# Middle School Math: Turn It On!!

NCTM 2013, Denver, CO

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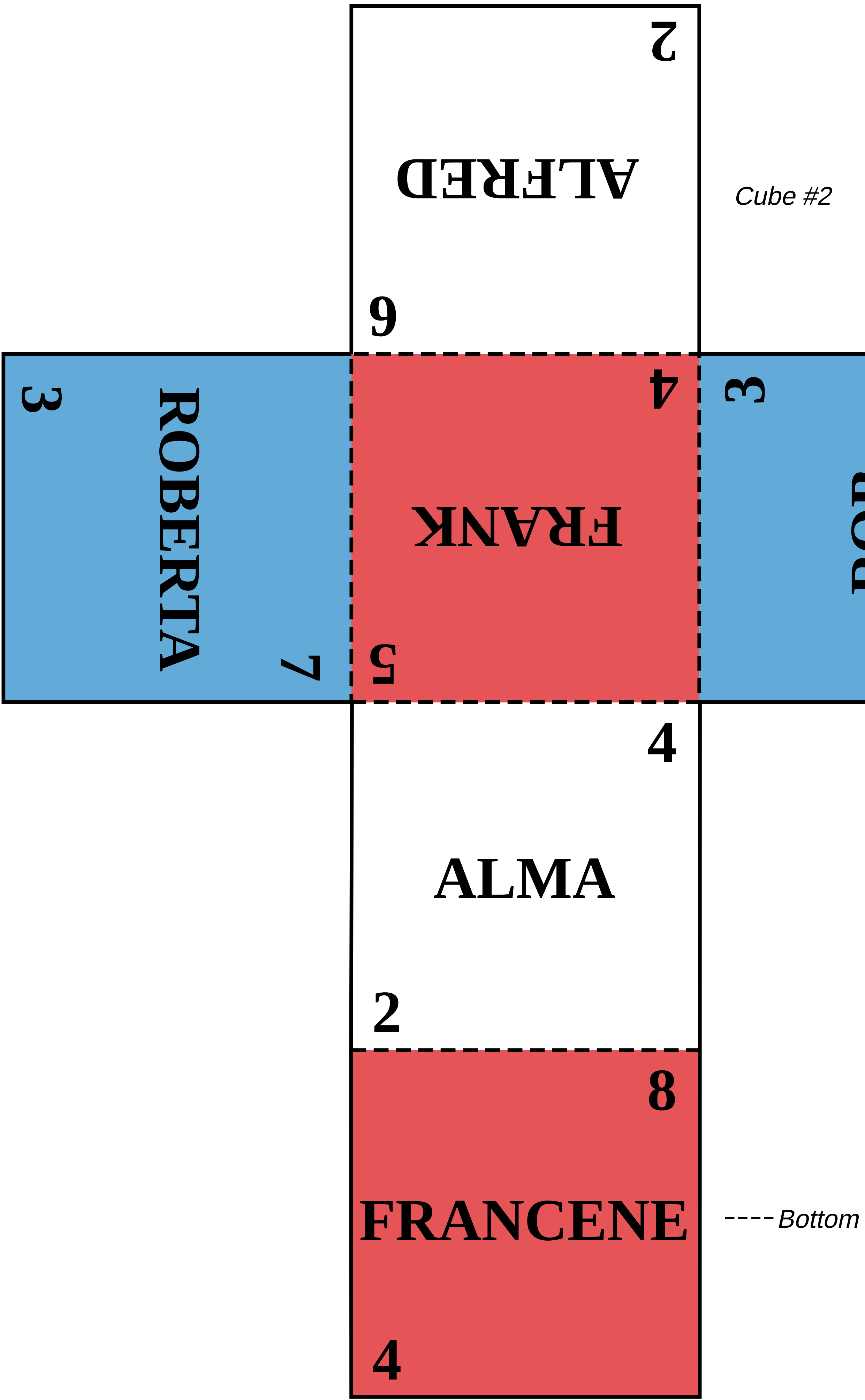
## Outline of Workshop

1. Experience Inquiry
    a. Cube Activity
    b. Inquiry in Motion: http://iim-web.clemson.edu/?page_id=3
2. Look at Technology Possibilities
    a. The Profit Makers Investigation
    b. Polygon Car Convoys Investigation
3. Make Mathematical Connections
    a. What's My Rule?
    b. Bo D. Bilder Competition
    c. Dog Sitting
    d. Twizzlers Munch

A complete handout with investigations and materials used in the workshop is available through NCTM. The handout and The Geometer's Sketchpad document are available at the following website (or scan QR code): http://north-morris.net.temp.guardedhost.com/jennifer-north-morris.html

*Cube #2*


ALFRED
2
9
ROBERTA
3
7
FRANK
4
5
ROB
3
3
ALMA
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FRANCENE
8
4
Bottom

# Finding Proof on the Cubes

## Proof that can be found on cube 1:

The cube has six sides.

The cube has five exposed sides.

The dots are black.

The exposed sides have numbers 1, 3, 4, 5, and 6.

The opposite sides add up to seven.

The even numbers have an opposite side that is odd.

## Proof that can be found on cube 2:

Names and numbers are in black.

Exposed sides have either a male or female name.

Opposing sides have a male name on one side and a female name on the other.

Names on opposite sides begin with the same letters.

The number in the upper-right corner of each side corresponds to the number of letters in the name on that side.

The number in the lower-left corner of each side corresponds to the number of the first letter that the names on opposite sides have in common.

The number of letters in the names on the five exposed sides progresses from three (Rob) to seven (Roberta).

Adapted from http://www.phreneticus.com/leftbrain/lessons/inquiry/inq_less/inq_lesson.htm

## *INVESTIGATION 1.9: THE PROFIT MAKERS*

***Did you know*** *the business that was rated the most charitable company in 2011 was the Kroger Company? The Kroger Company's headquarters is in Cincinnati, Ohio. They are one of the largest grocery retailers, with sales of $82.2 billion. In 2009, they gave 10.9% of their profits to charity, about $64 million. Reference: http://detroit.cbslocal.com/2011/12/02/forbes-kroger-number-one-among-americas-most-generous-companies/; www.thekrogerco.com/*

Bernesha, Jasmine, and Marcus are going into business together and dream of making $1,000,000 in profit. Bernesha notices that they wouldn't be able to receive exactly the same amount in profit if they make $1,000,000, that someone will get an extra cent. Though they realize they can live with that, it gets them to thinking about when $1,000,000 can and cannot be divided evenly. Consequently they decide to investigate.

A. Determine whether the $1,000,000 in profits could be shared equally if there are 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, or 25 employees.

B. What are the prime factors of 1,000,000? Explain.

C. Determine if there is a relationship between the prime factors for the number of employees, the prime factors of 1,000,000, and whether or not the profits can be divided equally. Explain what you find.

D. A unit fraction is a fraction with 1 in the numerator. Express all of the unit fractions with denominators from 2 to 25 as decimals. Which ones repeat and which ones terminate? Why?

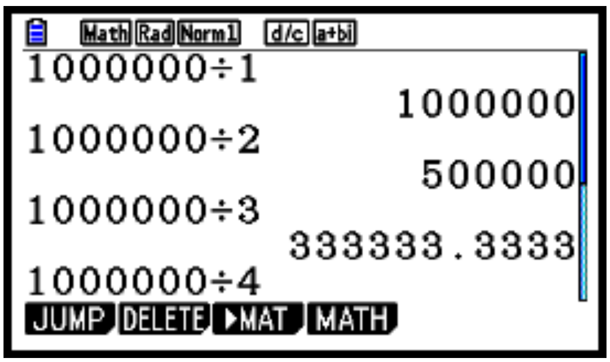
E. Discuss the relationship you find among the previous parts of this investigation.

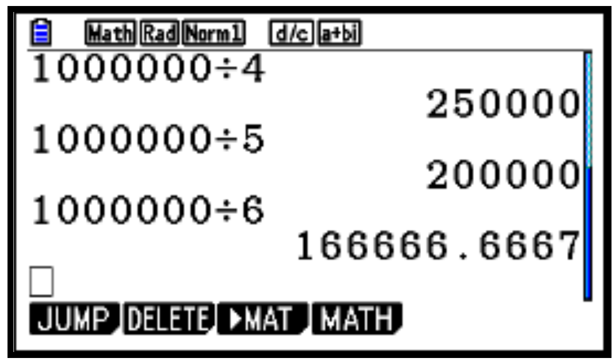
F. Generalize your results. In other words, state a hypothesis that will allow you to determine ahead of time whether any fraction (not just unit fractions) will repeat or terminate when expressed as a decimal. Test your hypothesis on at least 10 fractions. Explain why you believe your hypothesis is true.

## *SAMPLE SOLUTION: THE PROFIT MAKERS*

A. Determine whether the $1,000,000 in profits could be shared equally if there are 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, or 25 employees.

*To investigate this, we will use the Run-Matrix menu. We will divide our $1,000,000 in profits among the given number of people by performing division as shown below. If we have a decimal that goes more than two decimal places (past the hundredths, which represents cents), then we cannot share equally. If the decimal ends before that (which, of course, includes whole numbers), then we can share the profits equally. The first few divisions are shown below.*




*We summarize our results in the table below. The middle column indicates the value shown by the calculator, which in some cases has been automatically rounded.*

| *# OF EMPLOYEES* | *EACH PERSON'S SHARE* | *EQUAL AMOUNTS?* |
|---|---|---|
| *2* | *$500,000* | *Yes* |
| *3* | *$333,333.3333* | *No* |
| *4* | *$250,000* | *Yes* |
| *5* | *$200,000* | *Yes* |
| *6* | *$166,666.6667* | *No* |
| *7* | *$142,857.1429* | *No* |
| *8* | *$125,000* | *Yes* |
| *9* | *$111,111.1111* | *No* |
| *10* | *$100,000* | *Yes* |
| *11* | *$90,909.09091* | *No* |

| | | |
|---|---|---|
| *12* | *$83,333.33333* | *No* |
| *13* | *$76,923.07692* | *No* |
| *14* | *$71,428.57143* | *No* |
| *15* | *$66,666.66667* | *No* |
| *16* | *$62,500* | *Yes* |
| *17* | *$58,823.52941* | *No* |
| *18* | *$55,555.55556* | *No* |
| *19* | *$52,631.57895* | *No* |
| *20* | *$50,000* | *Yes* |
| *21* | *$47,619.04762* | *No* |
| *22* | *$45,454.54545* | *No* |
| *23* | *$43,478.26087* | *No* |
| *24* | *$41,666.66667* | *No* |
| *25* | *$40,000* | *Yes* |

*The numbers of people who could share $1,000,000 equally are 1, 2, 4, 5, 8, 10, 16, 20, and 25.*

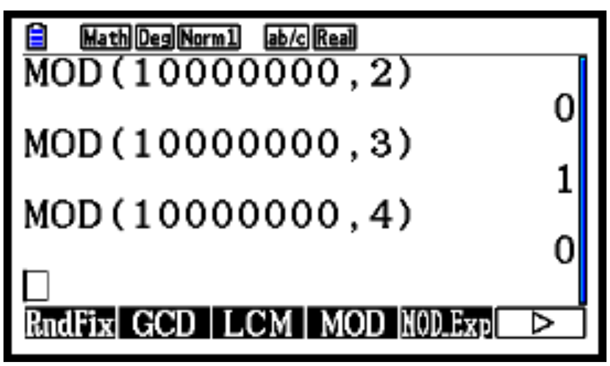
*The ones that cannot share equally are 3, 6, 7, 9, 11, 12, 13, 14, 15, 17, 18, 19, 21, 22, 23, and 24.*

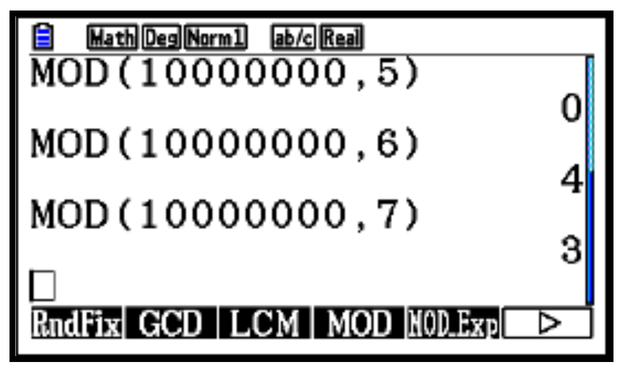
*An alternate way to view this is to use modular arithmetic. This is something teachers may or may not wish to address at this time. Modular arithmetic involves cyclical counting and can be thought of as remainders. The time of day, the days of the week, the months of the year, and many other measures are modular. For example, clocks operate on a 12-hour cycle. If we think of 12 o'clock as 0, then we can divide by 12 and look at the remainder to determine the time. For example to find the time 40 hours after 12:00, we see that dividing 40 by 12 leaves a remainder of 4, so it would be 4:00. To find out what time it would be 30 hours after 2:00, we can find the sum of 2 and 30 (of course, that's 32), divide by 12 and note that the remainder is 8. Thus it will be 8:00. If we want to distinguish between am and pm, we can use a 24-hour cycle rather than a 12-hour cycle.*

*For our problem here, to determine whether the profits can be equally shared, we want to find out if the remainder is 0 when we divide $1,000,000 by the number of employees. This would determine if there are any dollars left over. Because of our money system, we could actually determine if the remainder is 0 when we divide the number of cents in $1,000,000. This number is, of course 100,000,000.*

*What we will do is use what is called the modulo operation on the calculator. From the Run-Matrix*

- ☞ *Press OPTN, F6 for more options and F4 (NUMERIC).*
- ☞ *Select F6 for more options and F4 (MOD). In the parentheses, type in 100,000,000, a comma, and the number of employees. Then close the parentheses and press EXE. If the remainder is 0, then $1,000,000 can be divided equally. If the remainder is other than 0, then it cannot be. Below shows the results for 2, 3, 4, 5, 6, and 7 employees. It confirms that the money can be equally shared for 2, 4, and 5 employees, but not for 3, 6, and 7.*




B. What are the prime factors of 1,000,000? Explain.

1,000,000 = $10^6 = 10\times10\times10\times10\times10\times10.$

*Therefore,* $1{,}000{,}000 = (2\times5)^6 = 2\times2\times2\times2\times2\times2\times5\times5\times5\times5\times5\times5$

*When we break it down to primes, we find the only prime factors are 2 and 5.*

C. Determine if there is a relationship between the prime factors for the number of employees, the prime factors of 1,000,000, and whether or not the profits can be divided equally. Explain what you find.

*In the table below, we list the prime factors for the various number of employees along with whether or not the profits could be divided equally.*

| *# OF EMPLOYEES* | *PRIME FACTORS* | *EQUAL AMOUNTS?* |
|---|---|---|
| *2* | *2* | *Yes* |
| *3* | *3* | *No* |
| *4* | *2* | *Yes* |
| *5* | *5* | *Yes* |
| *6* | *2, 3* | *No* |
| *7* | *7* | *No* |
| *8* | *2* | *Yes* |
| *9* | *3* | *No* |
| *10* | *2, 5* | *Yes* |
| *11* | *11* | *No* |
| *12* | *2, 3* | *No* |
| *13* | *13* | *No* |
| *14* | *2, 7* | *No* |
| *15* | *3, 5* | *No* |
| *16* | *2* | *Yes* |
| *17* | *17* | *No* |
| *18* | *2, 3* | *No* |
| *19* | *19* | *No* |
| *20* | *2, 5* | *Yes* |
| *21* | *3, 7* | *No* |
| *22* | *2, 11* | *No* |
| *23* | *23* | *No* |
| *24* | *2, 3* | *No* |

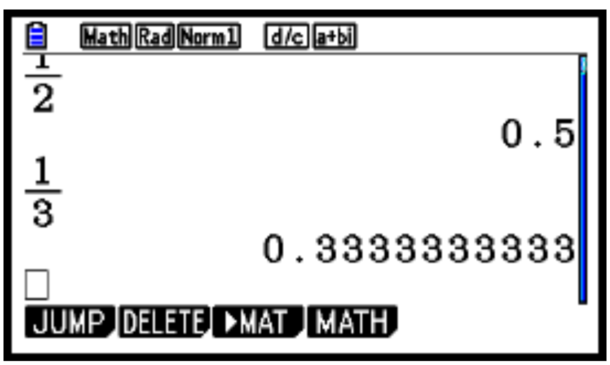
| | | |
|---|---|---|
| *25* | *5* | *Yes* |

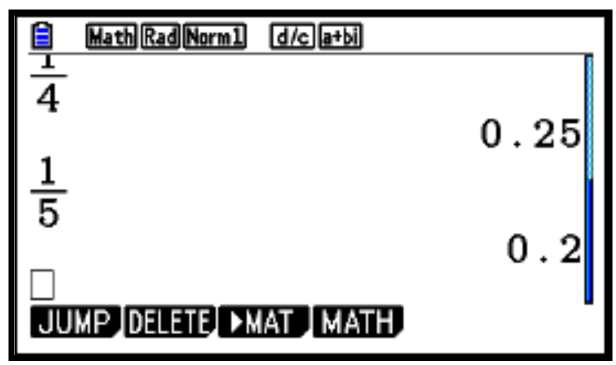
*What we want students to discover is that if the number of employees has any prime factors other than 2 or 5, which are the prime factors of 1,000,000, then the profits cannot be shared equally. If the prime factors have nothing other than 2 and 5 (and it needn't have both), then the profits can be divided equally.*

D. A unit fraction is a fraction with 1 in the numerator. Express all of the unit fractions with denominators from 2 to 25 as decimals. Which ones repeat and which ones terminate? Why?

*Here we'll take advantage of the natural display of the calculator. Again, we'll use the Run-Matrix menu.*

☞ *Enter the number using the fraction key and then press the F↔D key to see the number in decimal form. The decimals that continue past the decimal point are rounded by the calculator to fit the display.*




*Complete results as displayed on the calculator are shown in the table below. For repeating decimals, the calculator rounds to the $10^{th}$ or $11^{th}$ place past the decimal point.*

| *UNIT FRACTION* | *DECIMAL FORM* | *TERMINATE/REPEAT?* |
|---|---|---|
| $\frac{1}{2}$ | *0.5* | *Terminate* |
| $\frac{1}{3}$ | *0.3333333333* | *Repeat* |
| $\frac{1}{4}$ | *0.25* | *Terminate* |

| | | |
|---|---|---|
| $\frac{1}{5}$ | *0.2* | *Terminate* |
| $\frac{1}{6}$ | *0.1666666667* | *Repeat* |
| $\frac{1}{7}$ | *0.1428571429* | *Repeat* |
| $\frac{1}{8}$ | *0.125* | *Terminate* |
| $\frac{1}{9}$ | *0.1111111111* | *Repeat* |
| $\frac{1}{10}$ | *0.1* | *Terminate* |
| $\frac{1}{11}$ | *0.09090909091* | *Repeat* |
| $\frac{1}{12}$ | *0.08333333333* | *Repeat* |
| $\frac{1}{13}$ | *0.07692307692* | *Repeat* |
| $\frac{1}{14}$ | *0.07142857143* | *Repeat* |
| $\frac{1}{15}$ | *0.06666666667* | *Repeat* |
| $\frac{1}{16}$ | *0.0625* | *Terminate* |
| $\frac{1}{17}$ | *0.05882352941* | *Repeat* |
| $\frac{1}{18}$ | *0.05555555556* | *Repeat* |
| $\frac{1}{19}$ | *0.05263157895* | *Repeat* |
| $\frac{1}{20}$ | *0.05* | *Terminate* |
| $\frac{1}{21}$ | *0.04761904762* | *Repeat* |
| $\frac{1}{22}$ | *0.04545454545* | *Repeat* |
| $\frac{1}{23}$ | *0.04347826087* | *Repeat* |
| $\frac{1}{24}$ | *0.04166666667* | *Repeat* |
| $\frac{1}{25}$ | *0.04* | *Terminate* |

*Is it possible that some of the fractions that we have identified as repeating actually terminate? After all, there are a couple of decimals, such as 0.04347826087, for which we do not see what is repeating nor do we see an end. How do we know they truly* *repeat? We would be thrilled if students asked about this, and we would encourage more exploration as desired. It is certainly true that there are fractions that terminate more than 10 places past the decimal point. However, none of these fractions have denominators anywhere close to as small as the fractions we are studying in this investigation. We hope the reason becomes apparent as we continue with the problem.*

E. Discuss the relationship you find among the previous parts of this investigation.

*If the unit fraction terminates when represented as a decimal, the denominator of the fraction indicates a number of employees that can share equally in the $1,000,000 profit. If the fraction repeats when represented as a decimal, the denominator of the fraction indicates a number of employees that cannot share the $1,000,000 equally. Going a little further, if the denominator of the unit fraction has no other prime factors besides 2 and/or 5, the decimal representation terminates. If the denominator of the fraction has any prime factors other than 2 or 5, then its decimal representation will repeat.*

F. Generalize your results. In other words, state a hypothesis that will allow you to determine ahead of time whether any fraction (not just unit fractions) will repeat or terminate when expressed as a decimal. Test your hypothesis on at least 10 fractions. Explain why you believe your hypothesis is true.

*As long as a fraction is in reduced form, then what we have found above holds true, whether or not the fraction has 1 for its numerator. We need look only at the denominator. As discussed above, if the denominator has no prime factors other than 2 and/or 5, the decimal representation terminates. If the denominator does have prime factors other than 2 or 5, its decimal representation repeats.*

*Why does this happen? The places to the right of the decimal point are $10^{ths}$, then$100^{ths}$, then $1000^{ths}$, $10000^{ths}$, $100000^{ths}$, $1000000^{ths}$, and so on. The only prime factors that divide into these without remainders are 2's and 5's. A number with factors other than 2 and 5 cannot divide into any of these without a remainder. On the other hand, if the number has only 2's and/or 5's, then we can always find a power of 10 that will work.*

*For example consider $\frac{7}{40}$, which is a fraction in reduced form. By finding an appropriate form of 1 to multiply by, we can change the fraction so it has a denominator that is a power of 10, which in turn can be expressed as a terminating decimal as shown below.*

$$\frac{7}{40} = \frac{7}{2 \times 2 \times 2 \times 5} \times \frac{5 \times 5}{5 \times 5} = \frac{175}{1000} = 0.175$$

*We encourage teachers to have students try this multiple times with many different numbers.*

Name: ______________________________ Block: ______________

Group Members: ______________________________________________

# What’s My Rule?

1. Describe the situation through words and pictures. What is the story? What is the pattern?

2. What do we want to find out?

3. What are your variables?

4. Create a table.

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|---|---|
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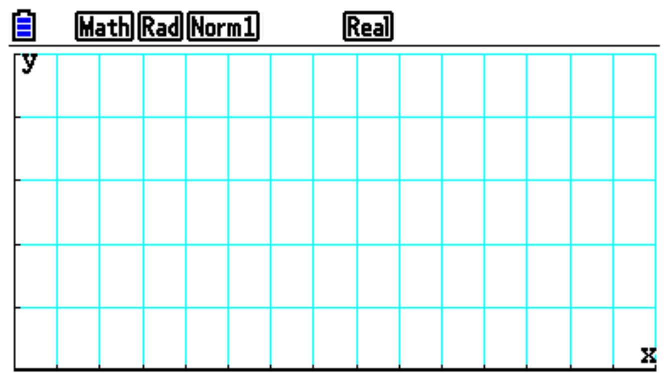
5. Graph your data.




6. Write the equation that represents the relationship of your situation. Describe how you figured this out.

7. What did you find out? Explain.

# Bo D. Bilder Competition

- Bo must build muscle!!!
- Needs to lift 35 pounds.
- Will add 2 pounds to barbell each week.
- How long will this take?
- Use the cubes to model this scenario
  – Table
  – Graph
  – Equation

# Dog Sitting

- Saving for the $49 video game
- Earn $7.50 per week.
- How long will this take?
- Examine this scenario
  - Table
  - Graph
  - Equation

# Twizzlers Munch

- Take approximately the same size bite (NOT ALL IN ONE BITE!!!!)
- Measure what is left.
- How many bites until it's gone?
- Use Twizzlers & rulers to examine this scenario
  - Table
  - Graph
  - Equation

## INVESTIGATION 3.1: POLYGON CAR CONVOYS

*Did you know that the simple geometric shape of a triangle is the key shape in making a geodesic dome? Probably the most famous geodesic dome is Disney World's Epcot Center. The dome at Epcot is 180 feet tall and is made up of more than 11,300 triangles.*

*disneyworld.orlandovacation.com/epcot-spaceship-earth.php*

Let's define a Polygon Car Convoy as a shape formed by congruent, regular polygons that share exactly one side. (Regular polygons are polygons whose sides are congruent and whose angles are congruent.) In this investigation, you will explore the relationship between the number of polygons and the perimeter of the convoys for regular triangles, quadrilaterals, pentagons, and hexagons.

A. Consider equilateral triangles. Why are they considered to be regular polygons? Sketch the first four convoys (the ones with 1, 2, 3, and 4 triangles). Assume the length of each side of the polygon is 1 unit. Create a table for which one column represents the number of equilateral triangles in the convoy and a second column represents the perimeter of the convoy. For the last row of your table, include the perimeter when the number of cars in the convoy is $x$ cars in the convoy. Explain how you obtain the values in the table.

B. What type of quadrilateral would be considered regular? Why? Sketch the first four convoys (the ones with 1, 2, 3, and 4 quadrilaterals). Then create a table similar to the one you created for equilateral triangles. Explain how you found the values in the table.

C. Create columns for the table to show the perimeters of pentagonal and hexagonal car convoys. Explain how you obtain the values in the table.

D. Create several graphs on the same axes based on the formulas you have found. For each of these graphs, $x$ should represent the number of cars in the convoy and $y$ should represent the perimeter. Explore these graphs and discuss the ways in which they are the same and the ways in which they are different.

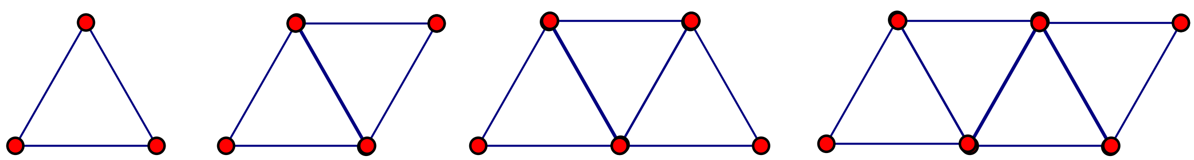
E. Challenge: What would the perimeter be for a regular N-Gon (a polygon with N sides) which has *x* cars in its convoy? Explain.

*SAMPLE SOLUTION: POLYGON CAR CONVOYS*

A. Consider equilateral triangles. Why are they considered to be regular polygons? Sketch the first four convoys (the ones with 1, 2, 3, and 4 triangles). Assume the length of each side of the polygon is 1 unit. Create a table for which one column represents the number of equilateral triangles in the convoy and a second column represents the perimeter of the convoy. For the last row of your table, include the perimeter when the number of cars in the convoy is *x* cars in the convoy. Explain how you obtain the values in the table.

*Equilateral triangles are regular polygons because all of their sides are congruent (note the name "equilateral") and all of the angles are congruent (all are 60 degrees). The convoy might look like the following:*

*CONVOY 1 CONVOY 2 CONVOY 3 CONVOY 4*



*The perimeter is the distance around the figure. Again, we assume the length of each side is 1 unit. Students should be given sufficient time to obtain the values and investigate the relationship.*

| *# TRIANGLES* | *PERIMETER* |
|---|---|
| *1* | *3* |
| *2* | *4* |
| *3* | *5* |
| *4* | *6* |
| *5* | *7* |
| *6* | *8* |
| *7* | *9* |

| | |
|---|---|
| *8* | *10* |
| *x* | *x + 2* |

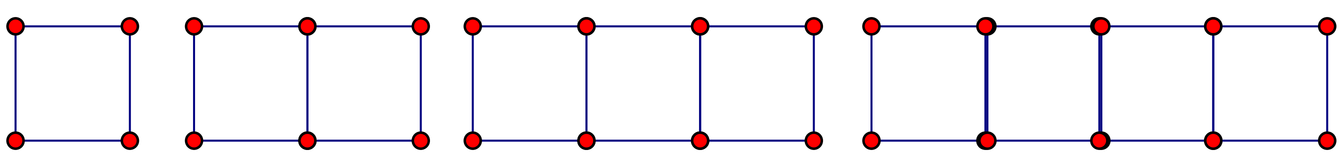
*To obtain the values in the table, we saw that as we added a triangle to the convoy, the perimeter increased by 1 unit. In creating a figure, we found that we could take the previous figure and add two sides to it. However, the perimeter increased by only 1 unit as one of the sides that had previously been on the exterior, and thus contributed to the perimeter, now was part of the interior, and did not contribute to the perimeter. Consequently, while we added two sides to the figure, we also subtracted one from the previous perimeter; the result is that we added only one side to the perimeter of the figure.*

*As for the perimeter, we saw that the perimeter was always 2 more than the number of triangles. Consequently, if there are x triangles, there are 2 more than that, of x + 2, triangles.*

B. What type of quadrilateral would be considered regular? Why? Sketch the first four convoys (the ones with 1, 2, 3, and 4 quadrilaterals). Then create a table similar to the one you created for equilateral triangles. Explain how you found the values in the table.

*A square is a regular quadrilateral. All of its sides are congruent, and all of its angles are congruent (90 degrees). The square convoy might look as follows.*

*CONVOY 1 CONVOY 2 CONVOY 3 CONVOY 4*



| *# SQUARES* | *PERIMETER* |
|---|---|
| *1* | *4* |
| *2* | *6* |

| | |
|---|---|
| *3* | *8* |
| *4* | *10* |
| *5* | *12* |
| *6* | *14* |
| *7* | *16* |
| *8* | *18* |
| *x* | *2x + 2* |

*As we added a car to form the next convoy, we found that we always had to draw three additional sides. However, one of the sides that had been on the perimeter (and thus contributed to the perimeter) became internal; therefore, the perimeter went up by 2 units, not 3, each time we added another square.*

*Teachers should give students plenty of time to try to come up with the formula for the perimeter when there are x cars in the convoy. Our strategy described here is certainly not the only approach to determining this. To obtain the formula for the row for which there were x cars, we thought that because it was growing by 2 each time, that we should have to multiply by 2. Going back to the beginning, when there was 1 square, we noted that if we double 1, we still had to add 2 to get the perimeter. When there were 2 squares, if we doubled 2, we still had to add 2 to get the perimeter. When there were 3 squares, if we doubled 3, once again we had to add 2 to get the perimeter. This was the pattern we noticed and it worked for each value. That helped us create our formula for the perimeter: double the number of squares and add 2. If there are x squares, then the perimeter is 2x + 2.*

C. Create columns for the table to show the perimeters of pentagonal and hexagonal car convoys. Explain how you obtain the values in the table.

*Drawing the convoys may be interesting as they cannot be placed in a straight line. This, however, does not interfere with our ability to determine their perimeters, which are shown in the table.*

| *# POLYGONS* | *PERIMETER: PENTAGONS* | *PERIMETER: HEXAGONS* |
|---|---|---|
| *1* | *5* | *6* |

| | | |
|---|---|---|
| *2* | *8* | *10* |
| *3* | *11* | *14* |
| *4* | *14* | *18* |
| *5* | *17* | *22* |
| *6* | *20* | *26* |
| *7* | *23* | *30* |
| *8* | *26* | *34* |
| *x* | *3x + 2* | *4x + 2* |

*When adding pentagons to its convoy, we have to draw 4 additional sides. One of the previously drawn sides becomes interior, so this adds only 3 units to the perimeter. Because this number increases by 3 each time, we thought that the number of polygons would have to be multiplied by 3; however, when we do this, we find that the value is always 2 less than the actual perimeter. This leads to our formula when there are x cars in the convoy: namely 3x + 2.*

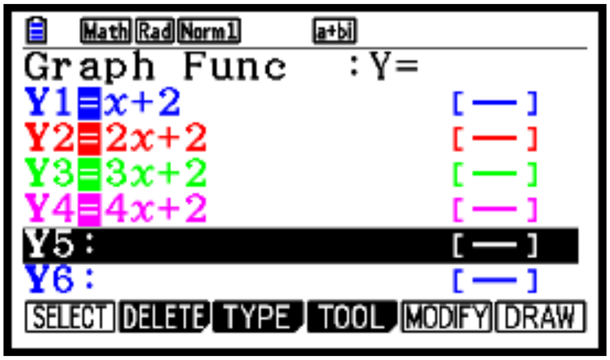
*When adding hexagons to its convoy, we have to draw 5 additional sides, but, once again, one of the previous sides moves from the exterior to the interior. Consequently our perimeter increases by 4 each time, not 5. Because we are always adding 4, we determined that the number of hexagons should be multiplied by 4; once again, we found we had to make an adjustment, adding 2 to the product. This gives us our formula when there are x cars in the convoy: 4x + 2.*

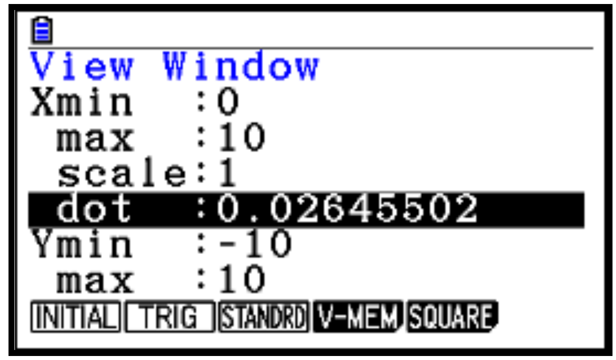
D. Create several graphs on the same axes based on the formulas you have found. For each of these graphs, *x* should represent the number of cars in the convoy and *y* should represent the perimeter. Explore these graphs and discuss the ways in which they are the same and the ways in which they are different.

*From the Main Menu, select 5 (GRAPH). Then,*

- ☞ *Enter the formula for equilateral triangles in Y1, for squares in Y2, for pentagons in Y3, and hexagons in Y4. See below left.*

☞ *Before viewing the graph, we will adjust the viewing window. We will set the X-values, but let the calculator adjust the Y-values automatically. Press SHIFT F3 (V-Window). Set the Xmin at 0, the Xmax at 10, and the scale at 1 as shown below right. Don't worry about what's stored in for the Ymin, Ymax, or Yscale just now. .*

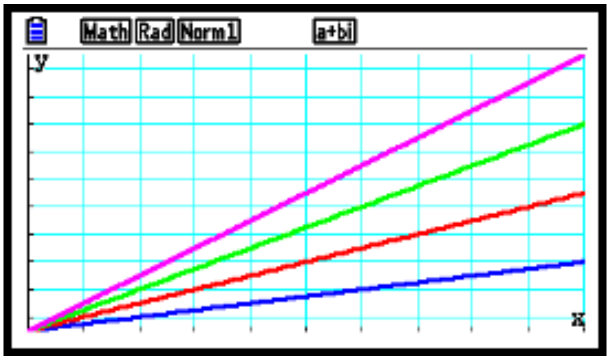


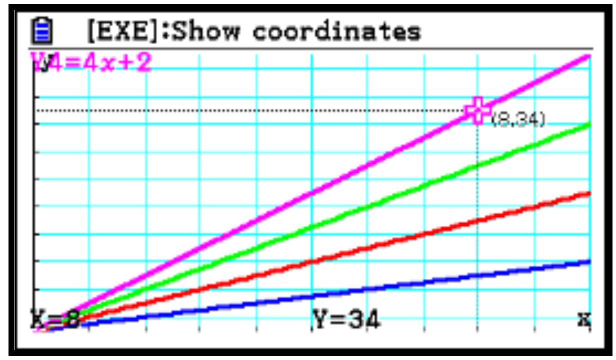




☞ *Press EXIT to back out of the View Window and F6 (DRAW) to create the graphs.*

☞ *Press SHIFT F2 (Zoom) and F5 (AUTO). The calculator automatically finds an appropriate Ymin and Ymax, but you may wish to change the Yscale at this time. To do so, press SHIFT F3 (V-Window), scroll down to the Yscale and set the desired value. We have chosen 4 for ours. Then press EXIT and F6 (DRAW) to recreate the graphs. See below.*

☞ *Press SHIFT F1 (Trace) to trace through the graphs. You can switch from one to another by toggling up or down. You can also move right or left with the roller, but you may prefer simply typing in an x-value in which you are interested. As long as that value is within your window, the calculator will find it for you. For example, below right the calculator shows that for Y4 (the hexagons), when there are 8 cars in the convoy, the perimeter is 34 units.*




*We find that all of the graphs are linear. Why? Whenever there is a constant change in the y-value for a constant change in the x-value, we always obtain a line. Here, for equilateral triangles, for each additional car there is an increase of 1 in the perimeter; for squares, the increase is always 2; for pentagons, the increase is always 3, and for hexagons, the increase is always 4.*

*We also find that all of the lines go through (0, 2). The point is not a realistic point, as it suggests that if there are 0 cars in the convoy, the perimeter would be 2. This does not mean that our formulas are wrong, but only that that a value of 0 doesn't make sense for them, an issue of domain that students will encounter later. The smallest x-value we should consider is 1, when there is 1 car in the convoy (which wouldn't be much of a convoy!). Note that in all of our formulas, we always had to add 2.*

*The main difference in the graphs is their steepness. The polygons with more sides have steeper graphs. This is a good opportunity for teachers to set the stage for students' future exploration into slope. The slope here is simply how much the perimeter changes for every additional polygon. For triangles the slope is 1, for quadrilaterals it's 2 for pentagons it's 3 and for hexagons it's 4. This is the factor by which the number of cars in the convoy is multiplied.*

E. Challenge: What would the perimeter be for a regular N-Gon (a polygon with N sides) which has *x* cars in its convoy? Explain.

*We will build on the idea described above. In the table below, we have shown the factor by which the number of cars in the convoy is multiplied for the various polygons. Our formula for n-polygons is based on the pattern we can observe.*

| *# OF SIDES IN POLYGON* | *INCREASE IN PERIMETER FOR EACH ADDITIONAL CAR* | *PERIMETER FOR x CARS IN CONVOY* |
|---|---|---|
| *3* | *1* | *x + 2* |
| *4* | *2* | *2x + 2* |
| *5* | *3* | *3x + 2* |
| *6* | *4* | *4x + 2* |
| *7* | *5* | *5x + 2* |
| *N* | *N - 2* | *X(N – 2) + 2* |

*The value in the last cell of the table answers our question. We found that we subtract 2 from the number of sides in the polygon and multiply this value by x, which represents the number of cars in the convoy. With all of the polygons, we had to adjust the product by adding 2. Working on this should enhance students' understanding of and facility with working with variables and expressions.*