathematical coherence is a critical component of any lesson, and considers the extent to which the teacher selects and enacts classroom activities in a clear way that leads to deeper student understanding.In the Trends in International Mathematics and Science Study (TIMSS) researchers found that mathematics lessons in the United States lacked logical order and coherence when compared to the other countries (Hiebert et al., 2005). These researchers suggested that mathematics lessons that are focused and conceptually coherent provide better opportunities for students to learn mathematics (Hiebert et al., 2005). *Structure of the lesson* refers to the extent to which the design of the lesson is organized to be conceptually coherent such that activities are related mathematically and build on one another in a logical manner. Also, knowledge of both the content and process of mathematics teaching and learning is essential for high quality mathematics instruction. Teachers' comfort and confidence with their knowledge of mathematics affects how they teach (Hill, Blunk, Charalambous, Lewis, Phelps, & Sleep, 2008). Their knowledge of mathematics impacts how tasks are enacted during teaching, the nature of the mathematical discourse, and their use of mathematical representations in their classrooms. *Mathematical Accuracy* reflects the extent to which mathematical concepts are presented clearly and accurately throughout the lesson (and the extent to which they are free of misconceptions and/or a teachers' effectiveness for handling misconceptions that arise).

M-Scan and Common Cores State Standards for Mathematics

The nine dimensions of M-Scan are theoretically linked to Standards for Mathematical Practices, which are student practices, in the Common Cores State Standards for Mathematics (CCSS-M). The Standards for Mathematical Practices explicate specific student practices that should be present in mathematics teaching and learning at all grade levels (NGA & CCSSO, 2010). These standards are based on the NCTM's (2000) Process Standards (problem solving, reasoning and proofs, communication, representation, and connections) and the National Research Council's strands of mathematical proficiency (adaptive reasoning, strategic competence, conceptual understanding, procedural fluency, and productive disposition). There are eight Standards for Mathematical Practices:

1. *Make sense of problems and persevere in solving them*. Mathematically proficient students are able to identify, apply, and adapt a variety of strategies to solve problems.
2. *Reason abstractly and quantitatively*. Mathematically proficient students make sense of and understand the meaning of quantities rather than just how to compute them.
3. *Construct viable arguments and critique the reasoning of others*. Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments.
4. *Model with mathematics*. Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace.
5. *Use appropriate tools strategically*. Mathematically proficient students consider the available tools (e.g., calculators, pattern blocks, fraction strips, counters, virtual tools) when solving a mathematical problem.
6. *Attend to precision*. Mathematically proficient students try to communicate precisely to others, use clear definitions, explain the meaning of symbols, specify units of measure, use accurate labels, and calculate accurately and efficiently.
7. *Look for and make use of structure*. Mathematically proficient students look closely to discern a pattern or structure.
8. *Look for and express regularity in repeated reasoning*. Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts.

Figure 2 shows the connections between M-Scan, which are teacher practices and the Standards for Mathematical Practice which are student practices. The purpose of figure two is to show how teacher practices interact and influences student practices. For example, the domain discourse in M-Scan is linked to student practices of reasoning abstractly and quantitatively, looking for and express regularity in repeated reasoning, and constructing viable arguments and critique the reasoning of others. Figure 2 is not exhaustive; that is, the links between M-Scan and CCSS-M are linked in many ways not represented in figure two. Figure 2 shows one way they are linked.

Figure 2: Connecting Teacher Practices with Student Practices

M-Scan (Teacher Practice)	CCSS-M Standards for Mathematical Practices (Student Practices)
Task • Cognitive Demand • Problem Solving • Connections and Application	• Make sense of problems and persevere in solving them. • Look for and make use of structure

Discourse • Mathematical Discourse Community • Explanation and Justification	• Reason abstractly and quantitatively. • Look for and express regularity in repeated reasoning • Construct viable arguments and critique the reasoning of others.
Representations • Use of Representations • Use of Mathematical Tools	• Use appropriate tools strategically • Model with mathematics.
Mathematical Coherence • Structure of Lesson • Mathematical Accuracy	• Attend to precision.

Existing Observational Measures

There are existing observationally-based measures that assess the aforementioned constructs of quality of teacher-student interactions and mathematics instructional quality. These existing measures have limitations for use with large scale research or do not capture the standards-based mathematics teaching practices that we were interested in examining. Analyses of these measures regarding each construct's dimensions or domains can be found in Table 1 and are outlined below. The *Classroom Assessment Scoring System* (CLASS; Pianta, La Paro et al., 2008) is a measure used to assess the quality of teacher-child interactions. To examine mathematics instruction, the *Reformed Teaching Observational Protocol* (RTOP; Piburn et al., 2000) is a widely used observational measure. Additionally, Table 1 includes the M-Scan.

Table 1

Content Analysis of Observational Measures

	Measure		
	M-Scan	*CLASS*	*RTOP*
Mathematics Instructional Quality			
Structure of the Lesson	X		
Use of Representations	X		X
Students' Use of Mathematical Tools	X		
Cognitive Demand	X		X
Mathematical Discourse Community	X	X	X
Explanation & Justification	X	X	
Problem Solving	X		X
Connections & Applications	X	X	
Mathematical Accuracy			
Teacher-Student Interaction Quality			
Emotional Support		X	
Classroom Organization		X	
Instructional Support	X	X	X

Classroom Assessment Scoring System. The CLASS focuses on the quality of interactions between students and teachers in a classroom (Pianta, La Paro et al., 2008). There are ten dimensions on the CLASS, each of which is scored on a 1-7 scale. The ten dimensions are used to measure quality in three domains: emotional support, classroom organization, and instructional support. Emotional support is assessed through scoring the nature of the climate (positive or negative), the sensitivity of the teacher, and the regard the teacher holds for various student perspectives, ideas, interests, and skills. Assessing behavior management, productivity, and format of instruction on the measure provides insights into classroom organization. The instructional support in classrooms is ascertained by looking at how concepts are developed, feedback is provided, and language is modeled. Researchers have examined the predictive validity of the CLASS, linking teacher-child interactions to child outcomes (Hamre & Pianta, 2005; Pianta et al., 2008). Theoretically, the CLASS domain of instructional support overlaps with three dimensions of mathematics instructional quality: discourse, explanation and justification, and connections and applications.

RTOP. The RTOP (Piburn et al., 2000) is a 25-item instrument that measures the extent to which instruction is standards or reform-based. The authors found multiple factors including one consisting of eleven items to load onto an inquiry-based instruction factor(Piburn et al., 2000). The RTOP is coded on a Likert scale of 0-4, with zero indicating the item "never occurred" to four indicating the lesson is "very descriptive" of the item. The authors of the RTOP examined construct and predictive validity and determined valid inferences can be drawn from RTOP scores (Piburn et al., 2000). Items on the abbreviated RTOP overlap to some degree with the instructional support domain, as conceptualized by the CLASS. Theoretically RTOP overlaps with four dimensions of mathematics instructional quality: use of representations, mathematical discourse community, cognitive demand, and problem solving.

Other measures. Two other observationally-based measures of mathematics instruction are used often, and thus, they warrant mention. The *Inside the Classroom* (ITC) Observation and Analytic Protocol (Horizon Research, 2000)addresses some dimensions of mathematics instructional quality (structure of the lesson, mathematical discourse, explanation and justification, and connections and applications); however, the protocol also requires synopses, descriptions, and analyses of the lesson, students, materials, and other classroom components. The length of the protocol and the required descriptions produce rich data but may not be conducive for use in large-scale studies because of the length of time required to gather data

The *Learning Mathematics for Teaching: Quality of Mathematics in Instruction* (LMT-QMI) instrument (Learning Mathematics for Teaching, 2006) is designed to measure teacher actions in regard to mathematical content, curriculum materials, and students. When using the measure, coders watch five-minute segments of videotaped mathematics instruction and document mathematical events in the segment. While the LMT-QMI includes some aspects of standards-based teaching practices (use of representations, students' use of mathematical tools, mathematical discourse community, explanation and justification, and connections and applications), the observation of five-minute segments may offer a more fragmented view than suited for some large-scale studies focused on examining standards-based mathematics teaching practices holistically.

Using the Mathematics Scan (M-Scan) Measure

M-Scan codes are based on a full mathematics lesson. Coders watch the first 30 minutes of the video-recorded lesson and take notes throughout the 30-minute segment to record what occurs during the lesson. Coders write their notes on the back of the coding sheet or on separate pieces of paper that can be stapled to the coding sheets. The notes are used as examples and references when completing the M-Scan coding for that segment. After the first 30 minutes, the video is paused to allow a period coders to reflect and mark "soft codes" (i.e. initial ratings) on coding sheet by underlining the number corresponding to the initial code. These marks will serve as indicators of what happened during the first part of the lesson.

After assigning "soft codes" for the first 30 minutes, coders continue watching the lesson, following the procedures done in the first 30 minute segment. Once coders have watched the entire lesson, final codes are assigned. Coders should refer to the coding guides while coding.

Training in the M-Scan

The M-Scan training is a five day process involving reading, listening to conversations about each coding dimension, watching videotapes, and coding practice tapes. Training in the M-Scan involves a four phase process and each phase has a corresponding letter of the acronym PTRD: 1) preparation, 2) training/mastery phase, 3) reliability phase, and 4) drift test phase. Master coders keep track and record trainee's progress of attaining and maintaining reliability through these phases.

Preparation Phase:

- Trainees read *Mathematics Teaching Today: Improving Practice, Improving Student Learning* (NCTM, 2007) and the *Principles and Standards for School Mathematics* (NCTM, 2000), as well as readings on cognitive demand, use of representations, mathematical tools, problem-solving, and discourse in mathematics teaching and learning. Trainees record notes and questions.

Training/Mastery Phase:

- Trainees meet with a master coder to discuss questions from the preparation phase and attend the training session on mathematics coding. During the training, they review and discuss the coding manual, observation forms, and highlights from the readings.

- Trainees practice with the expert on at least two full class mathematics videos. After the training session, trainees watch two videotaped classes independently and take notes. Afterward, ratingsare compared to those of the master coders.

- After trainees have watched and coded the assigned set of "training" videos, the master coders identify gaps and look for convergence. More training tapes are assigned if gaps are present. Trainees' progress to the reliability phase when ratings from the training videos converge with master codes.

Reliability Phase:

- Trainees watch and code six mathematics "reliability" video observations, without conferring with the master coder.

- Trainees meet with a master coder after watching the six mathematics "reliability" video observations. The master coder will identify gaps and look for convergence. Trainees' ratings are scrutinized carefully to figure out whether: 1) errors are systematic, for example, some constructs/items need further work, or 2) errors are not systematic, in which case the trainee and the master coder need to carefully review and discuss the codes together.

- Once the master coder has verified that the trainees reliable. The trainee is able to code video mathematics observations using the M-Scan.

Drift Test Phase:

- Twice a month, all coders meet to co-code one video mathematics lesson to check for drifts in coding. All coders confer to verify convergence with the master coders.

Early Findings using the M-Scan

The development of M-Scan comes from the work of a randomized controlled trial funded by the USDOE-Institute for Education Sciences (Rimm-Kaufman, Berry, Fan, 2007). The four-year Responsive Classroom Efficacy Study (RCES) examines the evidentiary basis of the *Responsive Classroom* (RC)Approach for improving elementary students' mathematics achievement. As part of RCES, more than 1000 videotapes of 360 third, fourth, and fifth grade teachers (videotaped) teaching a typical mathematics lesson three times during the school year; each lesson lasting approximately one hour. Handouts and other instructional materials were gathered corresponding to the lessons.

RCES enrolled 24 schools into two conditions, 13 intervention schools (i.e., *RC* Approach) and 11 control schools (i.e., business as usual). Roughly half of the 24 schools were selected (via stratified randomization) into the RC condition; the remaining schools as control schools. The sample is sufficiently diverse to support generalizability; 27% of the children receive free/reduced lunch; 57% of the children are ethnic minorities. Data from RCES have been used to examine the ability to achieve reliability in coding and validity of the M-Scan. A review by mathematics education and mathematics experts suggest that the dimensions of the M-Scan represent components of mathematics instructional quality (Walkowiak, et al. under review). Further, several substantive questions about predictors and correlates of quality of mathematics instructional quality have been addressed, as described below.

One-hundred eighty observations from a subset of 60 third grade teachers (83 female, 83% white, 6.8% African American, 10.2% other) at the 24 schools from RCES were used to examine the M-Scan validity and score reliability (Walkowiak, et al. under review). The coders' recorded rationales for their scores on each of the M-Scan constructs for the 60 mathematics lessons. These responses were qualitatively analyzed for alignment with coding guide descriptors. Analyses indicate 87.7% of the coders' rationales were aligned with coding guide descriptors for the entire M-Scan measure. Findings show that multiple observers can conduct observations reliably. Bivariate correlations between the M-Scan dimensions, RTOP, and CLASS domains were conducted. As hypothesized, M-Scan dimensions and RTOP converged on problem solving, mathematical discourse, and cognitive demand but low convergence on other dimensions. This suggests that M-Scan provides information distinct from the RTOP in relation to five dimensions (structure of the lesson, use of representations, students' use of mathematical tools, explanation and justification, connections and applications). The M-Scan dimensions and CLASS domains showed little convergence. This suggests that M-Scan captures indicators of quality that are unique to mathematics instruction. Generalizability theory was used to demonstrate more variability between classrooms than between coders observing the same classrooms. This finding indicates that M-Scan can be used with multiple coders.

The M-Scan has been used to examine teacher predictors of higher standards based mathematics teaching practices (Ottmar, Rimm-Kaufman,& Berry, under review). Findings show that teachers trained in the *RC* approach showed higher use of standards based mathematics teaching practices than those in a comparison condition, suggesting that efforts to improve teacher capacity in classroom organization (using the *RC* approach) had impact on the quality of mathematics instructional settings. Findings examining the relation between two types of classroom mathematics instructional efficacy (i.e., *personal mathematics teaching efficacy* and *mathematics teaching outcome expectancy*) showed that higher levels of personal mathematics teaching efficacy (but not mathematics teaching outcome expectancy) related to high mathematics instructional quality. Further analyses using a latent profile analysis were conducted to examine teacher profiles (Walkowiak, 2010). The first profile (51%) scored above the means (relative to the sample) on mathematics instructional quality, mathematical knowledge for teaching, and mathematics teaching efficacy; the second profile (41%) scored below average on the three constructs, and the third profile (8%) was comparable to the first profile with the exception of below average mathematics instructional quality. Also, research suggests that third grade students who receive free or reduced price lunch make greater gains in achievement than higher income peers when their teachers demonstrate greater use of standards based mathematics teaching practices (Merritt, Rimm-Kaufman, Walkowiak & Berry, under review).

The M-Scan measure has gone through the initial stages of translation into an instructional tool for pre-service and in-service teachers (Merritt, et al., 2010). Specifically, the dimensions have been described in detail to establish their connection to instructional practice and establish parameters for the professional development of pre-service and in-service teachers. M-Scan can be used to provide teachers and mathematics educators with a framework for understanding mathematics instructional quality.

Finding from the works using the M-Scan provide evidence of three noteworthy characteristics. First, M-Scan offers efficiency; the dimensions can be coded in a time-efficient manner, a necessary practicality for researchers in large-scale studies. Second, M-Scan gives a holistic assessment of mathematics lessons; it considers the entire lesson, beginning, middle, and end. Third, M-Scan provides a framework for understanding standards-based mathematics teaching practices.

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Appendix A

Cognitive Demand

1. Cognitive Demand: Cognitive demand refers to command of the central concepts or "big ideas" of the discipline, generalization from specific instances to larger concepts, and connections and relationships among mathematics concepts. This dimension considers two aspects of cognitive demand: task selection and teacher enactment. That is, it considers the extent to which the selected task is cognitively demanding and the extent to which the teacher consistently and effectively promotes cognitive depth (Stein & Lane, 1996).

	Low (1, 2)	**Medium (3, 4, 5)**	**High (6, 7)**
Task Selection	The tasks of the lesson are focused on memorization or procedures without connections to underlying concepts.	Some of the tasks are focused on memorization or procedures without connections to underlying concepts, *and* some of the tasks are focused on procedures with connections to underlying concepts or non-algorithmic, complex thinking.	The majority of the tasks of the lesson are focused on procedures with connections to underlying concepts or non-algorithmic, complex thinking.
	None of the tasks are open-ended.	Some of the tasks are open-ended.	Most of the tasks are open-ended.
Teacher Enactment	The teacher rarely provides feedback, modeling, or examples that promote complex thinking by students.	The teacher sometimes provides feedback, modeling, or examples that promote complex thinking by students.	The teacher often provides feedback, modeling, or examples that promote complex thinking by students.
	The teacher rarely encourages students to make conceptual connections.	The teacher sometimes encourages students to make conceptual connections.	The teacher often encourages students to make conceptual connections.

Problem Solving

2. Problem Solving: The extent to which instructional activities enable students to identify, apply, and adapt a variety of strategies to solve problems. The extent to which the problems that students solve are complex and allow for multiple solutions. NOTE: To receive a "High" rating, problems should not be routine or algorithmic; they should consistently require novel, challenging, and/or creative thinking.

[1] Student formulation of problems can improve the score for this domain, but scores should not decrease if this is not present.

[2] Student formulation of problems may involve students extending/following up on problems not originally formulated by students

	Low (1, 2)	**Medium (3, 4, 5)**	**High (6, 7)**
Students' Engagement with Problems	Students rarely engage in problems that allow them to grapple with mathematical concepts. Students often work on exercises for which they are practicing an already learned procedure.	Students sometimes engage in problems that allow them to grapple with mathematical concepts. Students sometimes work on exercises for which they are practicing an already learned procedure.	Students often engage in problems that allow them to grapple with mathematical concepts. Students rarely work on exercises for which they are practicing an already learned procedure.
Presence of Problem Solving with Multiple Strategies	Classroom activities encourage only one strategy to solve each problem.	Classroom activities sometimes encourage multiple strategies to solve each problem.	Classroom activities often encourage multiple strategies to solve each problem.
Student Formulation of Problems *(when applicable)*[1, 2]	If students formulate problems, they are generally procedural.	If students formulate problems, they are sometimes solved with multiple strategies.	If students formulate problems, they are generally solved with multiple strategies.

Connections/Applications

3. Connections/Applications: The extent to which the lesson helps students connect mathematics to other mathematical concepts, their own experience, to the world around them, and to other disciplines. The extent to which the lesson helps students apply mathematics to real world contexts and to problems in other disciplines. NOTE: The experiences may be teacher-generated or student-generated, but they should relate to the students' actual life situations.

	Low (1, 2)	**Medium (3, 4, 5)**	**High (6, 7)**
Connections	Meaningful connections between mathematics learned in the classroom and other math concepts, experiences, disciplines, orthe world are rarely made. The class work is not relevant to students' lives.	Meaningful connections between mathematics learned in the classroom and other math concepts, experiences, disciplines, orthe world are sometimes made. The class work is potentially relevant to the students' lives	Meaningful connections between mathematics learned in the classroom and other math concepts, experiences, disciplines, orthe world are often made. The class work is relevant to the students' lives.
Applications	Students are never asked to apply the math they learn to the world around them.	Students are sometimes asked to apply the math they learn to the world around them. .	Students are often asked to apply the math they learn to the world around them.

Use of Representations

4. Use of Representations:The extent to which the lesson promotes the use of and translation among multiple representations (pictures, graphs, symbols, words) to illustrate ideas and concepts. The use of and translation among representations should allow students to make sense of mathematical ideas or extend what they already understand.NOTE: Dimension includes both exposure (by teacher or curriculum) and use by students. As outlined in NCTM's *Principles and Standards for School Mathematics (2000),* "students in grades 3-5 should continue to develop the habit of representing problems and ideas to support and extend their reasoning. Such representations help to portray, clarify, or extend a mathematical idea" (p. 206).

	Low (1, 2)	**Medium (3, 4, 5)**	**High (6, 7)**
Presence of Representations	Teacher and/or students rarely use more than one representation of a mathematical concept.	Teacher and/or students sometimes use more than one representation of a mathematical concept.	Teacher and/or students often use more than one representation for a mathematical concept.
Teacher Translation among Representations	For the representation(s) used, the teacher does not make connections to concepts or between representations. (i.e., procedural approach to use of representations).	For the representation(s) used, the teacher makes some connections to concepts and between representations.	For the representation(s) used, the teacher often makes connections to concepts and between representations.
Student Translation among Representations	Students do not translate between representations.	Students sometimes translate back and forth between representations. They *do not* explain their representations.	Students translate back and forth between representations. They also explain their representations at times.

Students' Use of Mathematical Tools

5. Use of Mathematical Tools: The extent to which the lesson affords students the opportunity to use appropriate mathematical tools (e.g., calculators, pattern blocks, fraction strips, counters, virtual tools) and that these tools enable them to represent and develop abstract mathematical ideas.Tools are a way to represent abstract mathematical concepts through physical manipulation of objects. NOTE: When students use equipment and/or objects to collect data that are later used in exploring mathematical ideas, the equipment/objects are not considered to be mathematical tools unless they are also explicitly used to develop the mathematical ideas.

	Low (1, 2)	**Medium (3, 4, 5)**	**High (6, 7)**
Opportunity to Use Tools	Students do not use tools and/or are only permitted to use tools for help with procedural skills.	Students sometimes use tools to investigate concepts and solve problems.	Students often use tools to investigate concepts and solve problems.
Depth of Use	Students rarely make connections between tools and mathematical concepts.	Students sometimes make connections are between tools and mathematical concepts.	Students often make connections between tools and mathematical concepts.

Mathematical Discourse Community

6. Mathematical Discourse Community: The extent to which the classroom social norms foster a sense of community in which students can express their mathematical ideas openly. The extent to which the teacher and students "talk mathematics," and students are expected to communicate their mathematical thinking clearly to their peers and teacher, both orally and in writing, using the language of mathematics. NOTE: There is a "high bar" on this dimension because there is an expectation for students to have an active role in promoting discourse; this should not be only the teacher's role. This is in contrast to Explanation/Justification. The rating does take into account whether discourse focuses on mathematics content but not the cognitive depth of that content.
***Mathematical Thinking** = processes, strategies, and/or solutions.

	Low (1, 2)	**Medium (3, 4, 5)**	**High (6, 7)**
Teacher's Role in Discourse	The majority of math discussion in the classroom is directed from the teacher to the students. Students' ideas, questions, and input are rarely or never solicited.	Some of the math discussion in the classroom includes student participation, but some is teacher-initiated. Students' ideas, questions, and input are sometimes solicited.	Throughout the math discussion in the classroom, students consistently participate. Students' ideas, questions, and input are frequently solicited.
Sense of Mathematics Community through Student Talk	Student to student talk rarely or never occurs. When students talk, they rarely share mathematical thinking* and language.	Student to student talk sometimes occurs. When students talk, they sometimes share mathematical thinking* and language.	Student to student talk frequently occurs. When students talk, they often share mathematical thinking* and language.
Questions	Most of the teacher's questions have known/correct answers, and rarely encourage mathematical thinking*.	Some of the teacher's questions have known/correct answers, and some encourage mathematical thinking*.	Few of the teacher's questions have known/correct answers, and many encourage mathematical thinking*.

Explanation and Justification

7. Explanation and Justification: The extent to which the teacher expects and students provide explanations/justifications, both orally and on written assignments. NOTE: Simply "showing your work" on written assignments – i.e., writing the steps involved in calculating an answer – does not constitute an explanation.

	Low (1, 2)	**Medium (3, 4, 5)**	**High (6, 7)**
Presence of Explanation and Justification	Students rarely provide explanations or justify their reasoning.	Students sometimes provide explanations and/or justify their reasoning.	Students often provide explanations and/or justify their reasoning.
	Teachers rarely ask "what, how, why" questions or otherwise solicit student explanations/justifications.	Teachers sometimes ask "what, how, why" questions or otherwise solicit student explanations/justifications.	Teachers often ask "what, how, why" questions or otherwise solicit student explanations/justifications.
Depth of Explanation and Justification (procedural and conceptual)	Student explanations often focus on procedural steps *and rarely* include conceptual understanding of the topic(s).	Student explanations sometimes focus on procedural steps *and sometimes* include conceptual understanding of the topic(s).	Student explanations rarely focus on procedural steps *and often* focus on conceptual understanding of the topic(s).

Structure of the Lesson

8. Structure of the Lesson: The extent to which the design of the lesson is organized to be conceptually coherent such that activities are connected mathematically and build on one another in a logical manner. Coherence in a lesson is defined as "the implicit and explicit interrelation of all mathematical components of the lesson", (Hiebert et al., 2005, p. 124). NOTE 1: Ratings of observations should take into account interruptions for procedural activities that are not part of the instructional unit, when these interruptions consume a non-trivial amount of time. NOTE 2: If a warm-up (i.e., brief, possibly unrelated segment at beginning of class) does not interfere with overall flow of lesson, it should not count against the overall score for this measure.

	Low (1, 2)	**Medium (3, 4, 5)**	**High (6, 7)**
Logical Sequence	Overall, the components of the math lesson do not appear to be logically organized.	Some components of the math lesson are logically organized, but others do not seem to fit.	All components of the math lesson are logically organized.
Mathematical Coherence	The components of the lesson are not mathematically connected or coherent.	Some components of the lesson are mathematically connected and coherent.	All components of the lesson are mathematically connected and coherent.
Promotion of Deeper Understanding	The structure of the lesson does not appear to lead students to a deeper understanding of the mathematical concept(s) presented.	The structure of the lesson appears to lead students toward partial depth of understanding of mathematical concepts, or the structure of the lesson appears to lead students toward a deeper understanding of some concepts, but not others.	The structure of the lesson appears to lead students to a deeper understanding of the concept(s) presented.

Mathematical Accuracy

9. Mathematical Accuracy The extent to which the mathematical concepts are presented clearly and accurately throughout the lesson. The extent to which student misconceptions are present, and whether teachers handle student misconceptions in a way that clarifies conceptual understanding.

	Low (1, 2)	**Medium (3, 4, 5)**	**High (6, 7)**
Accuracy in Teacher Presentation	A few of the concepts and procedures presented to the students by the teacher are mathematically accurate. Most of the concepts and procedures are mathematically inaccurate.	Most of the concepts and procedures presented to students by the teacher are mathematically accurate, but on a few occasions, the concepts and procedures are mathematically inaccurate.	The concepts and procedures presented to the students by the teacher are mathematically accurate.
Clarity of Mathematical Concepts	The mathematical concepts are not articulated clearly by the teacher. There is ambiguity in presentation of key mathematical concepts.	The mathematical concepts may be articulated with some clarity by the teacher. There is some ambiguity in presentation of key mathematical concepts.	The mathematical concepts are articulated clearly by the teacher. There is no ambiguity in presentation of key mathematical concepts.
Responsiveness to Student Mathematical Thinking ***(code this portion NA if no misconceptions are observed)***	Student misconceptions are obvious in the lesson. Teacher response appears to leads to ambiguity or confusion about mathematical concepts	One or more student misconceptions are observed during the lesson. Teacher responses appear to lead to further clarity, but some ambiguity or confusion by students may still be present.	Student misconceptions may or may not have been observed during the lesson. Teacher responses lead to improved clarity about mathematical concepts for all students.

M-Scan Observational Measure

	Low			Medium			High
Cognitive Demand *Task selection* *Teacher enactment*	1	2	3	4	5	6	7
Problem Solving *Students' engagement w/problems* *Presence of multiple strategies* *Student formulation of problems*	1	2	3	4	5	6	7
Connections/Applications *Connections* *Applications*	1	2	3	4	5	6	7
Representations *Presence of representations* *Teacher translation among representations* *Student translation among representations*	1	2	3	4	5	6	7
Use of Mathematical Tools *Opportunity to use tools* *Depth of use*	1	2	3	4	5	6	7
Mathematical Discourse Community *Teachers' use of discourse* *Sense of mathematics community* *through student talk* *Questions*	1	2	3	4	5	6	7
Explanation and Justification *Presence of expl/just* *Depth of expl/just*	1	2	3	4	5	6	7
Structure of the Lesson *Logical sequence* *Mathematical coherence* *Promotion of deeper understanding*	1	2	3	4	5	6	7
Mathematical Accuracy *Accuracy in teacher presentation* *Clarity of mathematical concepts* *Responsiveness to student mathematical thinking*	1	2	3	4	5	6	7

Did the teacher present any content that was incorrect or communicate any misconceptions? _____Yes _____No
If yes, please describe: